Finally, Mathematics has an extremely rich history in India spanning thousands of years. India is where the place value number system (including zero), that we all use today to write numbers, was first developed and used and is where many of the key foundations of algebra, geometry, trigonometry, and calculus were laid. By learning about the development of Mathematics in India as well as throughout the world, the rootedness in India can be enhanced, along with a more general appreciation of the history of Mathematics, and of the remarkable evolution and development of mathematical concepts through time (and India's critical roles in these developments).

3.4.1 Curricular Goals and Competencies

3.4.1.1 Preparatory Stage

CG-1 Understands numbers (counting numbers and fractions), represents whole numbers using the Indian place value system, understands and carries out the four basic operations with whole numbers, and discovers and recognises patterns in number sequences	C-1.1	Represents numbers using the place value structure of the Indian number system, compares whole numbers, and knows and can read the names of very large numbers
	C-1.2	Represents and compares commonly used fractions in daily life (such as ½, ¼) as parts of unit wholes, as locations on number lines and as divisions of whole numbers
	C-1.3	Understands and visualises arithmetic operations and the relationships among them, knows addition and multiplication tables at least up to 10 × 10 (*Pahade*) and applies the four basic operations on whole numbers to solve daily life problems
	C-1.4	Recognises, describes, and extends simple number patterns such as odd numbers, even numbers, square numbers, cubes, powers of 2, powers of 10, and Virahanka–Fibonacci numbers.
CG-2 Analyses the characteristics and properties of two - and three-dimensional geometric shapes, specifies locations and describes spatial relationships, and recognises and creates shapes that have symmetry	C-2.1	Identifies, compares, and analyses attributes of two- and three-dimensional shapes and develops vocabulary to describe their attributes or properties
	C-2.2	Describes location and movement using both common language and mathematical vocabulary; understands the notion of map (*Najri Naksha*)
	C-2.3	Recognises and creates symmetry (reflection, rotation) in familiar 2D and 3D shapes
	C-2.4	Discovers, recognises, describes, and extends patterns in 2D and 3D shapes

CG-3 Understands measurable attributes of objects and the units, systems, and processes of such measurement, including those related to distance, length, weight, area, volume, and time using non-standard and standard units	C-3.1	Measures in non-standard and standard units and evaluates the need for standard units
	C-3.2	Uses an appropriate unit and tool for the attribute (like length, perimeter, time, weight, volume) being measured
	C-3.3	Carries out simple unit conversions, such as from centimetres to metres, within a system of measurement
	C-3.4	Understands the definition and formula for the area of a square or rectangle as length times breadth
	C-3.5	Devises strategies for estimating the distance, length, time, perimeter (for regular and irregular shapes), area (for regular and irregular shapes), weight, and volume and verifies the same using standard units
	C-3.6	Deduces that shapes having equal areas can have different perimetres and shapes having equal perimetres can have different areas
	C-3.7	Evaluates the conservation of attributes like length and volume, and solves daily-life problems related to them
CG-4 Develops problem-solving skills with procedural fluency to solve mathematical puzzles as well as daily-life problems, and as a step towards developing computational thinking	C-4.1	Solves puzzles and daily-life problems involving one or more operations on whole numbers (including word puzzles and puzzles from 'recreational' areas, such as the construction of magic squares)
	C-4.2	Learns to systematically count and list all possible permutations or combination given a constraint, in simple situations (e.g., how to make a committee of two people from a group of five people)
	C-4.3	Selects appropriate methods and tools for computing with whole numbers, such as mental computation, estimation, or paper-pencil calculation, in accordance with the context
CG-5 Knows and appreciates the development in India of the decimal place value system that is used around the world today	C-5.1	Understands the development of zero in India and the Indian place value system for writing numerals, the history of its transmission to the world, and its modern impact on our lives and in all technology

3.4.1.2 Middle Stage

CG-1 Understands numbers and sets of numbers (whole numbers, fractions, integers, rational numbers, and real numbers), looks for patterns, and appreciates relationships between numbers	C-1.1	Develops a sense for and an ability to manipulate (e.g., read, write, form, compare, estimate, and apply operations) and name (in words) large whole numbers of up to 20 digits, and expresses them in scientific notation using exponents and powers
	C-1.2	Discovers, identifies, and explores patterns in numbers and describes rules for their formation (e.g., multiples of 7, powers of 3, prime numbers), and explains relations between different patterns
	C-1.3	Learns about the inclusion of zero and negative quantities as numbers, and the arithmetic operations on them, as given by Brahmagupta
	C-1.4	Explores and understands sets of numbers, such as whole numbers, fractions, integers, rational numbers, and real numbers, and their properties, and visualises them on the number line
	C-1.5	Explores the idea of percentage and applies it to solve problems
	C-1.6	Explores and applies fractions (both as ratios and in decimal form) in daily-life situations
CG-2 Understands the concepts of variable, constant, coefficient, expression, and (one-variable) equation, and uses these concepts to solve meaningful daily-life problems with procedural fluency	C-2.1	Understands equality between numerical expressions and learns to check arithmetical equations
	C-2.2	Extends the representation of a number in the form of a variable or an algebraic expression using a variable
	C-2.3	Forms algebraic expressions using variables, coefficients, and constants and manipulates them through basic operations
	C-2.4	Poses and solves linear equations to find the value of an unknown, including to solve puzzles and word problems
	C-2.5	Develops own methods to solve puzzles and problems using algebraic thinking
CG-3 Understands, formulates, and applies properties and theorems regarding simple geometric shapes (2D and 3D)	C-3.1	Describes, classifies, and understands relationships among different types of two- and three-dimensional shapes using their defining properties/attributes
	C-3.2	Outlines the properties of lines, angles, triangles, quadrilaterals, and polygons and applies them to solve related problems
	C-3.3	Identifies attributes of three-dimensional shapes (cubes, parallelepipeds, cylinders, cones), works hands-on with material to construct these shapes, and also uses two-dimensional representations of three-dimensional objects to visualise and solve problems
	C-3.4	Draws and constructs geometric shapes, such as lines, parallel lines, perpendicular lines, angles, and simple triangles, with specified properties using a compass and straightedge
	C-3.5	Understands congruence and similarity as it applies to geometric shapes and identifies similar and congruent triangles

CG-4 Develops understanding of perimeter and area for 2D shapes and uses them to solve day-to-day life problems	C-4.1	Discovers, understands, and uses formulae to determine the area of a square, triangle, parallelogram, and trapezium and develops strategies to find the areas of composite 2D shapes
	C-4.2	Learns the Baudhayana-Pythagoras theorem on the lengths of the sides of a right-angled triangle, and discovers a geometric proof using areas of squares erected on the sides of the triangle, and other related geometric constructions from the *Sulba-Sutras*
	C-4.3	Constructs various designs (using tiling) on a plane surface using different 2D shapes and appreciates their appearances in art in India and around the world
	C-4.4	Develops familiarity with the notion of fractal and identifies and appreciates the appearances of fractals in nature and art in India and around the world
CG-5 Collects, organises, represents (graphically and in tables), and interprets data/ information from daily-life experiences	C-5.1	Collects, organises, and interprets the data using measures of central tendencies such as average/mean, mode, and median
	C-5.2	Selects, creates, and uses appropriate graphical representations (e.g., pictographs, bar graphs, histograms, line graphs, and pie charts) of data to make interpretations
CG-6 Develops mathematical thinking and the ability to communicate mathematical ideas logically and precisely	C-6.1	Applies both inductive and deductive logic to formulate definitions and conjectures, evaluate and produce convincing arguments or proofs to turn these definitions and conjectures into theorems or correct statements, particularly in the areas of algebra, elementary number theory, and geometry
CG-7 Engages with puzzles and mathematical problems and develops own creative methods and strategies to solve them	C-7.1	Demonstrates creativity in discovering one's own solutions to puzzles and other problems, and appreciates the work of others in finding their own, possibly different, solutions
	C-7.2	Engages in and appreciates the artistry and aesthetics of puzzle-making and puzzle-solving
CG-8 Develops basic skills and capacities of computational thinking, namely, decomposition, pattern recognition, data representation, generalisation, abstraction, and algorithms in order to solve problems where such techniques of computational thinking are effective	C-8.1	Approaches problems using programmatic thinking techniques such as iteration, symbolic representation, and logical operations and reformulates problems into series of ordered steps (i.e., algorithmic thinking)
	C-8.2	Learns systematic counting and listing, systematic reasoning about counts and iterative patterns, and multiple data representations; learns to devise and follow algorithms, with an eye towards understanding correctness, effectiveness, and efficiency of algorithms

CG-9 Knows and appreciates the development of mathematical ideas over a period of time and the contributions of past and modern mathematicians from India and across the world	C-9.1	Recognises how concepts (like counting numbers, whole numbers, negative numbers, rational numbers, zero, concepts of algebra, geometry) evolved over a period of time in different civilisations.
	C-9.2	Knows and appreciates the contributions of specific Indian mathematicians, such as, Baudhayana, Pingala, Aryabhata, Brahmagupta, Virahanka, Bhaskara, and Ramanujan.
CG-10 Knows about and appreciates the interaction of Mathematics with each of their other school subjects	C-10.1	Recognises interaction of Mathematics with multiple subjects across Science, Social Science, Visual Arts, Music, Vocational Education, and Sports

3.4.1.3 Secondary Stage

CG-1 Understands numbers (natural, whole, integer, rational, irrational, and real), ways of representing numbers, relationships amongst numbers, and number sets	C-1.1	Develops understanding of numbers, including the set of real numbers and its properties
CG-2 Builds deductive and inductive logic to prove theorems related to numbers and their relationships (such as '$\sqrt{2}$ is an irrational number', recursion relation for *Virahanka* numbers, formula for the sum of the first *n* square numbers)	C-2.1	Extends the understanding of powers (radical powers) and exponents
CG-3 Discovers and proves algebraic identities and models real-life situations in the form of equations to solve them	C-3.1	States and motivates/proves remainder theorem, factor theorem, and division algorithm
	C-3.2	Models and solves contextualised problems using equations (e.g., simultaneous linear equations in two variables or single polynomial equations) and draws conclusions about a situation being modelled
	C-3.3	Learns Brahmagupta's quadratic formula (in both symbolic and poetic form) and its derivation, and uses it to solve some of the poetic puzzles of Bhaskara as well as modern-day problems

CG-4 Analyses characteristics and properties of two-dimensional geometric shapes and develops mathematical arguments to explain geometric relationships	C-4.1	Describes relationships including congruence of two-dimensional geometric shapes (such as lines, angles, triangles) to make and test conjectures and solve problems
	C-4.2	Proves theorems using Euclid's axioms and postulates for triangles and quadrilaterals, and applies them to solve geometric problems
	C-4.3	Proves theorems about the geometry of a circle, including its chords, subtended angles, inscribed polygons, and area in terms of π
	C-4.4	Understands the irrationality of π, the best approximations to π discovered over human history, and the first exact formula (infinite series) for π given by Madhava
	C-4.5	Specifies locations and describes spatial relationships using coordinate geometry, e.g., plotting a pair of linear equations and graphically finding the solution, or finding the area of triangle with given coordinates as vertices
	C-4.6	Understands the definitions of the basic trigonometric functions, their history and motivation (including the introduction of the *sin* and *cos* functions by Aryabhata using chords), and their utility across the sciences
CG-5 Derives and uses formulae to calculate areas of plane figures, and surface areas and volumes of solid objects	C-5.1	Visualises, represents, and calculates the area of a triangle using Heron's formula and its generalisation to cyclic quadrilaterals given by Brahmagupta's formula
	C-5.2	Visualises and uses mathematical thinking to discover formulae to calculate surface areas and volumes of solid objects (cubes, cuboids, spheres, hemispheres, right circular cylinders or cones, and their combinations)
CG-6 Analyses and interprets data using statistical concepts (such as measures of central tendency, standard deviations) and probability	C-6.1	Applies measures of central tendencies such as mean, median, and mode
	C-6.2	Applies concepts from probability to solve problems on the likelihood of everyday events

CG-7 Begins to perceive and appreciate the axiomatic and deductive structure of Mathematics	C-7.1	Proves mathematical statements and carries out geometric constructions using stated assumptions, axioms, postulates, definitions, and mathematics vocabulary
	C-7.2	Visualises and appreciates geometric proofs for algebraic identities and other 'proofs without words'
	C-7.3	Proves theorems using Euclid's axioms and postulates – for angles, triangles, quadrilaterals, circles, area-related theorems for triangles and parallelograms
	C-7.4	Constructs different geometrical shapes like bisectors of line segments, angles and their bisectors, triangles, and other polygons, satisfying given constraints
CG-8 Builds skills such as visualisation, optimisation, representation, and mathematical modelling along with their application in daily life	C-8.1	Models daily-life phenomena and uses representations such as graphs, tables, and equations to draw conclusions
	C-8.2	Uses two-dimensional representations of three-dimensional objects to visualise and solve problems such as those involving surface area and volume
	C-8.3	Employs optimisation strategies to maximise desired quantities (such as area, volume, or other output) under given constraints
CG-9 Develops computational thinking, i.e., deals with complex problems and is able to break them down into a series of simple problems that can then be solved by suitable procedures/ algorithms	C-9.1	Decomposes a problem into sub problems
	C-9.2	Describes and analyses a sequence of instructions being followed
	C-9.3	Analyses similarities and differences among problems to make one solution or procedure work for multiple problems
	C-9.4	Engages in algorithmic problem solving to design such solutions
CG-10 Knows and appreciates important contributions of mathematicians from India and around the world	C-10.1	Recognises the important contributions made by mathematicians (Indian and others) in the field of Mathematics (such as the evolution of numbers, geometry, algebra)
	C-10.2	Recognises modern contributions to Mathematics made in both India and abroad, and understands the next frontiers and next major open questions in the field of Mathematics
CG-11 Explores connections of Mathematics with other subjects	C-11.1	Applies mathematical knowledge and tools to analyse problems or situations in multiple subjects across Science, Social Science, Visual Arts, Music, Vocational Education, and Sports

3.4.2 Rationale for Selection of Concepts

The Learning Standards, the Curricular Goals and Competencies defined here make choices for the concepts that will be taught and learnt in each of the stages. The key principles that underlie these choices are described here: