

UNIT 1

INTEGERS

(A) Main Concepts and Results

- Representation of integers on the number line and their addition and subtraction.
- Properties of integers:
 - Integers are closed under addition, subtraction and multiplication.
 - Addition and multiplication are commutative for integers, i.e., $a + b = b + a$ and $a \times b = b \times a$ for any two integers a and b .
 - Addition and multiplication are associative for integers, i.e., $(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$ for any three integers a , b and c .
 - Zero (0) is an additive identity for integers, i.e., $a + 0 = 0 + a = a$ for any integer a .
 - 1 is multiplicative identity for integers, i.e., $a \times 1 = 1 \times a = a$ for any integer a .
 - Integers show distributive property of multiplication over addition, i.e., $a \times (b + c) = a \times b + a \times c$ for any three integers a , b and c .
- Product of a positive integer and a negative integer is a negative integer, i.e., $a \times (-b) = -ab$, where a and b are positive integers.
- Product of two negative integers is a positive integer, i.e., $(-a) \times (-b) = ab$, where a and b are positive integers.
- Product of even number of negative integers is positive, where as the product of odd number of negative integers is negative, i.e.,

$$\underbrace{(-a) \times (-b) \times \dots \times (-p)}_{\text{even number } 2m \text{ times}} = a \times b \times \dots \times p \text{ and}$$

$$\underbrace{(-a) \times (-b) \times \dots \times (-q)}_{\text{odd number } (2m+1) \text{ times}} = -(a \times b \times \dots \times q), \text{ where } a, b, \dots, p, q \text{ and } m \text{ are positive integers.}$$

- When a positive integer is divided by a negative integer or vice-versa and the quotient obtained is an integer then it is a negative integer, i.e.,

$$a \div (-b) = (-a) \div b = -\frac{a}{b}, \text{ where } a \text{ and } b \text{ are positive integers and } -\frac{a}{b} \text{ is an integer}$$

- When a negative integer is divided by another negative integer to give an integer then it gives a positive integer, i.e., $(-a) \div (-b) = \frac{a}{b}$, where a and b are positive integers and $\frac{a}{b}$ is also an integer.
- For any integer a , $a \div 1 = a$ and $a \div 0$ is not defined.

(B) Solved Examples

In Examples 1 to 3, there are four options, out of which one is correct. Write the correct answer.

Example 1: Madhre is standing in the middle of a bridge which is 20 m above the water level of a river. If a 35 m deep river is flowing under the bridge (see Fig. 1.1), then the vertical distance between the foot of Madhre and bottom level of the river is:

- (a) 55 m (b) 35 m (c) 20 m (d) 15 m

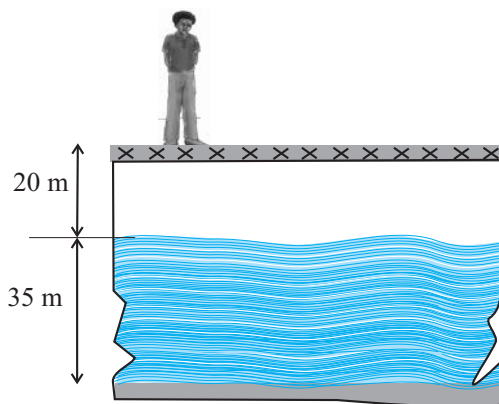


Fig. 1.1

Solution: The correct answer is (a)
[Vertical distance = 20 m + 35 m = 55 m]

Example 2: $[(-10) \times (+9)] + (-10)$ is equal to
(a) 100 (b) -100 (c) -80 (d) 80

Solution: Correct answer is (b)

Example 3: $-16 \div [8 \div (-2)]$ is equal to
(a) -1 (b) 1 (c) 4 (d) -4

Solution: Correct answer is (c),

In Examples 4 and 5, fill in the blanks to make the statements true.

Example 4: $(-25) \times 30 = -30 \times \underline{\hspace{2cm}}$.

Solution: 25

Example 5: $75 \div \underline{\hspace{2cm}} = -75$

Solution: -1

ADDING INTEGERS

If the signs are the same	If the signs are different
Find the sum of the values of integers without sign, and then use the same sign as the integers have.	Find the difference of the values of integers without sign (subtract lower value integer from greater value integer) and then use the sign of the integer with the greater value.

In Examples 6 and 7, state whether the statements are True or False.

Example 6: $(-5) \times (-7)$ is same as $(-7) \times (-5)$

Solution : True

Example 7: $(-80) \div (4)$ is not same as $80 \div (-4)$

Solution: False

Example 8: Find the odd one out* of the four options in the following:

(a) (-2, 24) (b) (-3, 10) (c) (-4, 12) (d) (-6, 8)

Solution: Here $-2 \times 24 = -48$,
 $-4 \times 12 = -48$ and
 $-6 \times 8 = -48$

* To find odd one out, you have to look for a pattern between the numbers and then find out which option is not on that pattern.

All the pairs i.e. $(-2, 24)$; $(-4, 12)$; $(-6, 8)$ give same answer on multiplication, whereas $-3 \times 10 = -30$, gives a different answer. So, odd one is (b).

Example 9: Find the odd one out of the four options given below:

(a) $(-3, -6)$ (b) $(+1, -10)$ (c) $(-2, -7)$ (d) $(-4, -9)$

Solution: Here $-3 + (-6) = -9$,
 $+1 + (-10) = -9$ and
 $-2 + (-7) = -9$

All the above pairs i.e. $(-3, -6)$; $(+1, -10)$; $(-2, -7)$ give same answer on adding, whereas $-4 + (-9) = -13$, gives a different answer. So, odd one out is (d).

Example 10: Match the integer in Column I to an integer in Column II so that the sum is between -11 and -4

Column I

- (a) -6
- (b) $+1$
- (c) $+7$
- (d) -2

Column II

- (i) -11
- (ii) -5
- (iii) $+1$
- (iv) -13

Solution: (a) \leftrightarrow (iii)
 because $-6 + (+1) = -5$, which lies between -11 and -4 .
 (b) \leftrightarrow (i)
 because $+1 + (-11) = -10$ which lies between -11 and -4 .
 (c) \leftrightarrow (iv)
 because $+7 + (-13) = -6$ which lies between -11 and -4 .
 (d) \leftrightarrow (ii)
 because $-2 + (-5) = -7$ which lies between -11 and -4 .

Example 11: If a is an integer other than 1 and -1 , match the following:

Column I

- (a) $a \div (-1)$
- (b) $1 \div (a)$
- (c) $(-a) \div (-a)$
- (d) $a \div (+1)$

Column II

- (i) a
- (ii) 1
- (iii) Not an integer
- (iv) $-a$

Solution: (a) \leftrightarrow (iv) (b) \leftrightarrow (iii) (c) \leftrightarrow (ii) (d) \leftrightarrow (i)

Example 12: Write a pair of integers whose sum is zero (0) but difference is 10.

Solution: Since sum of two integers is zero, one integer is the additive inverse of other integer, like $-3, 3; -4, 4$ etc. But the difference has to be 10. So, the integers are 5 and -5 as $5 - (-5)$ is 10.

SUBTRACTING INTEGERS

Words	Numbers	Formula
To subtract an integer, add its inverse	$3 - 7 = 3 + (-7)$ $5 - (-8) = 5 + 8$	$a - b = a + (-b)$ $a - (-b) = a + b$

Example 13: Write two integers which are smaller than -3 , but their difference is greater than -3 .

Solution: -5 and -4 are smaller than -3 but their difference is $(-4) - (-5) = 1$ which is greater than -3 .

or

-6 and -10 are smaller than -3 but their difference is $(-6) - (-10) = 4$ which is greater than -3 .

Example 14: Write a pair of integers whose product is -15 and whose difference is 8.

Solution: There are few pairs of integers whose product is -15 .

$$\begin{aligned} \text{e.g.} \quad & -1 \times 15 \\ & -3 \times 5 \\ & 3 \times (-5) \\ & 15 \times (-1) \end{aligned}$$

but difference of -3 and 5 or -5 and 3 is 8. So the required pair of integers is $-3, 5$ and $-5, 3$.

Example 15: If Δ is an operation such that for integers a and b we have $a \Delta b = a \times a + b \times b - a \times b$, then find $(-3) \Delta 2$.

Solution : $-3 \Delta 2 = (-3) \times (-3) + 2 \times 2 - (-3) \times 2$
 $= 9 + 4 - (-6) = 13 + 6 = 19$.

Example 16: In an objective type test containing 25 questions. A student is to be awarded $+5$ marks for every correct answer, -5 for every incorrect answer and zero for not writing any answer. Mention the ways of scoring 110 marks by a student.

Solution: Marks scored = $+110$

So, minimum correct responses = $110 \div (+5) = 22$

Case 1

Correct responses = 22

Marks for 1 correct response = + 5

Marks for 22 correct responses = +110 (22 × 5)

Marks scored = +110

Marks obtained for incorrect answer = 0

So, no incorrect response

And, therefore, 3 were unattempted

Case 2

Correct responses = 23

Marks from 23 correct responses = + 115 (23 × 5)

Marks scored = + 110

Marks obtained for incorrect answers = $110 - (+115)$
= -5

Marks for 1 incorrect answer = -5

Number of incorrect responses = $(-5) \div (-5)$
= 1

So, 23 correct, 1 incorrect and 1 unattempted.

Case 3

Correct responses = 24

Marks from 24 correct responses = + 120 (24 × 5)

Marks scored = + 110

Marks obtained for incorrect answers = $+110 - (+120)$
= -10

Number of incorrect responses = $(-10) \div (-5)$
= 2

Thus the number of questions = $24 + 2 = 26$. Whereas, total number of questions is 25. So, this case is not possible.

So, the possible ways are:

- 22 correct, 0 incorrect, 3 unattempted
- 23 correct, 1 incorrect, 1 unattempted.

MULTIPLYING AND DIVIDING TWO INTEGERS

If the signs are the same, the sign of the answer is **positive**.

If the signs are different, the sign of the answer is **negative**.

Application on Problem Solving Strategy



Example 17

A boy standing on the third stair on a staircase goes up by five more stairs. Which stair is he standing at now? At which step will he be after he comes down by 2 stairs?

Solution:



Understand and Explore the Problem

- What do you know?
The current position of the boy.
- What are we trying to find?
The new position of the boy after he takes five more stairs in the same direction.



Plan a Strategy

- Going up one stairs is equal to $+1$ and coming down by one stairs is equal to -1 . Ground level is taken as 0.



Solve

- He is currently at the third stair i.e. at $(+3)$.
- He goes up 5 stairs in the same direction.
- Since $3 + 5 = 8$. Therefore, he is at 8th stair on the staircase.

Now, the boy comes down by 2 stairs. Since he comes down in opposite direction i.e. downwards by 2 stairs (i.e. -2), so $8 + (-2) = 8 - 2 = 6$. He is at 6th step now.



Revise

Since the boy moves 5 stairs up and then 2 stairs down, therefore, as a result he moves $(5 - 2)$ stairs i.e. 3 stairs up. As he was on stair 3 and since he goes up by 3 stairs (i.e. $+3$), so, he is at $(3 + 3)$ the i.e 6th step now.

Think and Discuss

1. Can you find the position of the boy if he comes down further by 3 more stairs?

(C) Exercise

In the Questions 1 to 25, there are four options, out of which only one is correct. Write the correct one.

- When the integers 10, 0, 5, -5, -7 are arranged in descending or ascending order, then find out which of the following integers always remains in the middle of the arrangement.
(a) 0 (b) 5 (c) -7 (d) -5
- By observing the number line (Fig. 1.2), state which of the following statements is not true.

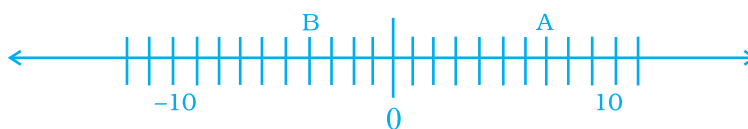


Fig. 1.2

- B is greater than -10 (b) A is greater than 0
 - B is greater than A (d) B is smaller than 0
- By observing the above number line (Fig. 1.2), state which of the following statements is true.
(a) B is 2 (b) A is -4
(c) B is -13 (d) B is -4

Think and Discuss

- Compare** the sums $10 + (-22)$ and $-10 + 22$.
- Describe** how to add the following addition expressions on a number line, $9 + (-13)$ and $-13 + 9$. Then compare the sums.

- Next three consecutive numbers in the pattern 11, 8, 5, 2, --, --, -- are
(a) 0, -3, -6 (b) -1, -5, -8
(c) -2, -5, -8 (d) -1, -4, -7

5. The next number in the pattern $-62, -37, -12$ _____ is

- (a) 25 (b) 13 (c) 0 (d) -13

6. Which of the following statements is not true?

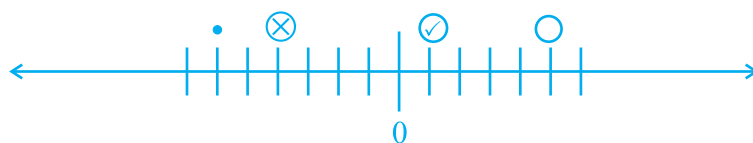
- (a) When two positive integers are added, we always get a positive integer.
 (b) When two negative integers are added we always get a negative integer.
 (c) When a positive integer and a negative integer is added we always get a negative integer.
 (d) Additive inverse of an integer 2 is (-2) and additive inverse of (-2) is 2.

7. On the following number line value 'Zero' is shown by the point



- (a) X (b) Y (c) Z (d) W

8. If \otimes , \circ , \ominus and \bullet represent some integers on number line, then descending order of these numbers is



- (a) $\bullet, \otimes, \ominus, \circ$ (b) $\otimes, \bullet, \ominus, \circ$ (c) $\circ, \ominus, \otimes, \bullet$ (d) $\circ, \bullet, \otimes, \ominus$

9. On the number line, the value of $(-3) \times 3$ lies on right hand side of

- (a) -10 (b) -4 (c) 0 (d) 9

10. The value of $5 \div (-1)$ does not lie between

- (a) 0 and -10 (b) 0 and 10 (c) -4 and -15 (d) -6 and 6

11. Water level in a well was 20m below ground level. During rainy season, rain water collected in different water tanks was drained

into the well and the water level rises 5 m above the previous level. The wall of the well is 1m 20 cm high and a pulley is fixed at a height of 80 cm. Raghu wants to draw water from the well. The minimum length of the rope that he can use is

- (a) 17 m (b) 18 m (c) 96 m (d) 97 m

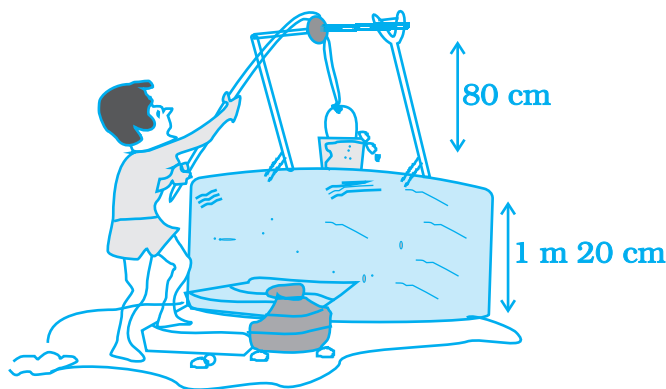


Fig. 1.3

12. $(-11) \times 7$ is not equal to
 (a) $11 \times (-7)$ (b) $-(11 \times 7)$ (c) $(-11) \times (-7)$ (d) $7 \times (-11)$
13. $(-10) \times (-5) + (-7)$ is equal to
 (a) -57 (b) 57 (c) -43 (d) 43
14. Which of the following is not the additive inverse of a ?
 (a) $-(-a)$ (b) $a \times (-1)$ (c) $-a$ (d) $a \div (-1)$

Vocabulary	1. _____ is the _____ of addition.	• Addition
	2. The expression 3×4 and 4×3 are equal by the _____.	• Associative Property
	3. The expressions $1 + (2 + 3)$ and $(1 + 2) + 3$ are equal by the _____.	• Commutative Property
	4. Multiplication and _____ are opposite operations.	• Division
	5. _____ and _____ are commutative.	• Multiplication
		• Opposite Operation
		• Subtraction

15. Which of the following is the multiplicative identity for an integer a ?

- (a) a (b) 1 (c) 0 (d) -1

16. $[(-8) \times (-3)] \times (-4)$ is not equal to

- (a) $(-8) \times [(-3) \times (-4)]$ (b) $[(-8) \times (-4)] \times (-3)$
(c) $[(-3) \times (-8)] \times (-4)$ (d) $(-8) \times (-3) - (-8) \times (-4)$

17. $(-25) \times [6 + 4]$ is not same as

- (a) $(-25) \times 10$ (b) $(-25) \times 6 + (-25) \times 4$
(c) $(-25) \times 6 \times 4$ (d) -250

18. -35×107 is not same as

- (a) $-35 \times (100 + 7)$ (b) $(-35) \times 7 + (-35) \times 100$
(c) $-35 \times 7 + 100$ (d) $(-30 - 5) \times 107$

19. $(-43) \times (-99) + 43$ is equal to

- (a) 4300 (b) -4300 (c) 4257 (d) -4214

20. $(-16) \div 4$ is not same as

- (a) $(-4) \div 16$ (b) $-(16 \div 4)$ (c) $16 \div (-4)$ (d) -4

21. Which of the following does not represent an integer?

- (a) $0 \div (-7)$ (b) $20 \div (-4)$ (c) $(-9) \div 3$ (d) $(-12) \div 5$

22. Which of the following is different from the others?

- (a) $20 + (-25)$ (b) $(-37) - (-32)$
(c) $(-5) \times (-1)$ (d) $(45) \div (-9)$

23. Which of the following shows the maximum rise in temperature?

- (a) 23° to 32° (b) -10° to $+1^\circ$ (c) -18° to -11° (d) -5° to 5°

24. If a and b are two integers, then which of the following may not be an integer?

- (a) $a + b$ (b) $a - b$ (c) $a \times b$ (d) $a \div b$

25. For a non-zero integer a which of the following is not defined?

- (a) $a \div 0$ (b) $0 \div a$ (c) $a \div 1$ (d) $1 \div a$

Encircle the odd one of the following (Questions 26 to 30).

26. (a) $(-3, 3)$ (b) $(-5, 5)$ (c) $(-6, 1)$ (d) $(-8, 8)$

27. (a) $(-1, -2)$ (b) $(-5, +2)$ (c) $(-4, +1)$ (d) $(-9, +7)$

28. (a) $(-9) \times 5 \times 6 \times (-3)$ (b) $9 \times (-5) \times 6 \times (-3)$
 (c) $(-9) \times (-5) \times (-6) \times 3$ (d) $9 \times (-5) \times (-6) \times 3$
29. (a) $(-100) \div 5$ (b) $(-81) \div 9$
 (c) $(-75) \div 5$ (d) $(-32) \div 9$
30. (a) $(-1) \times (-1)$ (b) $(-1) \times (-1) \times (-1)$
 (c) $(-1) \times (-1) \times (-1) \times (-1)$ (d) $(-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1)$

In Questions 31 to 71, fill in the blanks to make the statements true.

31. $(-a) + b = b +$ Additive inverse of _____.
32. _____ $\div (-10) = 0$
33. $(-157) \times (-19) + 157 =$ _____
34. $[(-8) + \text{_____}] + \text{_____} = \text{_____} + [(-3) + \text{_____}] = -3$
35. On the following number line, $(-4) \times 3$ is represented by the point _____.



36. If x , y and z are integers then $(x + \text{_____}) + z = \text{_____} + (y + \text{_____})$
37. $(-43) + \text{_____} = -43$
38. $(-8) + (-8) + (-8) = \text{_____} \times (-8)$
39. $11 \times (-5) = -(\text{_____} \times \text{_____}) = \text{_____}$
40. $(-9) \times 20 = \text{_____}$
41. $(-23) \times (42) = (-42) \times \text{_____}$
42. While multiplying a positive integer and a negative integer, we multiply them as _____ numbers and put a _____ sign before the product.
43. If we multiply _____ number of negative integers, then the resulting integer is positive.
44. If we multiply six negative integers and six positive integers, then the resulting integer is _____.

45. If we multiply five positive integers and one negative integer, then the resulting integer is _____.
46. _____ is the multiplicative identity for integers.
47. We get additive inverse of an integer a when we multiply it by _____.
48. $(-25) \times (-2) =$ _____
49. $(-5) \times (-6) \times (-7) =$ _____
50. $3 \times (-1) \times (-15) =$ _____
51. $[12 \times (-7)] \times 5 =$ _____ $\times [(-7) \times$ _____ $]$
52. $23 \times (-99) =$ _____ $\times (-100 +$ _____ $) = 23 \times$ _____ $+ 23 \times$ _____
53. _____ $\times (-1) = -35$
54. _____ $\times (-1) = 47$

Think and Discuss

- Explain** why $10 - (-10)$ does not equal $-10 - 10$.
- Describe** the answer that you get when you subtract a greater number from a lesser number.

55. $88 \times$ _____ $= -88$
56. _____ $\times (-93) = 93$
57. $(-40) \times$ _____ $= 80$
58. _____ $\times (-23) = -920$
59. When we divide a negative integer by a positive integer, we divide them as whole numbers and put a _____ sign before quotient.
60. When -16 is divided by _____ the quotient is 4.

61. Division is the inverse operation of _____
62. $65 \div (-13) = \underline{\hspace{2cm}}$
63. $(-100) \div (-10) = \underline{\hspace{2cm}}$
64. $(-225) \div 5 = \underline{\hspace{2cm}}$
65. $\underline{\hspace{2cm}} \div (-1) = -83$
66. $\underline{\hspace{2cm}} \div (-1) = 75$
67. $51 \div \underline{\hspace{2cm}} = -51$
68. $113 \div \underline{\hspace{2cm}} = -1$
69. $(-95) \div \underline{\hspace{2cm}} = 95$
70. $(-69) \div (69) = \underline{\hspace{2cm}}$
71. $(-28) \div (-28) = \underline{\hspace{2cm}}$

In Questions 72 to 108, state whether the statements are True or False.

72. $5 - (-8)$ is same as $5 + 8$.
73. $(-9) + (-11)$ is greater than $(-9) - (-11)$.
74. Sum of two negative integers always gives a number smaller than both the integers.
75. Difference of two negative integers cannot be a positive integer.
76. We can write a pair of integers whose sum is not an integer.
77. Integers are closed under subtraction.
78. $(-23) + 47$ is same as $47 + (-23)$.
79. When we change the order of integers, their sum remains the same.
80. When we change the order of integers their difference remains the same.
81. Going 500 m towards east first and then 200 m back is same as going 200 m towards west first and then going 500 m back.
82. $(-5) \times (33) = 5 \times (-33)$
83. $(-19) \times (-11) = 19 \times 11$

84. $(-20) \times (5 - 3) = (-20) \times (-2)$
85. $4 \times (-5) = (-10) \times (-2)$
86. $(-1) \times (-2) \times (-3) = 1 \times 2 \times 3$
87. $-3 \times 3 = -12 - (-3)$
88. Product of two negative integers is a negative integer.
89. Product of three negative integers is a negative integer.
90. Product of a negative integer and a positive integer is a positive integer.
91. When we multiply two integers their product is always greater than both the integers.
92. Integers are closed under multiplication.
93. $(-237) \times 0$ is same as $0 \times (-39)$
94. Multiplication is not commutative for integers.
95. (-1) is not a multiplicative identity of integers.
96. 99×101 can be written as $(100 - 1) \times (100 + 1)$
97. If a, b, c are integers and $b \neq 0$ then, $a \times (b - c) = a \times b - a \times c$
98. $(a + b) \times c = a \times c + a \times b$
99. $a \times b = b \times a$
100. $a \div b = b \div a$
101. $a - b = b - a$

Think and Discuss

- List** all possible multiplication and division statements for the integers with 5, -5, 6, -6 and 30, -30.
For example, $5 \times 6 = 30$.
- Compare** the sign of the product of two negative integers with the sign of the sum of two negative integers.
- Suppose** the product of two integers is positive. What do you know about the signs of the integers?

- 102.** $a \div (-b) = -(a \div b)$
- 103.** $a \div (-1) = -a$
- 104.** Multiplication fact $(-8) \times (-10) = 80$ is same as division fact $80 \div (-8) = (-10)$
- 105.** Integers are closed under division.
- 106.** $[(-32) \div 8] \div 2 = -32 \div [8 \div 2]$
- 107.** The sum of an integer and its additive inverse is zero (0).
- 108.** The successor of $0 \times (-25)$ is $1 \times (-25)$
- 109.** Observe the following patterns and fill in the blanks to make the statements true:
- (a) $-5 \times 4 = -20$
 $-5 \times 3 = -15 = -20 - (-5)$
 $-5 \times 2 = \underline{\hspace{2cm}} = -15 - (-5)$
 $-5 \times 1 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 $-5 \times 0 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
 $-5 \times -1 = 5 = \underline{\hspace{2cm}}$
 $-5 \times -2 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
- (b) $7 \times 4 = 28$
 $7 \times 3 = \underline{\hspace{2cm}} = 28 - 7$
 $7 \times 2 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} - 7$
 $7 \times 1 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} - 7$
 $7 \times 0 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} - \underline{\hspace{2cm}}$
 $7 \times -1 = -7 = \underline{\hspace{2cm}} - \underline{\hspace{2cm}}$
 $7 \times -2 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} - \underline{\hspace{2cm}}$
 $7 \times -3 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} - \underline{\hspace{2cm}}$
- 110. Science Application:** An atom consists of charged particles called electrons and protons. Each proton has a charge of +1 and each electron has a charge of -1. Remember number of electrons is equal to number of protons, while answering these questions:
- (a) What is the charge on an atom?
- (b) What will be the charge on an atom if it loses an electron?
- (c) What will be the charge on an atom if it gains an electron?
- 111.** An atom changes to a charged particle called ion if it loses or gains electrons. The charge on an ion is the charge on electrons plus charge

on protons. Now, write the missing information in the table given below:

Name of Ion	Proton Charge	Electron Charge	Ion Charge
Hydroxide ion	+9	—	-1
Sodium ion	+11	—	+1
Aluminium ion	+13	-10	—
Oxide ion	+8	-10	—



Plan a Strategy

- Some problems contain a lot of information. Read the entire problem carefully to be sure you understand all of the facts. You may need to read it over several times perhaps aloud so that you can hear yourself.
- Then decide which information is the most important (prioritise). Is there any information that is absolutely necessary to solve the problem? This information is the most important.
- Finally, put the information in order (sequence). Use comparison words like *before*, *after*, *longer*, *shorter*, and so on to help you. Write down the sequence before you try to solve the problem.

Read the problem given below and then answer the questions that follow:

- Five friends are standing in a line waiting for the opening of a show. They are in line according to their arrival. Shreya arrived 3 minutes after Sachin. Roy took his place in line at 9:01 P.M. He was 1 minute behind Reena and 7 minutes ahead of Shreya. The first person arrived at 9:00 P.M. Babu showed up 6 minutes after the first person. List the time of each person's arrival.
 - (a) Whose arrival information helped you to determine arrival time of each?
 - (b) Can you determine the order without the time?
 - (c) List the friends' order from the earliest arrival to the last arrival.

112. Social Studies Application: Remembering that 1AD came immediately after 1BC, while solving these problems take 1BC as -1 and 1AD as $+1$.

- The Greeco-Roman era, when Greece and Rome ruled Egypt started in the year 330 BC and ended in the year 395 AD. How long did this era last?
- Bhaskaracharya was born in the year 1114 AD and died in the year 1185 AD. What was his age when he died?
- Turks ruled Egypt in the year 1517 AD and Queen Nefertis ruled Egypt about 2900 years before the Turks ruled. In what year did she rule?
- Greek mathematician Archimedes lived between 287 BC and 212 BC and Aristotle lived between 380 BC and 322 BC. Who lived during an earlier period?

113. The table shows the lowest recorded temperatures for each continent. Write the continents in order from the lowest recorded temperature to the highest recorded temperature.

The Lowest Recorded Temperatures	
Continent	Temperature (in Fahrenheit)
Africa	-11°
Antarctica	-129°
Asia	-90°
Australia	-9°
Europe	-67°
North America	-81°
South America	-27°

114. Write a pair of integers whose product is -12 and there lies seven integers between them (excluding the given integers).

115. From given integers in Column I match an integer of Column II so that their product lies between -19 and -6 :

Column I

-5
6
 -7
8

Column II

1
 -1
3
 -2

116. Write a pair of integers whose product is -36 and whose difference is 15 .

117. Match the following

Column I

- (a) $a \times 1$
- (b) 1
- (c) $(-a) \div (-b)$
- (d) $a \times (-1)$
- (e) $a \times 0$
- (f) $(-a) \div b$
- (g) 0
- (h) $a \div (-a)$
- (i) $-a$

Column II

- (i) Additive inverse of a
- (ii) Additive identity
- (iii) Multiplicative identity
- (iv) $a \div (-b)$
- (v) $a \div b$
- (vi) a
- (vii) $-a$
- (viii) 0
- (ix) -1

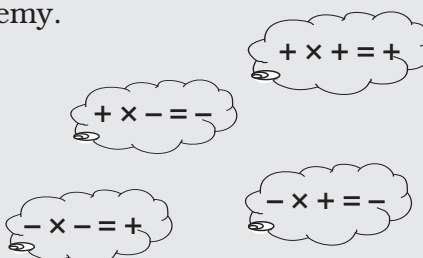
118. You have ₹ 500 in your savings account at the beginning of the month. The record below shows all of your transactions during the month. How much money is in your account after these transactions?

Cheque No.	Date	Transaction Description	Payment	Deposit
384102	4/9	Jal Board	₹ 120	₹ 200
275146	12/9	Deposit		
384103	22/9	LIC India	₹ 240	₹ 150
801351	29/9	Deposit		

Think and Discuss

Is it not true? If $+$ is a friend and $-$ is an enemy.

- Your friend's friend is your friend.
- Your friend's enemy is your enemy.
- Your enemy's friend is your enemy.
- Your enemy's enemy is your friend.



- 119.** (a) Write a positive integer and a negative integer whose sum is a negative integer.
- (b) Write a positive integer and a negative integer whose sum is a positive integer.
- (c) Write a positive integer and a negative integer whose difference is a negative integer.
- (d) Write a positive integer and a negative integer whose difference is a positive integer.
- (e) Write two integers which are smaller than -5 but their difference is -5 .
- (f) Write two integers which are greater than -10 but their sum is smaller than -10 .
- (g) Write two integers which are greater than -4 but their difference is smaller than -4 .
- (h) Write two integers which are smaller than -6 but their difference is greater than -6 .
- (i) Write two negative integers whose difference is 7 .
- (j) Write two integers such that one is smaller than -11 , and other is greater than -11 but their difference is -11 .
- (k) Write two integers whose product is smaller than both the integers.
- (l) Write two integers whose product is greater than both the integers.

120. What's the Error? Ramu evaluated the expression $-7 - (-3)$ and came up with the answer -10 . What did Ramu do wrong?

121. What's the Error? Reeta evaluated $-4 + d$ for $d = -6$ and gave an answer of 2 . What might Reeta have done wrong?

122. The table given below shows the elevations relative to sea level of four locations.

Taking sea level as zero, answer the following questions:

Location	Elevation (in m)
A	-180
B	1600
C	-55
D	3200

- (a) Which location is closest to sea level?
- (b) Which location is farthest from sea level?
- (c) Arrange the locations from the least to the greatest elevation.

123. You are at an elevation 380 m above sea level as you start a motor ride. During the ride, your elevation changes by the following metres: 540 m, -268 m, 116 m, -152 m, 490 m, -844 m, 94 m. What is your elevation relative to the sea level at the end of the ride?

124. Evaluate the following, using distributive property.

- (i) -39×99
- (ii) $(-85) \times 43 + 43 \times (-15)$
- (iii) $53 \times (-9) - (-109) \times 53$
- (iv) $68 \times (-17) + (-68) \times 3$

125. If $*$ is an operation such that for integers a and b we have

$$a * b = a \times b + (a \times a + b \times b)$$

then find (i) $(-3) * (-5)$

(ii) $(-6) * 2$

126. If Δ is an operation such that for integers a and b we have

$$a \Delta b = a \times b - 2 \times a \times b + b \times b \quad (-a) \times b + b \times b$$

then find (i) $4 \Delta (-3)$

(ii) $(-7) \Delta (-1)$

Also show that $4 \Delta (-3) \neq (-3) \Delta 4$

and $(-7) \Delta (-1) \neq (-1) \Delta (-7)$

127. Below u , v , w and x represent different integers, where $u = -4$ and $x \neq 1$. By using following equations, find each of the values:

$$u \times v = u$$

$$x \times w = w$$

$$u + x = w$$

- (a) v
- (b) w
- (c) x

Explain your reasoning using the properties of integers.

128. Height of a place A is 1800 m above sea level. Another place B is 700 m below sea level. What is the difference between the levels of these two places?

129. The given table shows the freezing points in $^{\circ}\text{F}$ of different gases at sea level. Convert each of these into $^{\circ}\text{C}$ to the nearest integral value using the relation and complete the table,

$$C = \frac{5}{9} (F - 32)$$

Gas	Freezing Point at Sea Level (°F)	Freezing Point at Sea Level (°C)
Hydrogen	-435	
Krypton	-251	
Oxygen	-369	
Helium	-458	
Argon	-309	

- 130.** Sana and Fatima participated in an apple race. The race was conducted in 6 parts. In the first part, Sana won by 10 seconds. In the second part she lost by 1 minute, then won by 20 seconds in the third part and lost by 25 seconds in the fourth part, she lost by 37 seconds in the fifth part and won by 12 seconds in the last part. Who won the race finally?
- 131.** A green grocer had a profit of ₹ 47 on Monday, a loss of ₹ 12 on Tuesday and loss of ₹ 8 on Wednesday. Find his net profit or loss in 3 days.
- 132.** In a test, +3 marks are given for every correct answer and -1 mark are given for every incorrect answer. Sona attempted all the questions and scored +20 marks though she got 10 correct answers.
- How many incorrect answers has she attempted?
 - How many questions were given in the test?
- 133.** In a true-false test containing 50 questions, a student is to be awarded 2 marks for every correct answer and -2 for every incorrect answer and 0 for not supplying any answer. If Yash secured 94 marks in a test, what are the possibilities of his marking correct or wrong answer?
- 134.** A multistorey building has 25 floors above the ground level each of height 5m. It also has 3 floors in the basement each of height 5m. A lift in building moves at a rate of 1m/s. If a man starts from 50m above the ground, how long will it take him to reach at 2nd floor of basement?
- 135.** Taking today as zero on the number line, if the day before yesterday is 17 January, what is the date 3 days after tomorrow?

Think and Discuss

- Explain** how integers are used in real life to manage a bank account.
- Explain** whether -1, -4, and 5 are additive inverses.

- 136.** The highest point measured above sea level is the summit of Mt. Everest which is 8,848m above sea level and the lowest point is challenger Deep at the bottom of Mariana Trench which is 10911m below sea level. What is the vertical distance between these two points?

(D) Application

Puzzle 1

Fill in the blank space of the following magic square so that the sum of the numbers in each row, each column and each of the diagonals is -6 .

(i)

-1		
3	-2	
	5	

(ii) In this magic square, sum of the numbers in every row, column and each of the diagonals is -2 . Fill in the blank:

7		-6	
		1	
0	-2		
-5		6	-8

Puzzle 2

If $a * b$ means $a \times b + 2$ and

$a \# b$ means $-a + b - (-3)$,

then find the value of the following:

- | | |
|----------------------|----------------------|
| (i) $-4 * 3$ | (ii) $(-3) * (-2)$ |
| (iii) $(-7) \# (-3)$ | (iv) $2 \# (-4)$ |
| (v) $7 * (-5)$ | (vi) $(-7 * 2) \# 3$ |

Next, match these answers with suitable letters by looking at the table below and arrange them in increasing order of integers to decode the name of the mathematician:

Integers	-9	14	-3	4	-10	8	-33	-21	7	18
Letters	P	Y	C	T	U	I	E	G	L	D

Puzzle 3

'Equinoxes' are the two days of the year when the sun is directly above the earth's equator, due to which the days and nights are of nearly equal length everywhere on the earth.

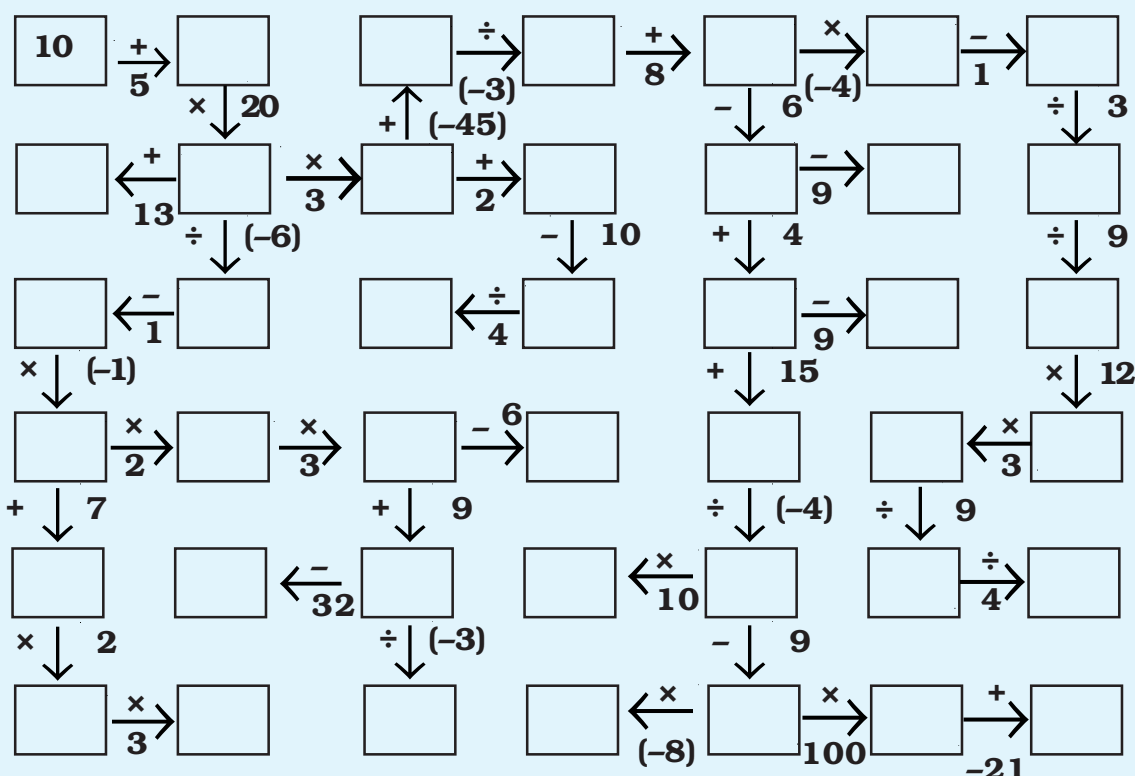
Find the name of the month of autumn equinox using suitable properties of integers by solving the following questions. Match your answer with the letter given in the table and fill it in the box provided in each question.

- (a) $(-1) \times (-2) \times (-3) \times (-4) \times (-5)$
- (b) $18946 \times 99 - (-18946)$
- (c) $-1 + (-2) + (-3) + (-9) + (-8)$
- (d) $15 \times (-99)$
- (e) $-143 + 600 - 257 + 400$
- (f) $0 \div (-12)$
- (g) $-125 \times 9 - 125$
- (h) $\underbrace{(-1) \times (-1) \times \dots \times (-1)}_{20 \text{ times}}$
- (i) $\underbrace{-4 + 4 - 4 + 4 - \dots - 4}_{21 \text{ times}}$

1	E
-1485	T
-120	S
-30	P
-4	R
-1250	B
1894600	E
600	E
0	M

Puzzle 4

Complete the number grids by following the direction of arrows.


Puzzle 5

Solve the following riddles.

- (a) Minus of minus six

Minus minus-minus-seven

What do you get if this is **added** to

minus-minus-seven again?

- (b) Now **add** the value in riddle (a) to minus four and then minus two you **take away**

Divide this by minus two

What is this value can you say?

- (c) Take the result of riddle (b) and **subtract** from it minus six

Multiply this by minus two

What will the answer be?

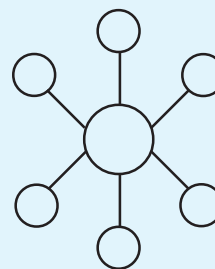


Fig. 1.4

Puzzle 6

Use the integers -2, 4, -5, -12, 20, -25 and 50 just one each in the wheel shown in Fig. 1.4 to make the product 1200 along each line.

UNIT 2

FRACTIONS AND DECIMALS

(A) Main Concepts and Results

- A fraction is either a proper fraction or an improper fraction.
- A proper fraction is a number representing a part of a whole. This whole may be a single object or a group of objects. An improper fraction is a number in which numerator is greater than denominator.
- A mixed fraction is a combination of a natural number and a proper fraction.
- Two fractions are multiplied by multiplying their numerators and denominators separately and writing the product as

$$\frac{\text{product of numerators}}{\text{product of denominators}}. \text{ For example, } \frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4} = \frac{6}{20}.$$

- A fraction acts as an operator 'of'. For example, $\frac{1}{3}$ of 3 is $\frac{1}{3} \times 3 = 1$.
- The product of two proper fractions is less than each of the fractions, For example, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ and $\frac{1}{6}$ is less than both $\frac{1}{2}$ and $\frac{1}{3}$.
- The product of a proper and an improper fraction is less than the improper fraction and greater than the proper fraction. For example, $\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$ and $\frac{3}{4}$ is less than $\frac{3}{2}$ but greater than $\frac{1}{2}$.
- The product of two improper fractions is greater than the two fractions. For example, $\frac{3}{2} \times \frac{7}{4} = \frac{21}{8}$ and $\frac{21}{8}$ is greater than both $\frac{3}{2}$ and $\frac{7}{4}$.

- The reciprocal of a non-zero fraction is obtained by interchanging its numerator and denominator. For example, reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$.
- While dividing a whole number by a fraction, we multiply the whole number with the reciprocal of that fraction. For example, $3 \div \frac{1}{2} = 3 \times \frac{2}{1}$.
- While dividing a fraction by a natural number, we multiply the fraction by the reciprocal of the natural number. For example, $\frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2}$.
- While dividing one fraction by another fraction, we multiply the first fraction by the reciprocal of the other. For example, $\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \times \frac{3}{1}$.
- While multiplying two decimal numbers, first multiply them as whole numbers. Count the number of digits to the right of the decimal point in both the decimal numbers. Add the number of digits counted. Put the decimal point in the product by counting the number of digits equal to sum obtained from its rightmost place. For example, $1.2 \times 1.24 = 1.488$.
- To multiply a decimal number by 10, 100 or 1000, we move the decimal point in the number to the right by as many places as many zeros (0) are the right of one. For example, $1.33 \times 10 = 13.3$.
- To divide a decimal number by a natural number, we first take the decimal number as natural number and divide by the given natural number. Then place the decimal point in the quotient as in the decimal number. For example, $\frac{1.2}{4} = 0.3$
- To divide a decimal number by 10, 100 or 1000, shift the decimal point in the decimal number to the left by as many places as there are zeros over 1, to get the quotient. For example, $\frac{1.34}{100} = 0.0134$
- While dividing one decimal number by another, first shift the decimal points to the right by equal number of places in both, to convert the divisor to a natural number and then divide. For example $\frac{1.44}{1.2} = \frac{14.4}{12} = 1.2$.

(B) Solved Examples

In Examples 1 to 11, there are four options, out of which one is correct. Write the correct one.

Example 1: Savita is dividing $1\frac{3}{4}$ kg of sweets equally among her seven friends. How much does each friend receive?

- (a) $\frac{3}{4}$ kg (b) $\frac{1}{4}$ kg (c) $\frac{1}{2}$ kg (d) $\frac{3}{28}$ kg

Solution: Correct answer is (b)

Example 2: If $\frac{3}{4}$ of a number is 12, the number is

- (a) 9 (b) 16 (c) 18 (d) 32

Solution: Correct answer is (b)

Example 3: Product of fractions $\frac{2}{7}$ and $\frac{5}{9}$ is

- (a) $\frac{2 \times 5}{7 + 9}$ (b) $\frac{2 + 5}{2 + 9}$ (c) $\frac{2 \times 9}{5 \times 7}$ (d) $\frac{2 \times 5}{7 \times 9}$

Solution: Correct answer is (d)

Example 4: Given that $0 < p < q < r < s$ and p, q, r, s are integers, which of the following is the smallest?

- (a) $\frac{p+q}{r+s}$ (b) $\frac{p+s}{q+r}$ (c) $\frac{q+s}{p+r}$ (d) $\frac{r+s}{p+q}$

Solution: Correct answer is (a)

Example 5: The next number of the pattern

60, 30, 15, _____ is

- (a) 10 (b) 5 (c) $\frac{15}{4}$ (d) $\frac{15}{2}$

Solution: Correct answer is (d)

Example 6: The decimal expression for 8 rupees 8 paise (in Rupees) is

- (a) 8.8 (b) 8.08 (c) 8.008 (d) 88.0

Solution: Correct answer is (b)

Example 7: Each side of a regular hexagon is 3.5cm long. The perimeter of the given polygon is

- (a) 17.5cm (b) 21cm (c) 18.3cm (d) 20cm

Solution: Correct answer is (b)

Example 8: $2.5 \div 1000$ is equal to

- (a) 0.025 (b) 0.0025 (c) 0.2500 (d) 25000

Solution : Correct answer is (b)

Example 9: Which of the following has the smallest value?

- (a) 0.0002 (b) $\frac{2}{1000}$ (c) $\frac{(0.2)^2}{2}$ (d) $\frac{2}{100} \div 0.01$

Solution: Correct answer is (a)

Example 10: Which of the following has the largest value?

- (a) $\frac{32}{0.05}$ (b) $\frac{0.320}{50}$ (c) $\frac{3.2}{0.05}$ (d) $\frac{3.2}{50}$

Solution: Correct answer is (a)

Example 11: The largest of the following is

- (a) 0.0001 (b) $\frac{1}{1000}$ (c) $(0.100)^2$ (d) $\frac{1}{10} \div 0.1$

Solution: Correct answer is (d)

In Examples 12 to 19, fill in the blanks to make the statement true.

Example 12: A fraction acts as an operator_____

Solution: of

Example 13: Fraction which is reciprocal of $\frac{2}{3}$ is _____.

Solution: $\frac{3}{2}$

Example 14: Product of a proper and improper fraction is _____ the improper fraction.

Solution: less than.

Example 15: The two non-zero fractions whose product is 1, are called the _____ of each other.

Solution: Reciprocal

Example 16: 5 rupees 5 paise = ₹ _____.

Solution: 5.05

Example 17: 45mm = _____ m.

Solution: 0.045

Example 18: $2.4 \times 1000 =$ _____.

Solution: 2400

Example 19: To divide a decimal number by 100, we shift the decimal point in the number to the _____ by _____ places.

Solution: left, two

In Examples 20 to 23 state whether the statements are True or False.

Example 20: Reciprocal of an improper fraction is an improper fraction.

Solution: False

Example 21: $2\frac{2}{5} \div 2\frac{1}{5} = 2$

Solution: False $\left[\text{because } 2\frac{2}{5} \div 2\frac{1}{5} = \frac{12}{5} \times \frac{5}{11} = \frac{12}{11} \right]$

Example 22: $0.04 \div 0.2 = 0.2$

Solution: True

Example 23: $0.2 \times 0.3 = 0.6$

Solution: False [as $0.2 \times 0.3 = 0.06$]

Example 24: Find $\frac{2}{3}$ of 6 using circles with shaded parts.

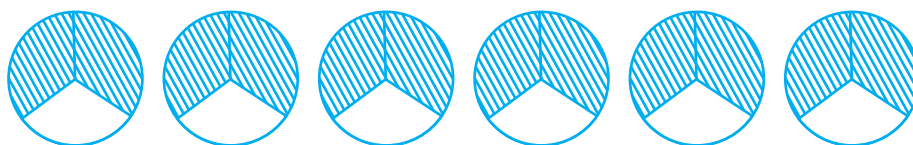


Fig. 2.1

Solution: From the following figure, try to find out $\frac{2}{3}$ of 6.

There are 12 shaded parts out of 18 parts which can be taken as shown

below (Fig. 2.2), which means 4 wholes. Thus $\frac{2}{3}$ of 6 is 4.

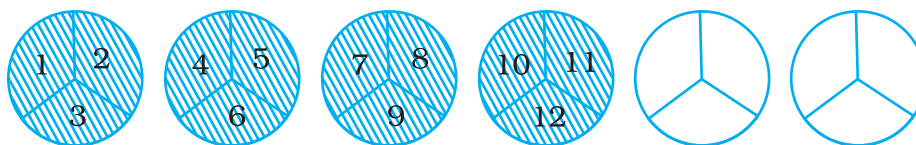


Fig. 2.2

Example 25: Find the value of

$$\frac{1}{4\frac{2}{7}} + \frac{1}{3\frac{11}{13}} + \frac{1}{\left(\frac{5}{9}\right)}$$

Solution: Given expression =

$$\begin{aligned} & \frac{1}{\left(\frac{30}{7}\right)} + \frac{1}{\left(\frac{50}{13}\right)} + \frac{1}{\left(\frac{5}{9}\right)} \\ &= \frac{7}{30} + \frac{13}{50} + \frac{9}{5} \end{aligned}$$

$$= \frac{35}{150} + \frac{39}{150} + \frac{270}{150} = \frac{35 + 39 + 270}{150} = \frac{172}{75}$$

Example 26: There is a $3 \times 3 \times 3$ cube which consists of twenty seven $1 \times 1 \times 1$ cubes (see Fig. 2.3). It is 'tunneled' by removing cubes from the coloured squares.

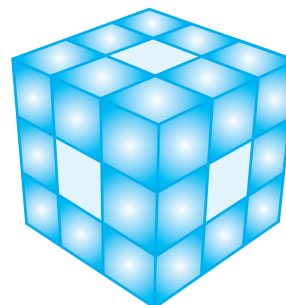


Fig. 2.3

Find:

- Fraction of number of small cubes removed to the number of small cubes left in given cube.
- Fraction of the number of small cubes removed to the total number of small cubes.
- What part is (ii) of (i)?

Solution: (i) Number of small cubes removed = $1 + 1 + 1 + 1 + 1 + 1 + 1 = 7$

So, required fraction = $\frac{7}{20}$

(ii) Required fraction = $\frac{7}{27}$

(iii) Required part is $\frac{7}{27} \div \frac{7}{20} = \frac{7}{27} \times \frac{20}{7} = \frac{20}{27}$

Example 27: Ramu finishes $\frac{1}{3}$ part of a work in 1 hour. How much part of the work will be finished in $2\frac{1}{5}$ hours?

Solution: The part of the work finished by Ramu in 1 hour = $\frac{1}{3}$

So, the part of the work finished by Ramu in $2\frac{1}{5}$ hours

$$= 2\frac{1}{5} \times \frac{1}{3} = \frac{11}{5} \times \frac{1}{3} = \frac{11 \times 1}{5 \times 3} = \frac{11}{15}$$

Ramu will finish $\frac{11}{15}$ part of the work in $2\frac{1}{5}$ hours.

Example 28: How many $\frac{2}{3}$ kg pieces can be cut from a cake of weight 4 kg?

Solution: Observe the following figure representing 4 cakes each of 1 kg and try to give the answer.

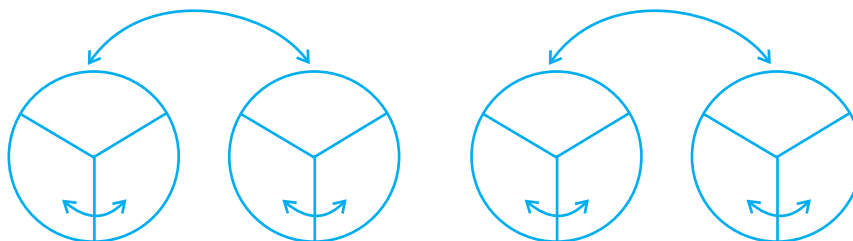


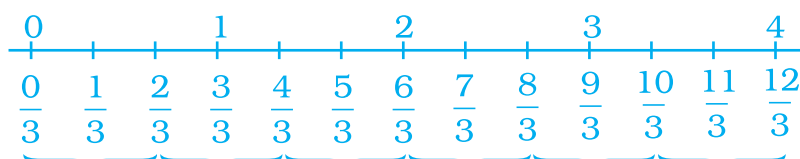
Fig. 2.4

In the above figure we look for ‘how many $\frac{2}{3}$ s are there in these 4 cakes?’

$$\text{That is, } 4 \div \frac{2}{3} = 4 \times \frac{3}{2} = 6$$

Alternate Method

This can be observed also in the following way.



We get the answer as 6.

Example 29: Harmeet purchased 3.5kg of potatoes at the rate of ₹13.75 per kg. How much money should she pay in nearest rupees?

Solution: Cost of 1 kg of potatoes = ₹ 13.75.

$$\text{Cost of 3.5 kg of potatoes} = ₹ 13.75 \times 3.5$$

$$\begin{array}{r} 13.75 \\ \times 3.5 \\ \hline 6875 \\ 4125 \times \\ \hline 48.125 \end{array}$$

So, cost of 3.5 kg of potatoes = ₹ 48, to the nearest rupees.

Example 30: Kavita had a piece of rope of length 9.5 m. She needed some small pieces of rope of length 1.9 m each. How many pieces of the required length will she get out of this rope?

Solution : The length of the rope = 9.5m
The length of a small piece of rope = 1.9m
Number of small pieces = $9.5 \text{ m} \div 1.9\text{m}$

$$= \frac{9.5}{1.9} = \frac{9.5 \times 10}{1.9 \times 10}$$

$$= \frac{95}{19} = 5$$

So, she will get 5 small pieces of rope.

Example 31: Three boys earned a total of ₹ 235.50. What was the average amount earned per boy?

Solution : Three boys earned = ₹ 235.50

The average amount earned per boy = ₹ $\frac{235.50}{3}$

$$\begin{array}{r} 78.50 \\ 3 \overline{) 235.50} \\ \underline{21} \\ 25 \\ \underline{24} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

The average amount earned per boy is ₹ 78.50.

Example 32: Find the product of

(i) $\frac{1}{2}$ and $\frac{5}{8}$ (ii) $\frac{1}{3}$ and $\frac{7}{5}$ (iii) $\frac{4}{3}$ and $\frac{5}{2}$

Solution : (i) $\frac{1}{2} \times \frac{5}{8} = \frac{1 \times 5}{2 \times 8} = \frac{5}{16}$

(ii) $\frac{1}{3} \times \frac{7}{5} = \frac{1 \times 7}{3 \times 5} = \frac{7}{15}$

$$(iii) \quad \frac{4}{3} \times \frac{5}{2} = \frac{4 \times 5}{3 \times 2} = \frac{20}{6} = \frac{10}{3}$$

Example 33: Observe the 3 products given in Example 32 and now give the answers of the following questions.

(i) Does interchanging the fractions in the example,

$$\frac{1}{2} \times \frac{5}{8}, \text{ affect the answer?}$$

(ii) Is the value of the fraction in the product greater or less than the value of either fraction?

Solution : (i) By interchanging $\frac{1}{2} \times \frac{5}{8}$ we get $\frac{5}{8} \times \frac{1}{2}$

$$\frac{5}{8} \times \frac{1}{2} = \frac{5 \times 1}{8 \times 2} = \frac{5}{16} \text{ which is same as the product we get}$$

in Example 32 by multiplying $\frac{1}{2}$ and $\frac{5}{8}$. This means that interchanging the fractions does not affect the answer.

(ii) By observing the 3 products given in the solution of Example 32, we come to know that the value of the fractions in the products are as follows

(a) The product of two fractions whose value is less than 1 i.e. the proper fractions is less than each of the fractions that are multiplied.

(b) The product of a proper and an improper fraction is less than the improper fractions and greater than the proper fraction.

(c) The product of two improper fractions is greater than each of the two fractions.

Example 34 : Reshma uses $\frac{3}{4}$ m of cloth to stitch a shirt. How many

shirts can she make with $2\frac{1}{4}$ m cloth?

Solution : Study the following figures :

Let \square represent $\frac{1}{4}$ m

Then,

$$\frac{\overbrace{\text{□ □ □ □}}^{1 \text{ m}} \quad \overbrace{\text{□ □ □ □}}^{1 \text{ m}} \quad \overbrace{\text{□}}^{\frac{1}{4} \text{ m}}}{\text{□ □ □}} = \frac{9 \text{ fourths}}{3 \text{ fourths}} = 3$$

In fact, we calculate that “how many $\frac{3}{4}$ are in $2\frac{1}{4}$?”

And it is calculated as,

$$2\frac{1}{4} \div \frac{3}{4} = \frac{9}{4} \div \frac{3}{4} = \frac{9}{4} \times \frac{4}{3} = \frac{9 \times 4}{4 \times 3} = \frac{9}{3} = 3$$

Thus, 3 shirts can be made with $2\frac{1}{4}$ m of cloth.

MATHEMATICS IN MUSIC

How Does a Music Composer Use Maths?

Music composers write notes that can vary in length. There are whole notes, half notes, quarter notes, eighth notes and sixteen notes.

Example 35 : If the fraction of the frequencies of two notes have a common factor between the numerator and denominator, the two notes are harmonious. Use the graphic below to find the fraction of frequency of notes D and B.



Frequency Chart

Fig. 2.5

Solution: Fraction of frequencies of notes D and B is

$$= \frac{\text{Frequency of note D}}{\text{Frequency of note B}} = \frac{297}{495} = \frac{3 \times 3 \times 3 \times 11}{3 \times 3 \times 5 \times 11}$$

So, the fraction of the frequencies of notes D and B is $\frac{3}{5}$.

Clearly, the notes D and B are harmonies. Find other pairs of notes which are harmonious.

Application on Problem Solving Strategy



Example 36

Khilona said that we have gone about 120km or $\frac{2}{3}$ of the way to the camp site. So, how much farther do we have to go?

Solution :



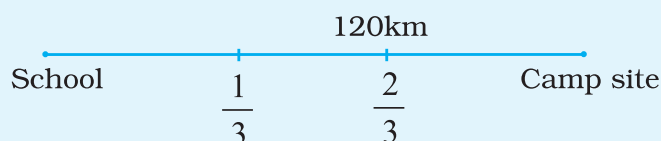
Understand and Explore the Problem

- What do you know?
- We know that 120km is about $\frac{2}{3}$ of the total distance.



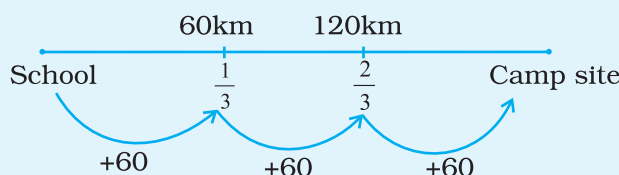
Plan a Strategy

- Draw a diagram showing the distance that Khilona has already gone and the fractional part that it represents.



Solve

- If $\frac{2}{3}$ of the distance is 120km, then $\frac{1}{3}$ of the distance would be $\frac{1}{2}$ of 120km i.e. 60km.



The total distance is $(120 + 60)$ km or 180km.



Revise :

Since $\frac{2}{3}$ of the total distance, denoted by x , equals 120km, so the equation $\frac{2x}{3} = 120$ represents this problem. By solving we get $x = 180$ km. Thus the solution is checked.

Think and Discuss

1. If $\frac{1}{3}$ of the total distance is 120 km, then how far is the camp site?
2. Apply both strategies i.e. by drawing and by using equation, to solve other problems and discuss with your friends that which method is easy.

(C) EXERCISE

In questions 1 to 20, out of four options, only one is correct. Write the correct answer.

1. $\frac{2}{5} \times 5\frac{1}{5}$ is equal to:

- (a) $\frac{26}{25}$ (b) $\frac{52}{25}$ (c) $\frac{2}{5}$ (d) 6

2. $3\frac{3}{4} \div \frac{3}{4}$ is equal to:

- (a) 3 (b) 4 (c) 5 (d) $\frac{45}{16}$

3. A ribbon of length $5\frac{1}{4}$ m is cut into small pieces each of length $\frac{3}{4}$ m.
Number of pieces will be:

- (a) 5 (b) 6 (c) 7 (d) 8

4. The ascending arrangement of $\frac{2}{3}, \frac{6}{7}, \frac{13}{21}$ is:

- (a) $\frac{6}{7}, \frac{2}{3}, \frac{13}{21}$ (b) $\frac{13}{21}, \frac{2}{3}, \frac{6}{7}$ (c) $\frac{6}{7}, \frac{13}{21}, \frac{2}{3}$ (d) $\frac{2}{3}, \frac{6}{7}, \frac{13}{21}$

5. Reciprocal of the fraction $\frac{2}{3}$ is:

- (a) 2 (b) 3 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

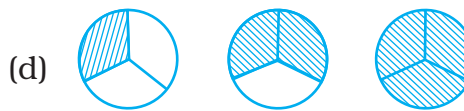
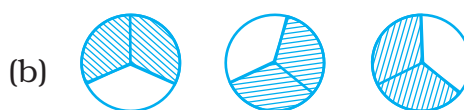
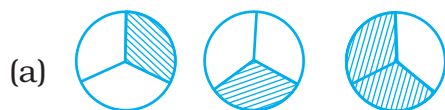
6. The product of $\frac{11}{13}$ and 4 is:

- (a) $3\frac{5}{13}$ (b) $5\frac{3}{13}$ (c) $13\frac{3}{5}$ (d) $13\frac{5}{3}$

7. The product of 3 and $4\frac{2}{5}$ is:

- (a) $17\frac{2}{5}$ (b) $\frac{24}{5}$ (c) $13\frac{1}{5}$ (d) $5\frac{1}{13}$

8. Pictorial representation of $3 \times \frac{2}{3}$ is:



9. $\frac{1}{5} \div \frac{4}{5}$ equal to:

- (a) $\frac{4}{5}$ (b) $\frac{1}{5}$ (c) $\frac{5}{4}$ (d) $\frac{1}{4}$

10. The product of 0.03×0.9 is:

- (a) 2.7 (b) 0.27 (c) 0.027 (d) 0.0027

11. $\frac{5}{7} \div 6$ is equal to:

- (a) $\frac{30}{7}$ (b) $\frac{5}{42}$ (c) $\frac{30}{42}$ (d) $\frac{6}{7}$

12. $5\frac{1}{6} \div \frac{9}{2}$ is equal to

- (a) $\frac{31}{6}$ (b) $\frac{1}{27}$ (c) $5\frac{1}{27}$ (d) $\frac{31}{27}$

13. Which of the following represents $\frac{1}{3}$ of $\frac{1}{6}$?

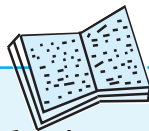
- (a) $\frac{1}{3} + \frac{1}{6}$ (b) $\frac{1}{3} - \frac{1}{6}$ (c) $\frac{1}{3} \times \frac{1}{6}$ (d) $\frac{1}{3} \div \frac{1}{6}$

14. $\frac{3}{7}$ of $\frac{2}{5}$ is equal to

- (a) $\frac{5}{12}$ (b) $\frac{5}{35}$ (c) $\frac{1}{35}$ (d) $\frac{6}{35}$

15. One packet of biscuits requires $2\frac{1}{2}$ cups of flour and $1\frac{2}{3}$ cups of sugar. Estimated total quantity of both ingredients used in 10 such packets of biscuits will be

- (a) less than 30 cups
(b) between 30 cups and 40 cups
(c) between 40 cups and 50 cups
(d) above 50 cups



Vocabulary

1. A number that consists of a whole number and a fraction is called a/an _____?
2. An _____ is a number that represents a part of a whole.
3. A fraction whose numerical (absolute) value is greater than 1 is called a/an _____, and a fraction whose numerical value is between 0 and 1 is called a/an _____.
4. _____ mean the same value.

- Equivalent Fraction
- Fraction
- Improper Fraction
- Mixed Number
- Proper Fraction

16. The product of 7 and $6\frac{3}{4}$ is

- (a) $42\frac{1}{4}$ (b) $47\frac{1}{4}$ (c) $42\frac{3}{4}$ (d) $47\frac{3}{4}$

17. On dividing 7 by $\frac{2}{5}$, the result is

- (a) $\frac{14}{2}$ (b) $\frac{35}{4}$ (c) $\frac{14}{5}$ (d) $\frac{35}{2}$

18. $2\frac{2}{3} \div 5$ is equal to

- (a) $\frac{8}{15}$ (b) $\frac{40}{3}$ (c) $\frac{40}{5}$ (d) $\frac{8}{3}$

19. $\frac{4}{5}$ of 5 kg apples were used on Monday. The next day $\frac{1}{3}$ of what was left was used. Weight (in kg) of apples left now is

- (a) $\frac{2}{7}$ (b) $\frac{1}{14}$ (c) $\frac{2}{3}$ (d) $\frac{4}{21}$

20. The picture



interprets

- (a) $\frac{1}{4} \div 3$ (b) $3 \times \frac{1}{4}$ (c) $\frac{3}{4} \times 3$ (d) $3 \div \frac{1}{4}$

In Questions 21 to 44, fill in the blanks to make the statements true.

21. Rani ate $\frac{2}{7}$ part of a cake while her brother Ravi ate $\frac{4}{5}$ of the remaining. Part of the cake left is _____

22. The reciprocal of $\frac{3}{7}$ is _____

23. $\frac{2}{3}$ of 27 is _____

24. $\frac{4}{5}$ of 45 is _____
25. $4 \times 6\frac{1}{3}$ is equal to _____
26. $\frac{1}{2}$ of $4\frac{2}{7}$ is _____
27. $\frac{1}{9}$ of $\frac{6}{5}$ is _____

Think and Discuss

- Explain** whether you need to find a common denominator to compare $\frac{2}{3}$ and $-\frac{1}{2}$.
- Describe** the steps you would use to compare 0.235 and 0.239.

28. The lowest form of the product $2\frac{3}{7} \times \frac{7}{9}$ is _____
29. $\frac{4}{5} \div 4$ is equal to _____
30. $\frac{2}{5}$ of 25 is _____
31. $\frac{1}{5} \div \frac{5}{6} = \frac{1}{5} \times \frac{6}{5}$
32. $3.2 \times 10 =$ _____
33. $25.4 \times 1000 =$ _____
34. $93.5 \times 100 =$ _____
35. $4.7 \div 10 =$ _____
36. $4.7 \div 100 =$ _____
37. $4.7 \div 1000 =$ _____
38. The product of two proper fractions is _____ than each of the fractions that are multiplied.
39. While dividing a fraction by another fraction, we _____ the first fraction by the _____ of the other fraction.
40. $8.4 \div \text{_____} = 2.1$

41. $52.7 \div \underline{\hspace{2cm}} = 0.527$

42. $0.5 \underline{\hspace{1cm}} 0.7 = 0.35$

43. $2 \underline{\hspace{1cm}} \frac{5}{3} = \frac{10}{3}$

44. $2.001 \div 0.003 = \underline{\hspace{2cm}}$

In each of the Questions 45 to 54, state whether the statement is True or False.

45. The reciprocal of a proper fraction is a proper fraction.

46. The reciprocal of an improper fraction is an improper fraction.

47. Product of two fractions = $\frac{\text{Product of their denominators}}{\text{Product of their numerators}}$

48. The product of two improper fractions is less than both the fractions.

49. A reciprocal of a fraction is obtained by inverting it upside down.

50. To multiply a decimal number by 1000, we move the decimal point in the number to the right by three places.

51. To divide a decimal number by 100, we move the decimal point in the number to the left by two places.

52. 1 is the only number which is its own reciprocal.

53. $\frac{2}{3}$ of 8 is same as $\frac{2}{3} \div 8$.

54. The reciprocal of $\frac{4}{7}$ is $\frac{4}{7}$.

55. If 5 is added to both the numerator and the denominator of the fraction $\frac{5}{9}$, will the value of the fraction be changed? If so, will the value increase or decrease?

56. What happens to the value of a fraction if the denominator of the fraction is decreased while numerator is kept unchanged?

57. Which letter comes $\frac{2}{5}$ of the way among A and J?

58. If $\frac{2}{3}$ of a number is 10, then what is 1.75 times of that number?
59. In a class of 40 students, $\frac{1}{5}$ of the total number of students like to eat rice only, $\frac{2}{5}$ of the total number of students like to eat chapati only and the remaining students like to eat both. What fraction of the total number of students like to eat both?
60. Renu completed $\frac{2}{3}$ part of her home work in 2 hours. How much part of her home work had she completed in $1\frac{1}{4}$ hours?
61. Reemu read $\frac{1}{5}$ th pages of a book. If she reads further 40 pages, she would have read $\frac{7}{10}$ th pages of the book. How many pages are left to be read?
62. Write the number in the box such that
- $$\frac{3}{7} \times \text{input} = \frac{15}{98}$$
63. Will the quotient $7\frac{1}{6} \div 3\frac{2}{3}$ be a fraction greater than 1.5 or less than 1.5? Explain.
64. Describe two methods to compare $\frac{13}{17}$ and 0.82. Which do you think is easier and why?

Think and Discuss

1. **Give an example** of an addition problem that involves connecting an improper fraction in the final step.
2. **Explain** why $\frac{7}{9} + \frac{7}{9}$ does not equal $\frac{14}{18}$.

- 65. Health:** The directions for a pain reliever recommend that an adult of 60 kg and over take 4 tablets every 4 hours as needed, and an adult who weighs between 40 and 50 kg take only $2\frac{1}{2}$ tablets every 4 hours as needed. Each tablet weighs $\frac{4}{25}$ gram.
- (a) If a 72 kg adult takes 4 tablets, how many grams of pain reliever is he or she receiving?
- (b) How many grams of pain reliever is the recommended dose for an adult weighing 46 kg?
- 66. Animals:** The label on a bottle of pet vitamins lists dosage guidelines. What dosage would you give to each of these animals?
- (a) a 18 kg adult dog
- (b) a 6 kg cat
- (c) a 18 kg pregnant dog

Do Good Pet Vitamins

- Adult dogs:
 $\frac{1}{2}$ tsp (tea spoon full) per 9kg body weight
- Puppies, pregnant dogs, or nursing dogs:
 $\frac{1}{2}$ tsp per 4.5kg body weight
- Cats:
 $\frac{1}{4}$ tsp per 1kg body weight



- 67.** How many $\frac{1}{16}$ kg boxes of chocolates can be made with $1\frac{1}{2}$ kg chocolates?

- 68.** Anvi is making bookmarker like the one shown in Fig. 2.6. How many bookmarker can she make from a 15 m long ribbon?

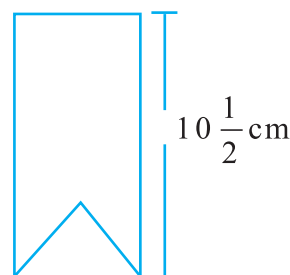


Fig. 2.6

- 69.** A rule for finding the approximate length of diagonal of a square is to multiply the length of a side of the square by 1.414. Find the length of the diagonal when :
- (a) The length of a side of the square is 8.3 cm.
 (b) The length of a side of the square is exactly 7.875 cm.
- 70.** The largest square that can be drawn in a circle has a side whose length is 0.707 times the diameter of the circle. By this rule, find the length of the side of such a square when the diameter of the circle is
- (a) 14.35 cm (b) 8.63 cm
- 71.** To find the distance around a circular disc, multiply the diameter of the disc by 3.14. What is the distance around the disc when :
- (a) the diameter is 18.7 cm?
 (b) the radius is 6.45 cm?
- 72.** What is the cost of 27.5 m of cloth at ₹ 53.50 per metre?
- 73.** In a hurdle race, Nidhi is over hurdle B and $\frac{2}{6}$ of the way through the race, as shown in Fig. 2.7.

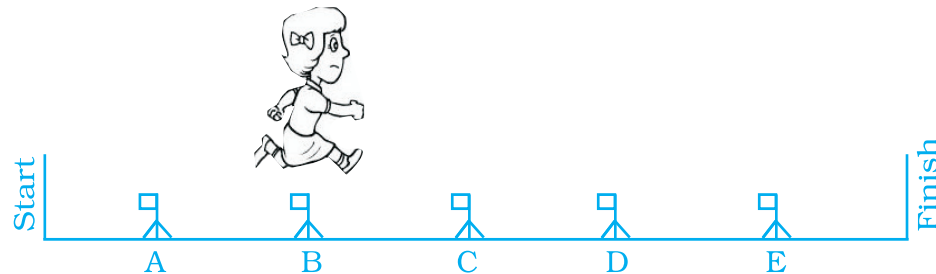


Fig. 2.7

Then, answer the following:

- (a) Where will Nidhi be, when she is $\frac{4}{6}$ of the way through the race?
- (b) Where will Nidhi be when she is $\frac{5}{6}$ of the way through the race?
- (c) Give two fractions to tell what part of the race Nidhi has finished when she is over hurdle C.
- 74.** Diameter of Earth is 12756000m. In 1996, a new planet was discovered whose diameter is $\frac{5}{86}$ of the diameter of Earth. Find the diameter of this planet in km.
- 75.** What is the product of $\frac{5}{129}$ and its reciprocal?

76. Simplify: $\frac{2\frac{1}{2} + \frac{1}{5}}{2\frac{1}{2} \div \frac{1}{5}}$

77. Simplify: $\frac{\frac{1}{4} + \frac{1}{5}}{1 - \frac{3}{8} \times \frac{3}{5}}$

78. Divide $\frac{3}{10}$ by $\left(\frac{1}{4} \text{ of } \frac{3}{5}\right)$

79. $\frac{1}{8}$ of a number equals $\frac{2}{5} \div \frac{1}{20}$. What is the number?

80. Heena's father paid an electric bill of ₹ 385.70 out of a 500 rupee note. How much change should he have received?

81. The normal body temperature is 98.6°F. When Savitri was ill her temperature rose to 103.1°F. How many degrees above normal was that?

Think and Discuss

1. **Name** the number of decimal places in the product of 5.625 and 2.75.
2. **Give an example** of two fractions whose product is an integer due to common factors.

82. **Meteorology:** One measure of average global temperature shows how each year varies from a base measure. The table shows results for several years.

Year	1958	1964	1965	1978	2002
Difference from Base	0.10°C	-0.17°C	-0.10°C	$\left(\frac{1}{50}\right)^{\circ}\text{C}$	0.54°C

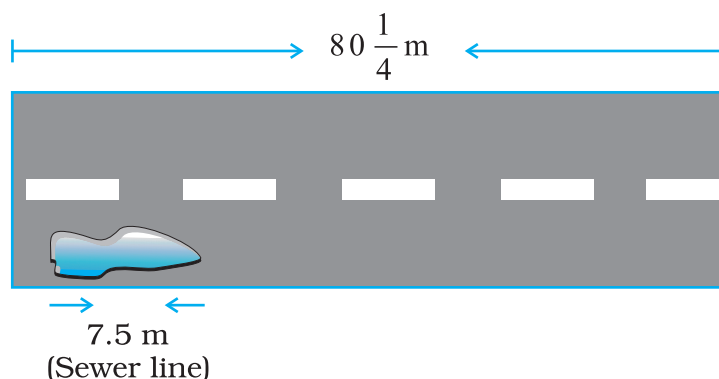
See the table and answer the following:

- (a) Order the five years from coldest to warmest.
- (b) In 1946, the average temperature varied by -0.03°C from the base measure. Between which two years should 1946 fall when the years are ordered from coldest to warmest?

Science Application

- 83.** In her science class, Jyoti learned that the atomic weight of Helium is 4.0030; of Hydrogen is 1.0080; and of Oxygen is 16.0000. Find the difference between the atomic weights of:
(a) Oxygen and Hydrogen
(b) Oxygen and Helium
(c) Helium and Hydrogen
- 84.** Measurement made in science lab must be as accurate as possible. Ravi measured the length of an iron rod and said it was 19.34 cm long; Kamal said 19.25 cm; and Tabish said 19.27 cm. The correct length was 19.33 cm. How much of error was made by each of the boys?
- 85.** When 0.02964 is divided by 0.004, what will be the quotient?
- 86.** What number divided by 520 gives the same quotient as 85 divided by 0.625?
- 87.** A floor is 4.5 m long and 3.6 m wide. A 6 cm square tile costs ₹ 23.25. What will be the cost to cover the floor with these tiles?
- 88.** Sunita and Rehana want to make dresses for their dolls. Sunita has $\frac{3}{4}$ m of cloth, and she gave $\frac{1}{3}$ of it to Rehana. How much did Rehana have?
- 89.** A flower garden is 22.50 m long. Sheela wants to make a border along one side using bricks that are 0.25 m long. How many bricks will be needed?
- 90.** How much cloth will be used in making 6 shirts, if each required $2\frac{1}{4}$ m of cloth, allowing $\frac{1}{8}$ m for waste in cutting and finishing in each shirt?
- 91.** A picture hall has seats for 820 persons. At a recent film show, one usher guessed it was $\frac{3}{4}$ full, another that it was $\frac{2}{3}$ full. The ticket office reported 648 sales. Which usher (first or second) made the better guess?
- 92.** For the celebrating children's students of Class VII bought sweets for ₹ 740.25 and cold drink for ₹ 70. If 35 students contributed equally what amount was contributed by each student?
- 93.** The time taken by Rohan in five different races to run a distance of 500 m was 3.20 minutes, 3.37 minutes, 3.29 minutes, 3.17 minutes and 3.32 minutes. Find the average time taken by him in the races.

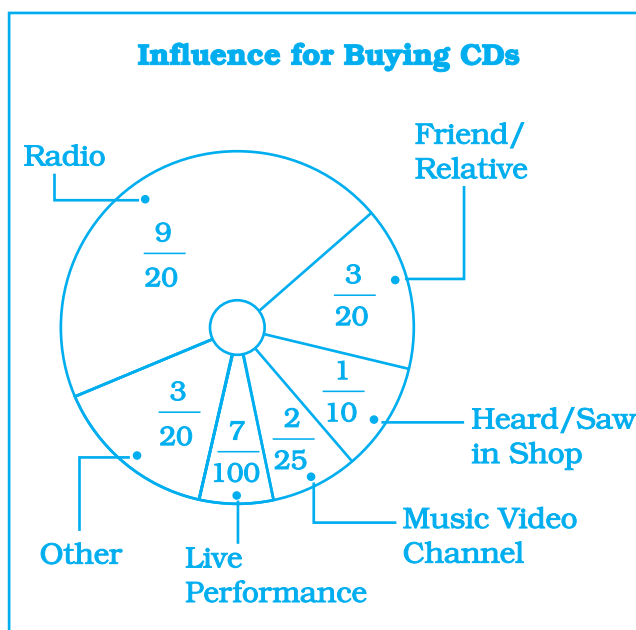
94. A public sewer line is being installed along $80\frac{1}{4}$ m of road. The supervisor says that the labourers will be able to complete 7.5 m in one day. How long will the project take to complete?



95. The weight of an object on moon is $\frac{1}{6}$ its weight on Earth. If an object weighs $5\frac{3}{5}$ kg on Earth, how much would it weigh on the moon?

96. In a survey, 200 students were asked what influenced them most to buy their latest CD. The results are shown in the circle graph.

- (a) How many students said radio influenced them most?
- (b) How many more students were influenced by radio than by a music video channel?



- (c) How many said a friend or relative influenced them or they heard the CD in a shop?

- 97.** In the morning, a milkman filled $5\frac{1}{2}$ L of milk in his can. He sold to Renu, Kamla and Renuka $\frac{3}{4}$ L each; to Shadma he sold $\frac{7}{8}$ L; and to Jassi he gave $1\frac{1}{2}$ L. How much milk is left in the can?
- 98.** Anuradha can do a piece of work in 6 hours. What part of the work can she do in 1 hour, in 5 hours, in 6 hours?
- 99.** What portion of a 'saree' can Rehana paint in 1 hour if it requires 5 hours to paint the whole saree? In $4\frac{3}{5}$ hours? In $3\frac{1}{2}$ hours?
- 100.** Rama has $6\frac{1}{4}$ kg of cotton wool for making pillows. If one pillow takes $1\frac{1}{4}$ kg, how many pillows can she make?
- 101.** It takes $2\frac{1}{3}$ m of cloth to make a shirt. How many shirts can Radhika make from a piece of cloth $9\frac{1}{3}$ m long?
- 102.** Ravi can walk $3\frac{1}{3}$ km in one hour. How long will it take him to walk to his office which is 10 km from his home?
- 103.** Raj travels 360 km on three fifths of his petrol tank. How far would he travel at the same rate with a full tank of petrol?
- 104.** Kajol has ₹ 75. This is $\frac{3}{8}$ of the amount she earned. How much did she earn?

Think and Discuss

- 1. Explain** how you can be sure that a fraction is simplified.
- 2. Give** the sign of a fraction in which the numerator is negative and the denominator is negative.

105. It takes 17 full specific type of trees to make one tonne of paper. If there are 221 such trees in a forest, then (i) what fraction of forest will be used to make;

(a) 5 tonnes of paper.

(b) 10 tonnes of paper.

(ii) To save $\frac{7}{13}$ part of the forest how much of paper we have to save.

106. Simplify and write the result in decimal form :

$$\left(1 \div \frac{2}{9}\right) + \left(1 \div 3\frac{1}{5}\right) + \left(1 \div 2\frac{2}{3}\right)$$

107. Some pictures (a) to (f) are given below. Tell which of them show:

(1) $2 \times \frac{1}{4}$

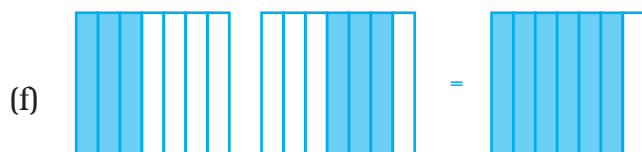
(2) $2 \times \frac{3}{7}$

(3) $2 \times \frac{1}{3}$

(4) $\frac{1}{4} \times 4$

(5) $3 \times \frac{2}{9}$

(6) $\frac{1}{4} \times 3$



108. Evaluate : $(0.3) \times (0.3) - (0.2) \times (0.2)$

109. Evaluate $\frac{0.6}{0.3} + \frac{0.16}{0.4}$

110. Find the value of : $\frac{(0.2 \times 0.14) + (0.5 \times 0.91)}{(0.1 \times 0.2)}$

111. A square and an equilateral triangle have a side in common. If side of triangle is $\frac{4}{3}$ cm long, find the perimeter of figure formed (Fig. 2.8).

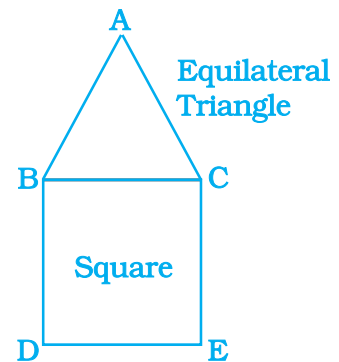


Fig. 2.8

112. Rita has bought a carpet of size $4 \text{ m} \times 6\frac{2}{3} \text{ m}$. But her room size is $3\frac{1}{3} \text{ m} \times 5\frac{1}{3} \text{ m}$. What fraction of area should be cut off to fit wall to wall carpet into the room?

113. Family photograph has length $14\frac{2}{5} \text{ cm}$ and breadth $10\frac{2}{5} \text{ cm}$. It has border of uniform width $2\frac{3}{5} \text{ cm}$. Find the area of framed photograph.

114. Cost of a burger is ₹ $20\frac{3}{4}$ and of Macpuff is ₹ $15\frac{1}{2}$. Find the cost of 4 burgers and 14 macpuffs.

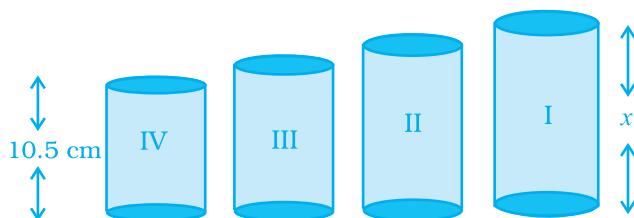
115. A hill, $101\frac{1}{3} \text{ m}$ in height, has $\frac{1}{4}$ th of its height under water. What is the height of the hill visible above the water?

116. *Sports:* Reaction time measures how quickly a runner reacts to the starter pistol. In the 100 m dash at the 2004 Olympic Games, Lauryn Williams had a reaction time of 0.214 second. Her total race time, including reaction time, was 11.03 seconds. How long did it take her to run the actual distance?

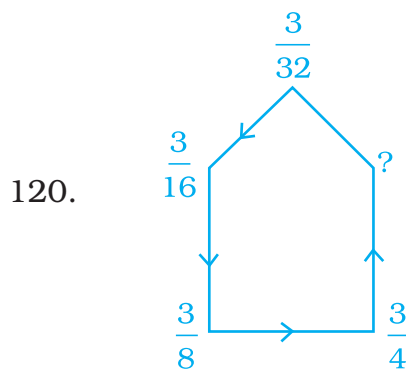
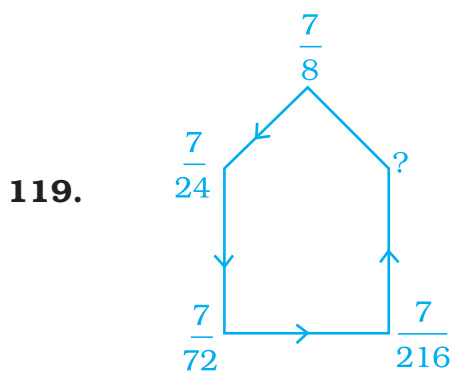
- 117.** State whether the answer is greater than 1 or less than 1. Put a '✓' mark in appropriate box.

Questions	Greater than 1	Less than 1
$\frac{2}{3} \div \frac{1}{2}$		
$\frac{2}{3} \div \frac{2}{1}$		
$6 \div \frac{1}{4}$		
$\frac{1}{5} \div \frac{1}{2}$		
$4\frac{1}{3} \div 3\frac{1}{2}$		
$\frac{2}{3} \times 8\frac{1}{2}$		

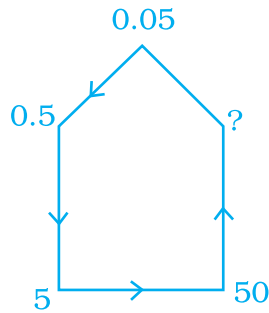
- 118.** There are four containers that are arranged in the ascending order of their heights. If the height of the smallest container given in the figure is expressed as $\frac{7}{25}x = 10.5$ cm. Find the height of the largest container.



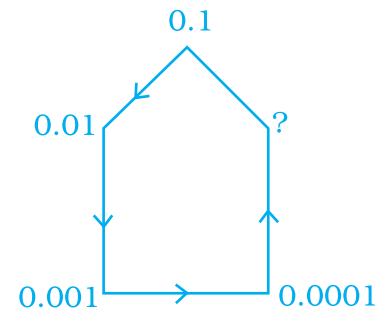
In Questions 119 to 122, replace '?' with appropriate fraction.



121.



122.



What is the Error in each of question 123 to 125?

- 123.** A student compared $-\frac{1}{4}$ and -0.3 . He changed $-\frac{1}{4}$ to the decimal -0.25 and wrote, "Since 0.3 is greater than 0.25 , -0.3 is greater than -0.25 ". What was the student's error?
- 124.** A student multiplied two mixed fractions in the following manner:
 $2\frac{4}{7} \times 3\frac{1}{4} = 6\frac{1}{7}$. What error the student has done?
- 125.** In the pattern $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots$ which fraction makes the sum greater than 1 (first time)? Explain.

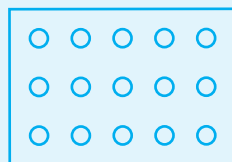
(D) Applications

Game 1:

Shade (i) $\frac{1}{3}$ of the circles in box (a)

(ii) $\frac{2}{5}$ of the triangles in box (b)

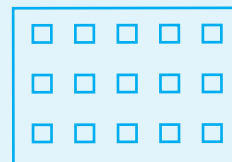
(iii) $\frac{1}{5}$ of the squares in box (c)



(a)



(b)



(c)

UNIT 3

DATA HANDLING

(A) Main Concepts and Results

- The information collected in the form of numbers is called **Data**.
- Data is organised and represented graphically so that it becomes easy to understand and interpret.
- The difference between the highest and lowest observations in a given data is called its **Range**.
- The average or **Arithmetic Mean** or mean of a given data is defined as :

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

- **Mode** is the observation that occurs most frequently in the data.
- If each of the values in a data are occurring one time (or equal number of times), then all are mode. Sometimes, we also say that this data has no mode since none of them is occurring frequently.
- When the given data is arranged in ascending (or descending) order, then the middle most observation is the **median** of the data.
- Mean, median and mode are the representative values of a group of observations. They are also called the **measures of central tendency** of the data.
- The representation of the data in the form of rectangles (bars) of uniform width is called a **Bar Graph**.
- A **double bar graph** can be used to compare informations related to two data.
- The situation that may or may not happen, have a chance of happening.

- The probability of an event which is certain to happen is '1'.
- The probability of an event which is impossible to happen is '0'.
- The probability of an event

$$= \frac{\text{Number of outcomes favourable to the event}}{\text{Total number of outcomes in the experiment}}$$

(B) Solved Examples

In Examples 1 to 3, there are four options, out of which only one is correct. Write the correct answer.

Example 1: The range of the data 14, 6, 12, 17, 21, 10, 4, 3 is
 (a) 21 (b) 17 (c) 18 (d) 11

Solution: Correct answer is (c)

Example 2: The mode of the data 23, 26, 22, 29, 23, 29, 26, 29, 22, 23 is
 (a) 23 and 29 (b) 23 only
 (c) 29 only (d) 26 only

Solution: Correct answer is (a)

Example 3: The median of the data 40, 50, 99, 68, 98, 60, 94 is
 (a) 40 (b) 60
 (c) 68 (d) 99

Solution: Correct answer is (c)

In Examples 4 and 5, fill in the blanks to make the statements true.

Example 4: The mean of first five prime numbers is _____.

Solution: 5.6

[Hint : First five prime numbers are 2, 3, 5, 7 and 11]

Example 5: The probability of getting a number greater than 2 on throwing a die once is _____.

Solution: $\frac{2}{3}$

In Examples 6, 7 and 8, state whether the statements are True or False .

Example 6: The mode of the observations 23, 26, 15, 12, 28, 38, 19, 23, 26, 23 is 28.

Solution: False.

Example 7:

Size of Sweater	Number of Sweaters Sold
40	15
42	17
44	13
46	14
48	11
Total	70

In the above table

- (a) The most popular size is 17.
- (b) 17 is the median for above data.

Solution: (a) False

(The numbers of sweater 17 tells us that 42 is the most common size. Thus, 17 is not mode rather 42 is mode.)

- (b) False

Example 8: Median of the data:

4, 5, 9, 2, 6, 8, 7 is 2

Solution: False

Example 9: Find the median of the data:

3, 11, 7, 2, 5, 9, 9, 2, 10, 15, 7

Solution: Arranging in ascending order.

2, 2, 3, 5, 7, 7, 9, 9, 10, 11, 15

Since number of observations is odd, the middle most value is the median. The middle most value is 7, so median is 7.

Example 10: Find the median of the data :

21, 15, 6, 25, 18, 13, 20, 9, 8, 12

Solution: Arranging in ascending order :

6, 8, 9, 12, 13, 15, 18, 20, 21, 25

Since number of observations is even, the median is given by finding the average or mean of the two middle most observations:

$$\text{So, median} = \frac{13+15}{2} = \frac{28}{2} = 14$$

Note : In this data, there are two middle most terms 13 and 15. So, median is the average of these observations.

Example 11: The cards bearing letters of the word “MATHEMATICS” are placed in a bag. A card is taken out from the bag without looking into the bag (at random).

- (a) How many outcomes are possible when a letter is taken out of the bag at random?
- (b) What is the probability of getting
 - (i) M?
 - (ii) Any vowel?
 - (iii) Any consonant?
 - (iv) X?

Solution: (a) There are 11 outcomes namely M, M, A, A, T, T, H, E, I, C, S.

(b) (i) Probability of getting 'M' = $\frac{2}{11}$

(ii) Probability of getting a vowel = $\frac{4}{11}$

(iii) Probability of getting a consonant = $\frac{7}{11}$

(iv) Probability of getting X = $0 = \frac{0}{11}$

Example 12: If the mean of 26, 28, 25, x , 24 is 27, find the value of x .

Solution: Mean = $\frac{\text{Sum of all observations}}{\text{Number of observations}}$

$$\text{or, } 27 = \frac{26 + 28 + 25 + x + 24}{5}$$

$$\text{or, } 27 = \frac{103 + x}{5}$$

$$\text{or, } 135 = 103 + x$$

$$\text{or, } x = 135 - 103$$

$$\text{So, } x = 32$$

Example 13: The mean of 10 observations was calculated as 40. It was detected on rechecking that the value of 45 was wrongly copied as 15. Find the correct mean.

Solution: Mean = $\frac{\text{Sum of all observations}}{\text{Number of observations}}$

$$\text{or, } 40 = \frac{\text{Sum of all observations}}{10}$$

So, sum of all observations = 400

But this is incorrect sum, since one observation was copied wrongly.

$$\begin{aligned} \text{So, correct sum} &= \text{Incorrect sum} - \text{Incorrect} \\ &\quad \text{observation} + \text{correct observation} \\ &= 400 - 15 + 45 \\ &= 430 \end{aligned}$$

$$\text{Correct Mean} = \frac{\text{Correct Sum}}{\text{Number of observations}} = \frac{430}{10} = 43$$

Example 14: The median of observations 11, 12, 14, 18, $x + 2$, 20, 22, 25, 61 arranged in ascending order is 21. Find the value of x .

Solution: Median from data = $x + 2$

$$\text{or, } 21 = x + 2$$

$$\text{or, } x = 21 - 2$$

$$\text{or, } x = 19$$

Example 15: Study the double bar graph given below and answer the questions that follow:

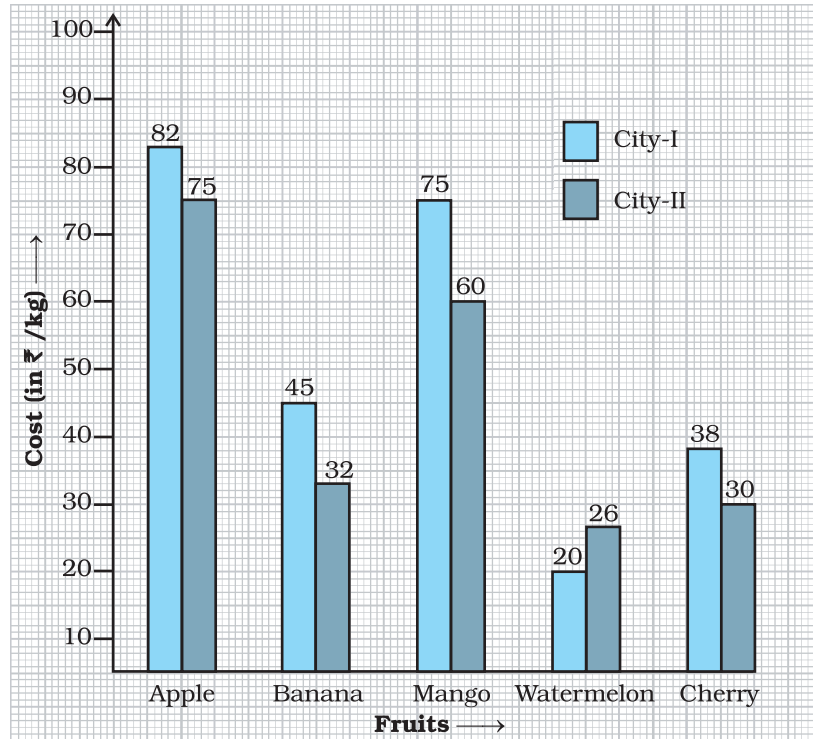


Fig. 3.1

- What information does the above double graph depict?
- Name the fruits for which cost of 1 kg is greater in City I as compared to City II.
- What is the difference of rates for apples in both the cities?
- Find the ratio of the cost of mangoes per kg in City I to the cost of mangoes per kg in City II.

Solution:

- The double bar graph compares the cost of different fruits per kg in Cities I and II.
- Apple, Banana, Mango and Cherry.
- Since ₹ 82 – ₹ 75 = ₹ 7 therefore, in both the cities the difference of rates of apples is ₹ 7/kg.
- ₹ 75 : ₹ 60 = 5 : 4

Example 16: The following double bar graph represents test matches results summary for Cricket Team of country X against different countries:

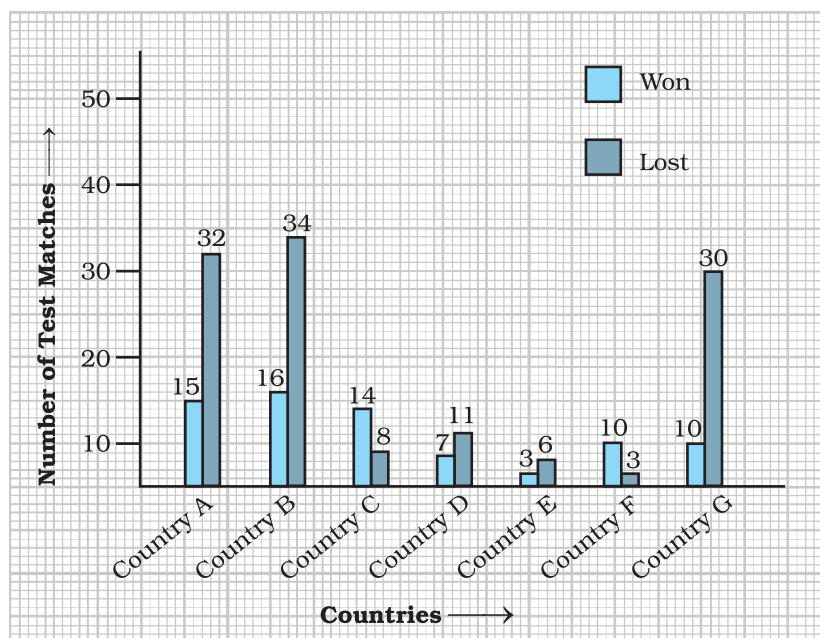


Fig. 3.2

Use the bar graph to answer the following questions:

- Which country has managed maximum wins against country X?
- The difference between the number of matches won and lost is highest for which country against country X?
- Number of wins of country E is the same as number of losses of which country against country X?

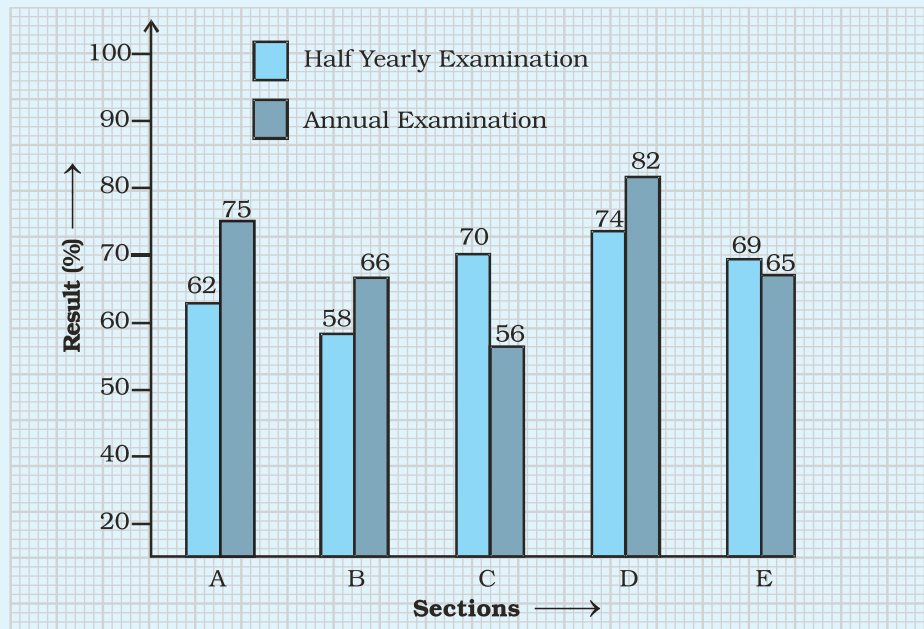
Solution: (a) Country B (b) Country G (c) Country F

Application on Problem Solving Strategy



Example 17

The double bar graph given below compares the class-averages in half yearly and annual examinations of 5 sections of Class VII.



Observe the graph carefully and tell which section showed the most improvement and by how much?

Solution:



Understand and Explore the Problem

- What information is given in the question?

The average result of half yearly and annual examinations of 5 different sections of Class VII are compared.

- What are you trying to find?

The section of Class VII that has showed the most improvement and the per cent of improvement shown.



Plan a Strategy

- Observe the graph and find out the sections in which the annual examination result is more than the half yearly result.

Improvement is in these sections only.

- Then only for these sections, compare the results graphically and locate the section for which the difference of results is the maximum.
- For this section, find the difference of the results.

**Solve**

- The sections in which the results of annual examination is more than half yearly examination are sections A, B and D.
- Observing the graph of these sections we locate that section A has the maximum difference between the results.

The difference of results of section A = $75 - 62 = 13$

Hence, section A has shown the maximum improvement and it is 13 per cent.

**Revise**

- Find the difference of the annual examination results and the half yearly examination results for each section.

Difference in results of section A = $75 - 62 = 13$

Difference in results of section B = $66 - 58 = 8$

Difference in results of section C = $56 - 70 = -14$

Difference in results of section D = $82 - 74 = 8$

Difference in results of section E = $65 - 69 = -4$

We see that the difference is maximum for Section A and the difference is 13, which is same as our answer.



Think and Discuss

1. **Can you compare** the ratio of difference of results of Sections B and D?
2. **From the graph**, can you observe the sections where there was no improvement?

(C) EXERCISE

In Questions 1 to 16, there are four options, out of which only one is correct. Write the correct answer.

- Let x, y, z be three observations. The mean of these observations is
 (a) $\frac{x \times y \times z}{3}$ (b) $\frac{x + y + z}{3}$ (c) $\frac{x - y - z}{3}$ (d) $\frac{x \times y + z}{3}$
- The number of trees in different parks of a city are 33, 38, 48, 33, 34, 34, 33 and 24. The mode of this data is
 (a) 24 (b) 34 (c) 33 (d) 48
- Which measures of central tendency get affected if the extreme observations on both the ends of a data arranged in descending order are removed?
 (a) Mean and mode (b) Mean and Median
 (c) Mode and Median (d) Mean, Median and Mode
- The range of the data : 21, 6, 17, 18, 12, 8, 4, 13 is
 (a) 17 (b) 12 (c) 8 (d) 15
- The median of the data : 3, 4, 5, 6, 7, 3, 4 is
 (a) 5 (b) 3 (c) 4 (d) 6
- Out of 5 brands of chocolates in a shop, a boy has to purchase the brand which is most liked by children. What measure of central tendency would be most appropriate if the data is provided to him?
 (a) Mean (b) Mode
 (c) Median (d) Any of the three
- There are 2 aces in each of the given set of cards placed face down. From which set are you certain to pick the two aces in the first go?



8. In the previous question, what is the probability of picking up an ace from set (d)?
- (a) $\frac{1}{6}$ (b) $\frac{2}{6}$ (c) $\frac{3}{6}$ (d) $\frac{4}{6}$
9. The difference between the highest and the lowest observations in a data is its
- (a) frequency (b) width (c) range (d) mode
10. In a school, only 2 out of 5 students can participate in a quiz. What is the chance that a student picked at random makes it to the competition?
- (a) 20% (b) 40% (c) 50% (d) 30%
11. Some integers are marked on a board. What is the range of these integers?
- (a) 31 (b) 37
(c) 20 (d) 3
12. On tossing a coin, the outcome is
- (a) only head
(b) only tail
(c) neither head nor tail
(d) either head or tail
13. The mean of three numbers is 40. All the three numbers are different natural numbers. If lowest is 19, what could be highest possible number of remaining two numbers?
- (a) 81 (b) 40 (c) 100 (d) 71
14. Khilona earned scores of 97, 73 and 88 respectively in her first three examinations. If she scored 80 in the fourth examination, then her average score will be
- (a) increased by 1 (b) increased by 1.5
(c) decreased by 1 (d) decreased by 1.5
15. Which measure of central tendency best represents the data of the most popular politician after a debate?
- (a) Mean (b) Median
(c) Mode (d) Any of the above

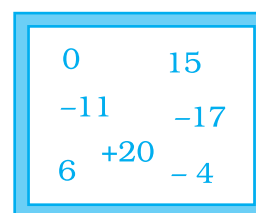


Fig. 3.3

16. Which of the following has the same mean, median and mode?

- (a) 6, 2, 5, 4, 3, 4, 1 (b) 4, 2, 2, 1, 3, 2, 3
(c) 2, 3, 7, 3, 8, 3, 2 (d) 4, 3, 4, 3, 4, 6, 4

In Questions 17 to 31, fill in the blanks to make the statements true.

- 17.** The difference between the highest and the lowest observations of a data is called _____.
- 18.** The mean of a data is defined as _____.
- 19.** In a set of observations, the observation that occurs the most often is called _____.
- 20.** In a given data, arranged in ascending or descending order, the middle most observation is called _____.
- 21.** Mean, Median, Mode are the measures of _____.
- 22.** The probability of an event which is certain to happen is _____.
- 23.** The probability of an event which is impossible to happen is _____.
- 24.** When a die is thrown, the probability of getting a number less than 7 is _____.
- 25.** In Throwing a die the number of possible outcomes is _____.
- 26.** _____ can be used to compare two collections of data.
- 27.** The representation of data with bars of uniform width is called _____.
- 28.** If the arithmetic mean of 8, 4, x , 6, 2, 7 is 5, then the value of x is _____.
- 29.** The median of any data lies between the _____ and _____ observations.
- 30.** Median is one of the observations in the data if number of observations is _____.

Think and Discuss

What is the difference between a bar graph and a histogram.

31. Rohit collected the data regarding weights of students of his class and prepared the following table:

Weight (in kg)	44 – 47	48 – 51	52 – 55	56 – 60
Number of Students	3	5	25	7

A student is to be selected randomly from his class for some competition. The probability of selection of the student is highest whose weight is in the interval _____.

In Questions 32 to 49, state whether the statements are True or False.

32. If a die is thrown, the probability of getting a number greater than 6 is 1.
33. When a coin is tossed, there are 2 possible outcomes.
34. If the extreme observations on both the ends of a data arranged in ascending order are removed, the median gets affected.
35. The measures of central tendency may not lie between the maximum and minimum values of data.
36. It is impossible to get a sum of 14 of the numbers on both dice when a pair of dice is thrown together.
37. The probability of the spinning arrow stopping in the shaded region (Fig. 3.4) is $\frac{1}{2}$.
38. A coin is tossed 15 times and the outcomes are recorded as follows :
H T T H T H H H T T H T H T T. The chance of occurrence of a head is 50 per cent.
39. Mean, Median and Mode may be the same for some data.
40. The probability of getting an ace out of a deck of cards is greater than 1.
41. Mean of the data is always from the given data.
42. Median of the data may or may not be from the given data.
43. Mode of the data is always from the given data.
44. Mean of the observations can be lesser than each of the observations.
45. Mean can never be a fraction.

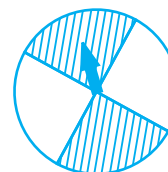


Fig. 3.4

- 46.** Range of the data is always from the data.
- 47.** The data 12, 13, 14, 15, 16 has every observation as mode.
- 48.** The range of the data 2, -5, 4, 3, 7, 6 would change if 2 was subtracted from each value in the data.
- 49.** The range of the data 3, 7, 1, -2, 2, 6, -3, -5 would change if 8 was added to each value in the data.
- 50.** Calculate the Mean, Median and Mode of the following data:
5, 10, 10, 12, 13.
Are these three equal ?
- 51.** Find the mean of the first ten even natural numbers.
- 52.** A data constitutes of heights (in cm) of 50 children. What do you understand by mode for the data?
- 53.** A car seller collects the following data of cars sold in his shop.

Colour of Car	Number of Cars Sold
Red	15
Black	20
White	17
Silver	12
Others	9

- (a) Which colour of the car is most liked?
- (b) Which measure of central tendency was used in (a)?
- 54.** The marks in a subject for 12 students are as follows:
31, 37, 35, 38, 42, 23, 17, 18, 35, 25, 35, 29
For the given data, find the
(a) Range (b) Mean (c) Median (d) Mode
- 55.** The following are weights (in kg) of 12 people.
70, 62, 54, 57, 62, 84, 75, 59, 62, 65, 78, 60
(a) Find the mean of the weights of the people.

- (b) How many people weigh above the mean weight?
 (c) Find the range of the given data.

56. Following cards are put facing down:

A E I O U

What is the chance of drawing out

- (a) a vowel (c) a card marked U
 (b) A or I (d) a consonant
- 57.** For the given data given below, calculate the mean of its median and mode.
 6, 2, 5, 4, 3, 4, 4, 2, 3
- 58.** Find the median of the given data if the mean is 4.5.
 5, 7, 7, 8, x , 5, 4, 3, 1, 2
- 59.** What is the probability of the sun setting tomorrow?
- 60.** When a spinner with three colours (Fig. 3.5) is rotated, which colour has more chance to show up with arrow than the others?
- 61.** What is the probability that a student chosen at random out of 3 girls and 4 boys is a boy?
- 62.** The letters written on paper slips of the word MEDIAN are put in a bag. If one slip is drawn randomly, what is the probability that it bears the letter D?
- 63.** Classify the following events as certain to happen, impossible to happen, may or may not happen:
 (a) Getting a number less than 1 on throwing a die.
 (b) Getting head when a coin is tossed.
 (c) A team winning the match.
 (d) Christmas will be on 25 December.
 (e) Today moon will not revolve around the earth.
 (f) A ball thrown up in the air will fall down after some time.
- 64.** A die was thrown 15 times and the outcomes recorded were
 5, 3, 4, 1, 2, 6, 4, 2, 2, 3, 1, 5, 6, 1, 2
 Find the mean, median and mode of the data.
- 65.** Find the mean of first six multiples of 4.
- 66.** Find the median of first nine even natural numbers.

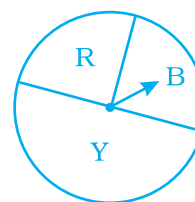


Fig. 3.5

- 67.** The mean of three numbers is 10. The mean of other four numbers is 12. Find the mean of all the numbers.
- 68.** Find the mode of the given data:
10, 8, 4, 7, 8, 11, 15, 8, 4, 2, 3, 6, 8
- 69.** Given below are heights of 15 boys of a class measured in cm:
128, 144, 146, 143, 136, 142, 138, 129, 140, 152, 144, 140, 150, 142, 154.
Find
(a) The height of the tallest boy.
(b) The height of the shortest boy.
(c) The range of the given data.
(d) The median height of the boys.
- 70.** Observe the data and answer the questions that follow:
16, 15, 16, 16, 8, 15, 17
(a) Which data value can be put in the data so that the mode remains the same?
(b) At least how many and which value(s) must be put in to change the mode to 15?
(c) What is the least number of data values that must be put in to change the mode to 17? Name them.
- 71.** Age (in years) of 6 children of two groups are recorded as below:

Age (in Years)	
Group A	Group B
7	7
7	9
9	11
8	12
10	12
10	12

- (a) Find the mode and range for each group.
(b) Find the range and mode if the two groups are combined together.

Measures of central tendency are used to describe the middle of a data set. Mean, median, and mode are measures of central tendency.

Measures of Central Tendency and Range

Description	
Mean	To find the mean (average), add the values in the data set. Then divide by the number of values in the set.
Median	The middle value or the mean of the two middle values, in an ordered (ascending or descending) set of data.
Mode	The value(s) that occur most frequently.
Range	The difference between the least and the greatest values in a data set.

72. Observe the given bar graph carefully and answer the questions that follow.

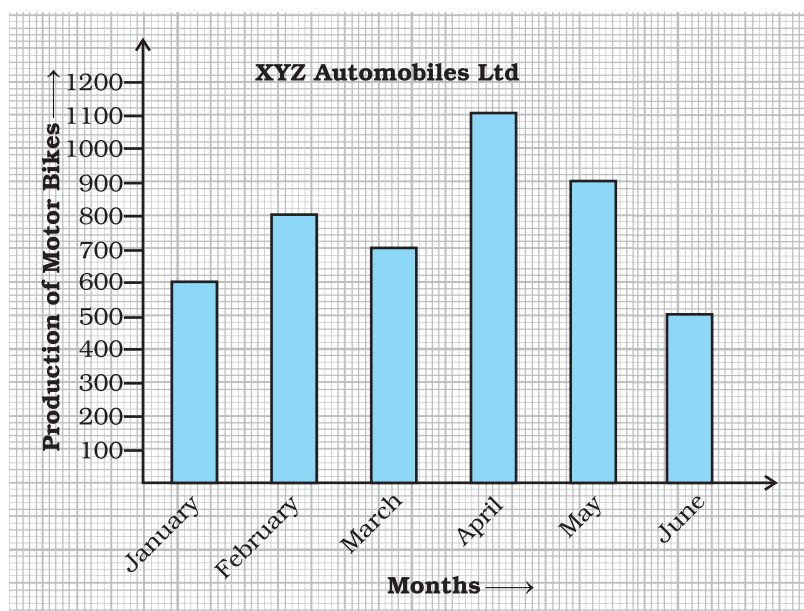


Fig. 3.6

- What information does the bar graph depict?
- How many motor bikes were produced in the first three months?
- Calculate the increase in production in May over the production in January.

- (d) In which month the production was minimum and what was it?
- (e) Calculate the average (mean) production of bikes in 6 months.

73. The bar graph given below shows the marks of students of a class in a particular subject:

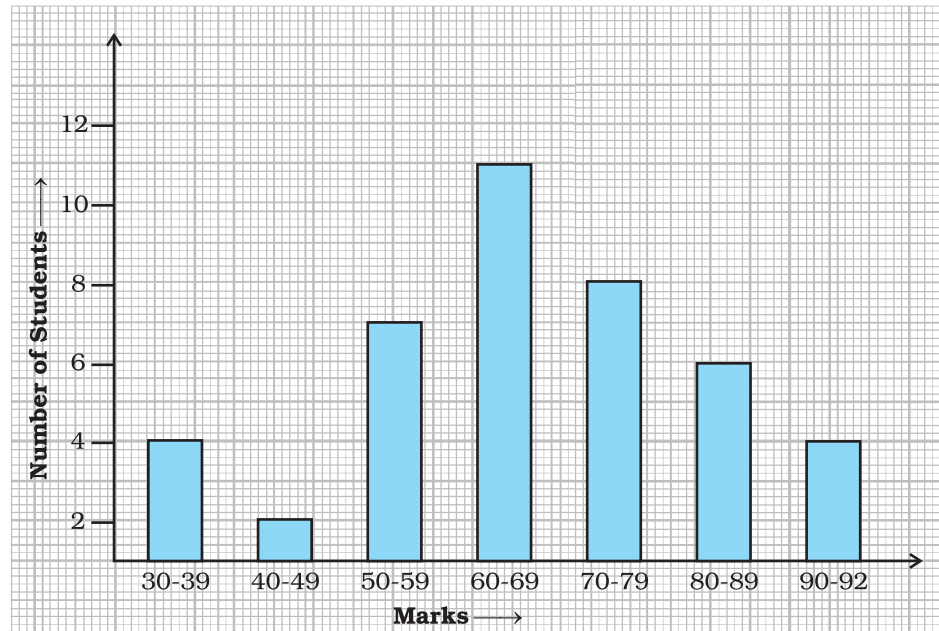


Fig. 3.7

Study the bar graph and answer the following questions:

- (a) If 40 is the pass mark, then how many students have failed?
- (b) How many students got marks from 50 to 69?
- (c) How many students scored 90 marks and above?
- (d) If students who scored marks above 80 are given merits then how many merit holders are there?
- (e) What is the strength of the class?

74. Study the bar graph given below and answer the questions that follow.

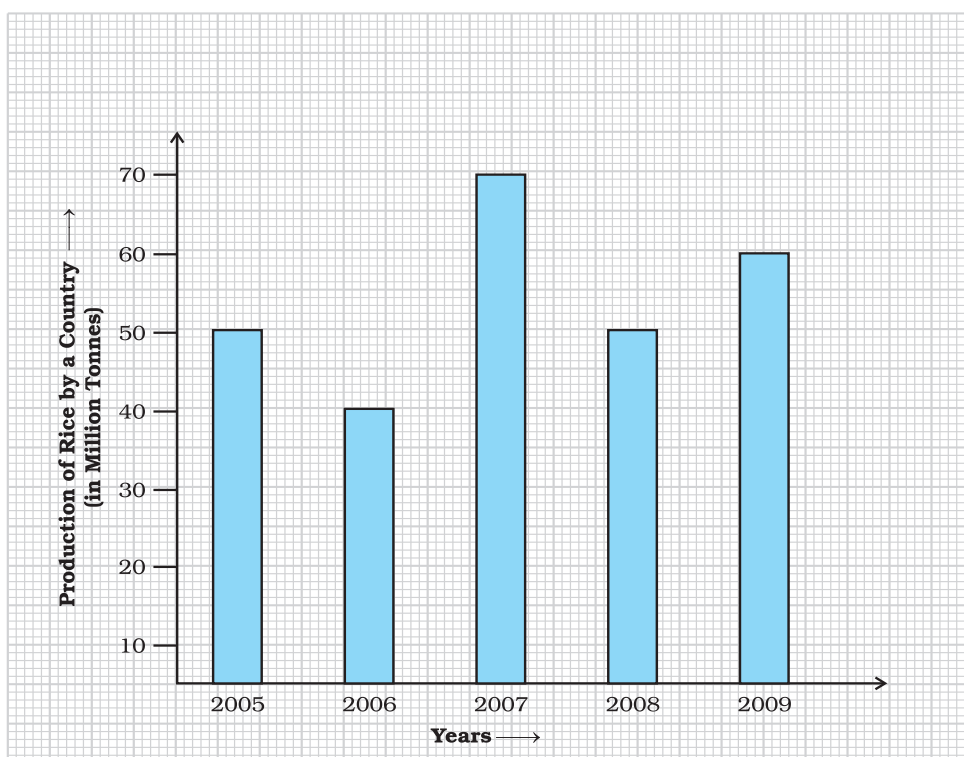
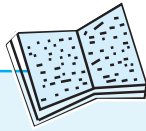


Fig. 3.8

- (a) What information does the above bar graph represent?
- (b) In which year was production the least?
- (c) After which year was the maximum rise in the production?
- (d) Find the average production of rice during the 5 years.
- (e) Find difference of rice production between years 2006 and 2008.

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, fill up the following:



Vocabulary

1. The population of an area is the total number of people living in that area. What might _____ mean in the process of gathering data?
2. The word median is derived from the Latin word medius, meaning “middle,” What might the _____ value in a set of data be?
3. When you sample a food, you taste a small portion. What might a _____ be in data collection?

- **Population**
- **Median**
- **Sample**

75. Study the bar graph given below and answer the questions that follow :

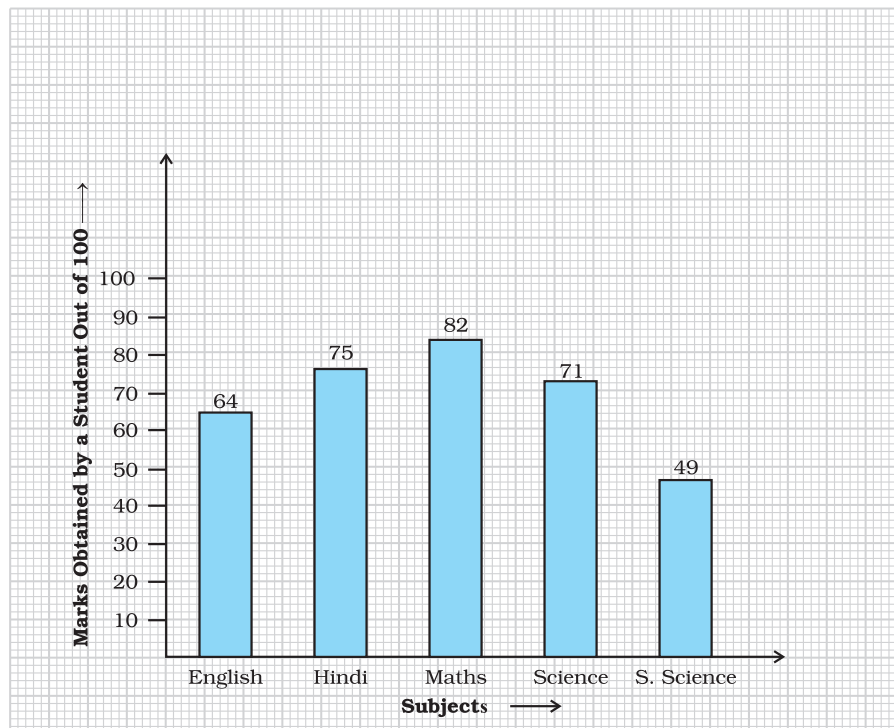


Fig. 3.9

- (a) What information is depicted from the bar graph?
- (b) In which subject is the student very good?
- (c) Calculate the average marks of the student.
- (d) If 75 and above marks denote a distinction, then name the subjects in which the student got distinction.
- (e) Calculate the percentage of marks the student got out of 500.

76. The bar graph given below represents the circulation of newspapers (dailies) in a town in six languages (the figures are approximated to hundreds).

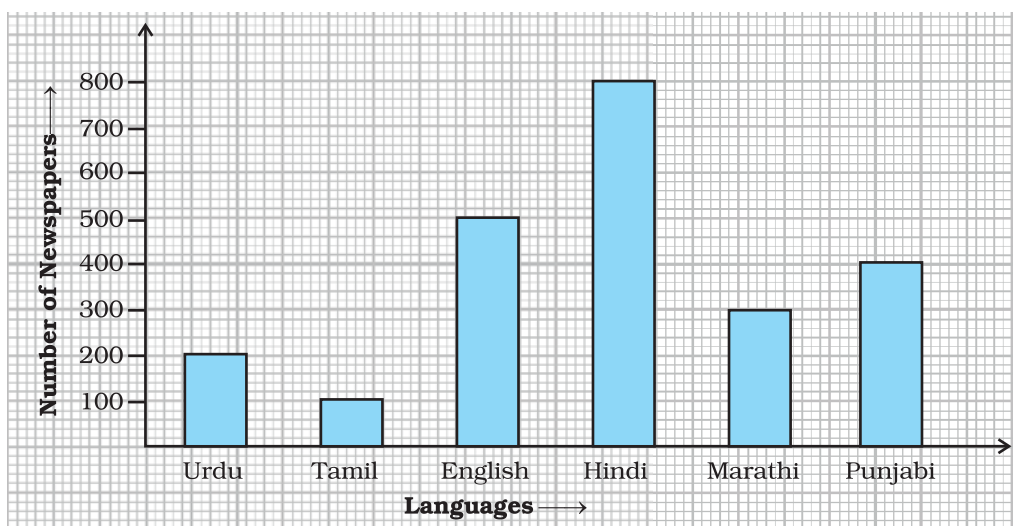


Fig. 3.10

Study the bar graph and answer the following questions:

- (a) Find the total number of newspapers read in Hindi, Punjabi, Urdu, Marathi and Tamil.
- (b) Find the excess number of newspapers read in Hindi than those in English.
- (c) Name the language in which the least number of newspapers are read.
- (d) Write the total circulation of newspapers in the town.

77. Study the double bar graphs given below and answer the following questions:

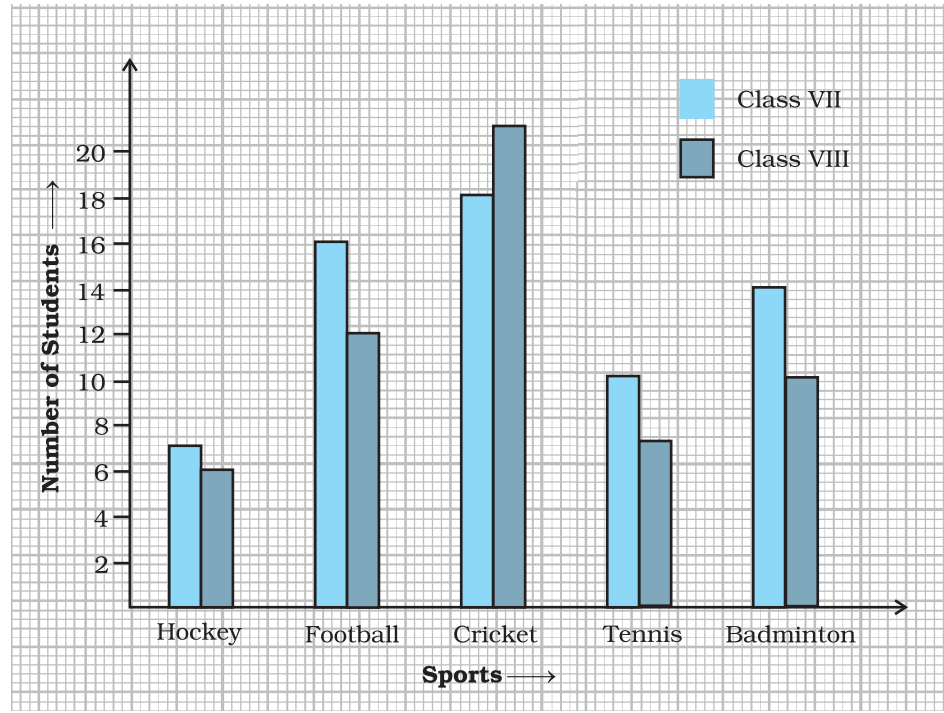


Fig. 3.11

- Which sport is liked the most by Class VIII students?
- How many students of Class VII like Hockey and Tennis in all?
- How many students are there in Class VII?
- For which sport is the number of students of Class VII less than that of Class VIII?
- For how many sports students of Class VIII are less than Class VII?
- Find the ratio of students who like Badminton in Class VII to students who like Tennis in Class VIII.

78. Study the double bar graph shown below and answer the questions that follow:

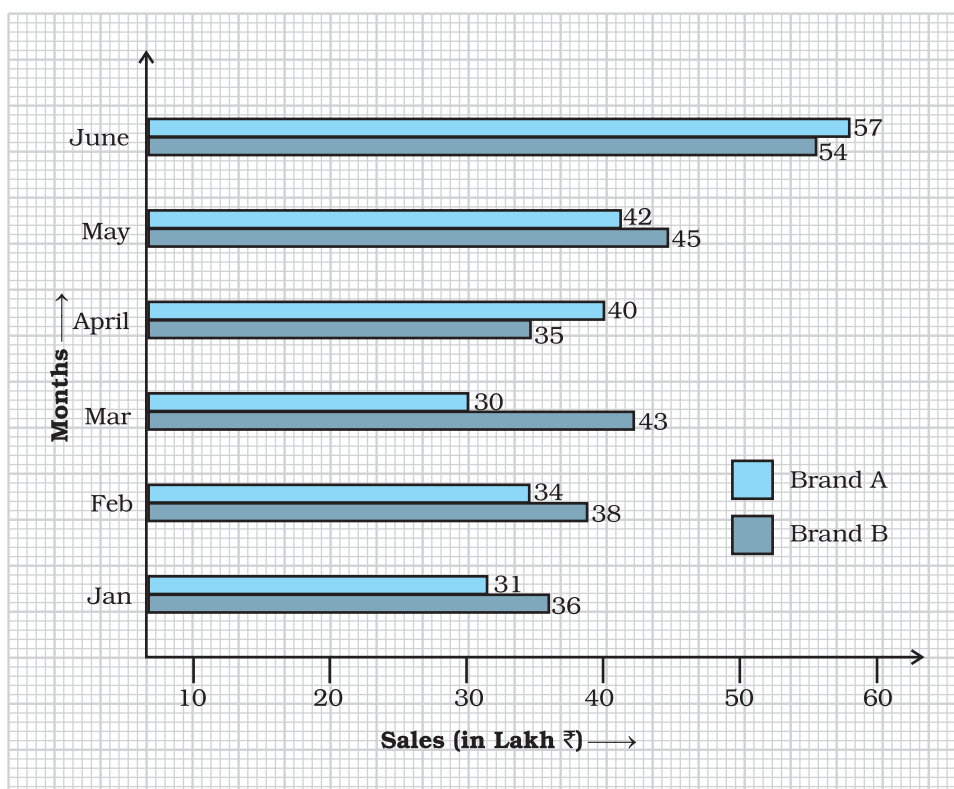


Fig. 3.12

- What information is represented by the above double bar graph?
- In which month sales of Brand A decreased as compared to the previous month?
- What is the difference in sales of both the Brands for the month of June?
- Find the average sales of Brand B for the six months.
- List all months for which the sales of Brand B was less than that of Brand A.
- Find the ratio of sales of Brand A as compared to Brand B for the month of January.

79. Study the double bar graph given below and answer the questions that follow:

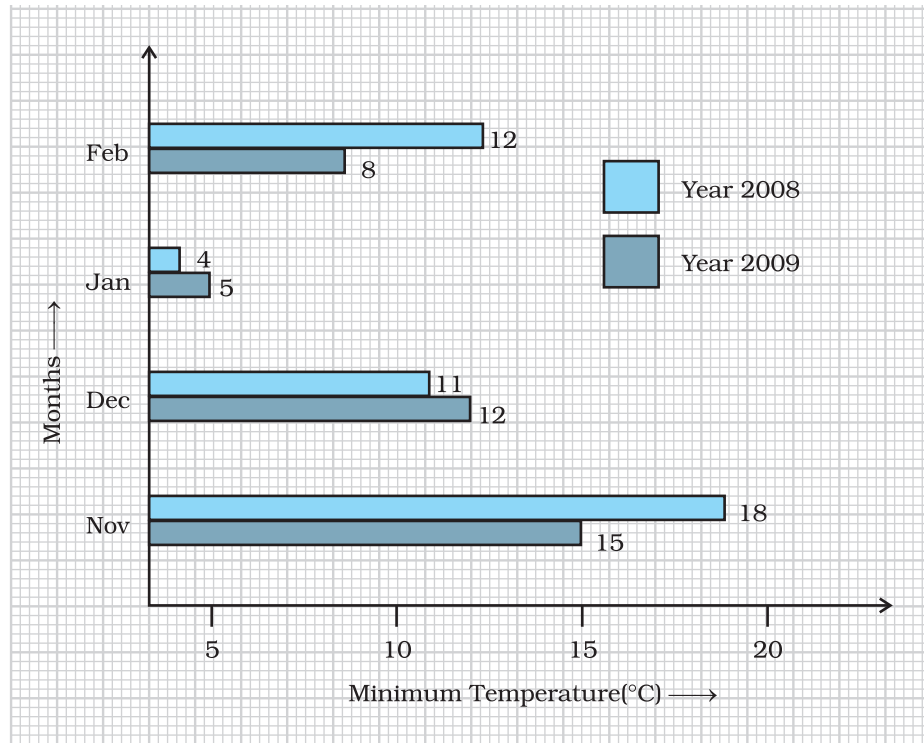


Fig. 3.13

- What information is compared in the above given double bar graph?
 - Calculate the ratio of minimum temperatures in the year 2008 to the year 2009 for the month of November.
 - For how many months was the minimum temperature in the year 2008 greater than that of year 2009? Name those months.
 - Find the average minimum temperature for the year 2008 for the four months.
 - In which month is the variation in the two temperatures maximum?
80. The following table shows the average intake of nutrients in calories by rural and urban groups in a particular year. Using a suitable scale for the given data, draw a double bar graph to compare the data.

Foodstuff	Rural	Urban
Pulses	35	49
Leafy vegetables	14	21
Other vegetables	51	89
Fruits	35	66
Milk	70	250
Fish and flesh foods	10	22
Fats and Oils	9	35
Sugar/Jaggery	19	31

81. Study the double bar graph and answer the questions that follow:

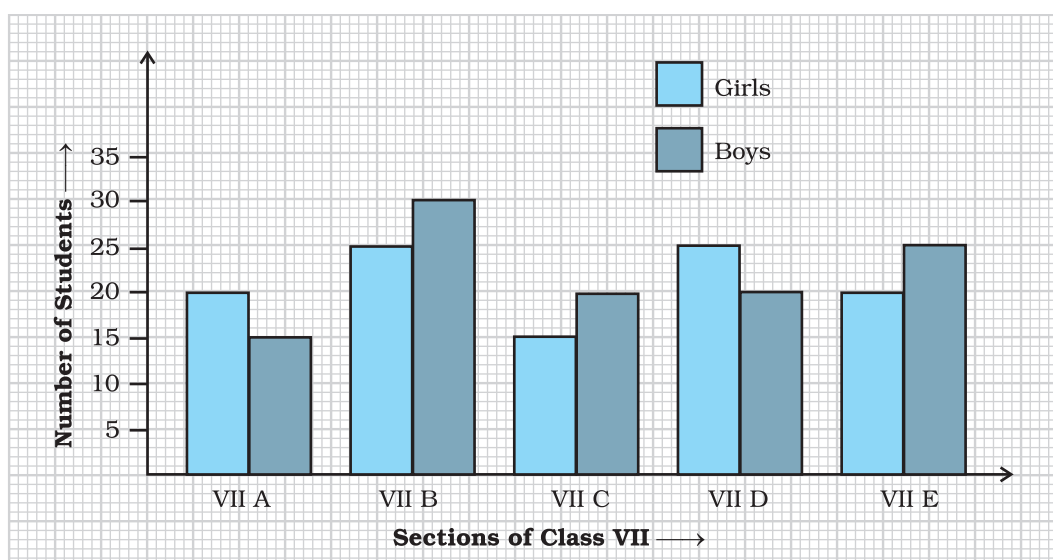


Fig. 3.14

- What information does the double bar graph represent?
- Find the total number of boys in all sections of Class VII.
- In which sections, the number of girls is greater than the number of boys?
- In which section, the number of boys is the maximum?
- In which section, the number of girls is the least?

- 82.** In a public library, the following observations were recorded by the librarian in a particular week:

Days	Mon	Tues	Wed	Thurs	Fri	Sat
Newspaper Readers	400	600	350	550	500	350
Magazine Readers	150	100	200	300	250	200

- (a) Draw a double bar graph choosing an appropriate scale.
 (b) On which day, the number of readers in the library was maximum?
 (c) What is the mean number of magazine readers?

- 83.** Observe the following data:

Government School, Chandpur		
Daily Attendance		Date : 15.4.2009
Class	Total Students	Number of Students Present on that Day
VI	90	81
VII	82	76
VIII	95	91
IX	70	65
X	63	62

- (a) Draw a double bar graph choosing an appropriate scale. What do you infer from the bar graph?
 (b) Which class has the maximum number of students?
 (c) In which class, the difference of total students and number of students present is minimum?
 (d) Find the ratio of number of students present to the total number of students of Class IX.
 (e) What per cent of Class VI students were absent?



Plan a Strategy

- Identify too much/too little Information.

When you read a problem, you must decide if the problem has too much or too little information. If the problem has too much information, you must decide what information to use to solve the problem. If the problem has too little information, then you should determine what additional information you need to solve the problem.

- Read the problems below and decide if there is too much or too little information in each problem. If there is too much information, tell what information you would use to solve the problem. If there is too little information, tell what additional information you would need to solve the problem.
- On Monday, 20 students took an examination. There were 10 students who scored above 85 and 10 students who scored below 85. What was the average score?
- Aayesha is practising for a marathon. She ran for 50 minutes on Monday, 70 minutes on Wednesday, and 45 minutes on Friday. On Tuesday and Thursday, she lifted weights at the gym for 45 minutes each day. She swam for 45 minutes over the weekend. What was the average amount of time per day Aayesha spent running last week?

84. Observe the given data:

Days of the Week	Mon	Tues	Wed	Thurs	Fri	Sat
Number of Mobile Phone Sets Sold	50	45	30	55	27	60

- Draw a bar graph to represent the above given information.
- On which day of the week was the sales maximum?
- Find the total sales during the week.
- Find the ratio of the minimum sale to the maximum sale.
- Calculate the average sale during the week.
- On how many days of the week was the sale above the average sales?

- 85.** Below is a list of 10 tallest buildings in India.

This list ranks buildings in India that stand at least 150m (492 ft.) tall, based on standard height measurement. This includes spires and architectural details but does not include antenna marks. Following data is given as per the available information till 2009. Since new buildings are always under construction, go on-line to check new taller buildings.

Use the information given in the table about sky scrapers to answer the following questions:

Name	City	Height	Floors	Year
Planet	Mumbai	181m	51	2009
UB Tower	Bengaluru	184 m	20	2006
Ashok Towers	Mumbai	193 m	49	2009
The Imperial I	Mumbai	249 m	60	2009
The Imperial II	Mumbai	249 m	60	2009
RNA Mirage	Mumbai	180 m	40	2009
Oberoi Woods Tower I	Mumbai	170 m	40	2009
Oberoi Woods Tower II	Mumbai	170 m	40	2009
Oberoi Woods Tower III	Mumbai	170 m	40	2009
MVRDC	Mumbai	156 m	35	2002

- (a) Find the height of each storey of the three tallest buildings and write them in the following table:

Building	Height	Number of Storeys	Height of Each Storey

- (b) The average height of one storey for the buildings given in (a) is _____.
- (c) Which city in this list has the largest percentage of skyscrapers? What is the percentage?
- (d) What is the range of data?
- (e) Find the median of the data.
- (f) Draw a bar graph for given data.

86. The marks out of 100 obtained by Kunal and Soni in the Half Yearly Examination are given below:

Subjects	English	Hindi	Maths	Science	S. Science	Sanskrit
Kunal	72	81	92	96	64	85
Soni	86	89	90	82	75	82

- Draw a double bar graph by choosing appropriate scale.
 - Calculate the total percentage of marks obtained by Soni.
 - Calculate the total percentage of marks obtained by Kunal.
 - Compare the percentages of marks obtained by Kunal and Soni.
 - In how many subjects did Soni get more marks than Kunal? Which are those subjects?
 - Who got more marks in S. Science and what was the difference of marks?
 - In which subject the difference of marks was maximum and by how much?
87. The students of Class VII have to choose one club from Music, Dance, Yoga, Dramatics, Fine arts and Electronics clubs. The data given below shows the choices made by girls and boys of the class. Study the table and answer the questions that follow:

Clubs	Music	Dance	Yoga	Dramatics	Fine Arts	Electronics
Girls	15	24	10	19	27	21
Boys	12	16	8	17	11	30

- Draw a double bar graph using appropriate scale to depict the above data.
- How many students are there in Class VII?
- Which is the most preferred club by boys?
- Which is the least preferred club by girls?
- For which club the difference between boys and girls is the least?
- For which club is the difference between boys and girls the maximum?

- 88.** The data given below shows the production of motor bikes in a factory for some months of two consecutive years.

Months	Feb	May	August	October	December
2008	2700	3200	6000	5000	4200
2007	2800	4500	4800	4800	5200

Study the table given above and answer the following questions:

- Draw a double bar graph using appropriate scale to depict the above information and compare them.
 - In which year was the total output the maximum?
 - Find the mean production for the year 2007.
 - For which month was the difference between the production for the two years the maximum?
 - In which month for the year 2008, the production was the maximum?
 - In which month for the year 2007, the production was the least?
- 89.** The table below compares the population (in hundreds) of 4 towns over two years:

Towns	A	B	C	D
2007	2900	6400	8300	4600
2009	3200	7500	9200	6300

Study the table and answer the following questions:

- Draw a double bar graph using appropriate scale to depict the above information.
 - In which town was the population growth maximum?
 - In which town was the population growth least?
- 90.** The table below gives the data of tourists visiting 5 hill stations over two consecutive years. Study the table and answer the questions that follow:

Hill stations	Nainital	Shimla	Manali	Mussoorie	Kullu
2008	4000	5200	3700	5800	3500
2009	4800	4500	4200	6200	4600

- Draw a double bar graph to depict the above information using appropriate scale.

- (b) Which hill station was visited by the maximum number of tourists in 2008?
- (c) Which hill station was visited by the least number of tourists in 2009?
- (d) In which hill stations was there increase in number of tourists in the year 2009?

91. The table below gives the flavours of ice cream liked by children (boys and girls) of a society.

Flavours	Vanilla	Chocolate	Strawberry	Mango	Butterscotch
Boys	4	9	3	8	13
Girls	8	12	7	9	10

Study the table and answer the following questions:

- (a) Draw a double bar graph using appropriate scale to represent the above information.
- (b) Which flavour is liked the most by the boys?
- (c) How many girls are there in all?
- (d) How many children like chocolate flavour of ice cream?
- (e) Find the ratio of children who like strawberry flavour to vanilla flavour of ice cream.

(D) Applications

Application 1: Create a table like the one shown

Object	Estimate (in cm)	Measure (in cm)
Length of a pen		
Length of an eraser		
Length of your palm		
Length of your geometry box		
Length of your math notebook		

Draw a double bar graph for the above. How accurate are your estimations?

Application 2: The Body Mass Index (BMI) is a statistical measurement which compares an individual's weight and height. It is a very useful tool to estimate a healthy body weight based on how tall an individual is. Indeed, it is the most widely used tool to identify the weight problem. BMI is very easy to measure and evaluate. With the help of BMI, one can come to know whether one is underweight, normal weight, over weight or in the category of obesity. Its value is measured in kg/m^2 .

BMI of any individual is calculated with the help of the following formula :

$$\text{Body Mass Index (BMI)} = \left(\frac{\text{Body Weight}}{\text{Height} \times \text{Height}} \right)$$

Here the weight of the individual is measured in kilograms and the height of that individual is taken in metres.

The categories in BMI are given in the following table :

Category	BMI
Under weight	<18.5
Normal weight	18.5 – 24.9
Over weight	25.0 – 29.9
Obesity Class I	30.0 – 34.9
Obesity Class II	35.0 – 39.9
Obesity Class III	≥ 40

After having a glance at the table given above, one can come to know the category in which any individual falls. Now fill the table given below using the data for the children of your class :

Sl. No.	Name of Student	Body Weight (in kg)	Height (in mtrs)	Value (BMI)	Cateogry

Also draw a bar graph for the data received.

Game 3: Collect the data from students of your class about their favourite programmes on television and prepare a table as shown below:

Sl.No.	Programmes	Number of Girls	Number of Boys
1.	Cartoons		
2.	Serials		
3.	Reality shows		
4.	Songs		
5.	Movies		
6.	News		
7.	Others		

- Represent the above information on a double bar graph using appropriate scale.
- Study the graph and find out the favourite programme of the most of students.
- Which programme is liked by most of the boys?
- Name the programme for which difference between likings of the number of boys and girls is the maximum.
- Calculate the percentage of boys who like to watch News.
- Calculate the percentage of girls who like to watch Cartoons.

Game 4: Throw a die 20 times and record the outcomes in the following table:

No. on Die	1	2	3	4	5	6
No. of Times (Frequency)						

Calculate the probability of getting the following numbers using your recorded data:

- 6
- greater than 6
- 3 or 4

Write about your observation about the certainty of getting any particular number by throwing a die.

Cross Word Puzzle 5

Solve the given crossword and then fill up the given blanks and then boxes. Clues are given below for across as well as downward filling. Also for across and down clues. Clue number is written at the corner of boxes. Answers of clues have to fill up their respective boxes.

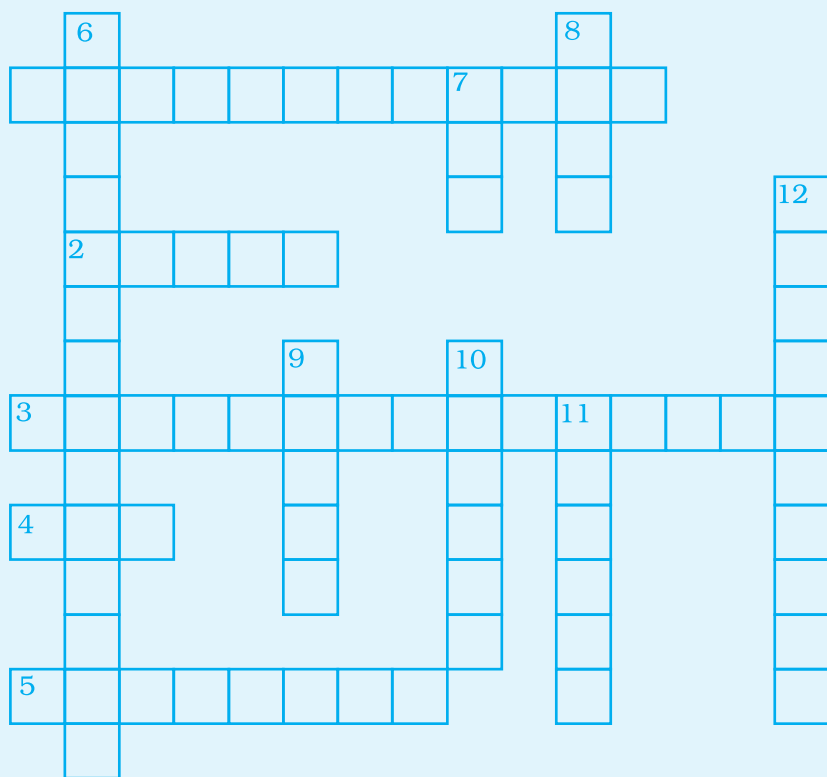
Clues

Across

1. Arranging the collected data in tabular form is called _____ of data.
2. Mean is defined as sum of all observations divided by _____ number of observations.
3. Mean, median and mode are collectively known as measures of _____.
4. Throwing a die gives _____ possible outcomes.
5. A _____ is the representation of data using bars of uniform width and varying heights.

Down

6. The most common representative value of a group of data is the _____.
7. Tossing a coin gives _____ outcomes.
8. The observation that occurs most often is called the _____.
9. The difference between the highest and lowest observations gives the _____.
10. _____ gives the middle observation of a given data.
11. A _____ bar graph helps in comparing two collections of data at a glance.
12. The number of times each observation occurs can be represented by _____.



UNIT 4

SIMPLE EQUATIONS

(A) Main Concepts and Results

- The word variable means something that can vary i.e., change and constant means that does not vary. The value of a variable is not fixed. Variables are denoted usually by letters of the English alphabets such as x , y , z , l , m , n , p , a etc.
- The expressions are formed by performing operations like addition, subtraction, multiplication and division on the variables and constants.
- An equation is a condition on a variable (or variables) such that two expressions in the variable (variables) have equal value.
- The value of the variable for which the equation is satisfied is called the solution or root of the equation.
- An equation remains the same if the LHS and the RHS are interchanged.
- In case of balanced equation if we (i) add the same number to both the sides, or (ii) subtract the same number from both the sides, or (iii) multiply both sides by the same non-zero number or (iv) divide both sides by the same non-zero number, the balance remains undisturbed.
- Transposing means moving from one side to the other. When a term is transposed from one side of the equation to the other side, its sign gets changed.
- Transposition of an expression can be carried out in the same way as the transposition of a term.

- To solve practical problems:
 - Read the problem carefully and denote the unknown quantity by variable x , y etc.
 - Form the equation according to the given conditions.
 - Solve the equation i.e., find the value of the unknown quantity (variable).

(B) Solved Examples

In Examples 1 to 3, there are four options, out of which one is correct. Choose the correct one.

Example 1: The solution of the equation $3x + 5 = 0$ is
 (a) $\frac{5}{3}$ (b) -5 (c) $-\frac{5}{3}$ (d) 5

Solution : Correct answer is (c).

Example 2: -1 is not a solution of the equation
 (a) $x + 1 = 0$ (b) $x - 1 = 2$ (c) $2y + 3 = 1$ (d) $2p + 7 = 5$

Solution : Correct answer is (b).

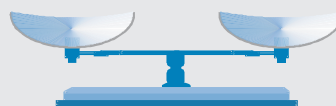
Example 3: Which of the following equations can be formed using the expression $x = 5$:

- (a) $2x + 3 = 13$ (b) $3x + 2 = 13$
 (c) $x - 5 = 1$ (d) $4x - 9 = 21$

Solution: Correct answer is (a).

[Hint: $x = 5$ on multiplying both sides by 2 gives $2x = 10$ which on adding 3 both sides gives $2x + 3 = 13$]

An **equation** is a mathematical sentence that uses an equality sign to show that two expressions have the same value. All of these are equations.



$$3 + 8 = 11$$

$$r + 6 = 14$$

$$-24 = x - 7$$

$$\frac{-100}{2} = -50$$

To solve an equation that contains a variable, find the value of the variable that makes the equation true. This value of the variable is called the solution of the equation.

In Examples 4 to 6, fill in the blanks to make it a true statement.

Example 4: Any value of the variable which makes both sides of an equation equal, is known as a _____ of the equation.

Solution: Solution

Example 5: The root of the equation $y - 13 = 9$ is _____.

Solution: 22

Example 6: $2x + \underline{\hspace{2cm}} = 11$ has the solution -4 .

Solution: 19

ADDITION PROPERTY OF EQUALITY

Words	Numbers	Algebra
You can add the same number to both sides of an equation, and the statement will still be true.	$\begin{array}{r} 2 + 3 = 5 \\ + 4 = +4 \\ \hline 2 + 7 = 9 \end{array}$	$\begin{array}{l} x = y \text{ implies} \\ x + z = y + z \end{array}$

In Examples 7 to 10, state whether the statements are True or False.

Example 7: 12 is a solution of the equation $4x - 5 = 3x + 10$.

Solution: False

$$[\text{LHS} = 4 \times 12 - 5 = 43]$$

$$\text{and RHS} = 3 \times 12 + 10 = 46 \text{ They are not equal.}]$$

Example 8: A number x divided by 7 gives 2 can be written as $\frac{x+1}{7} = 2$.

Solution: False.

Example 9: $x + 2 = 5$ and $3x - 1 = 8$ have the same solutions.

Solution: True

Example 10: The equation $3x + 7 = 10$ has 1 as its solution.

Solution: True

In each of the Examples 11 to 13, form an equation for each statement.

Example 11 : One fourth of a number is 20 less than the number itself.

Solution : Let the number be x .

So, one fourth of the number is $\frac{x}{4}$.

$\frac{x}{4}$ is 20 less than the number itself. So, the required equation is

$$\frac{x}{4} = x - 20.$$

Example 12 : On subtracting 13 from 3 times of a number, the result is 8.

Solution : Let the number be x .

So, 3 times the number = $3x$

On subtracting 13 from it, we get $3x - 13$.

Therefore, $3x - 13 = 8$ is the required equation.

Example 13 : Two times a number increased by 5 equals 9.

Solution : Let the required number be x .

So, 2 times this number = $2x$

When increased by 5, it gives the expression $2x + 5$

Thus, required equation is $2x + 5 = 9$.

Example 14 : 9 added to twice a number gives 13. Find the number.

Solution : Let the number be x .

As per the given condition,

$$2x + 9 = 13$$

$$\text{or } 2x = 4$$

$$\text{or } x = 2$$

PROBLEM SOLVING TIP

Always check the solution after solving equations.

Example 15 : 1 subtracted from one third of a number gives 1. Find the number.

Solution : Let the number be x .

According to the given condition,

$$\frac{1}{3}x - 1 = 1$$

$$\text{or } \frac{1}{3}x = 1 + 1$$

$$\text{or } \frac{1}{3}x = 2 \text{ or } x = 6.$$

Example 16: Correct the incorrect equation written in Roman numerals by moving only one tooth pick.

$$V - || = V|$$

Solution: By moving one tooth pick from numeral $||$, change the minus sign to plus, we get

$$V + | = V|$$

Example 16: Solve the riddle “What is too much fun for one, enough for two, and means nothing to three?” The answer to this is hidden in the equations given below.

$$\text{If } 4c = 16, \text{ then } c = ?$$

$$\text{If } 4e + 8 = 20, \text{ then } e = ?$$

$$\text{If } 2r - 3 = 7, \text{ then } r = ?$$

$$\text{If } 3t + 8 = 29, \text{ then } t = ?$$

$$\text{If } 2s + 4 = 4s, \text{ then } s = ?$$

To get the answer substitute the numbers for the letters it equals in the following:

$$\text{manner: } \frac{\square}{2}, \frac{\square}{3}, \frac{\square}{4}, \frac{\square}{5}, \frac{\boxed{e}}{\square}, \frac{\square}{7}$$

Solution : Solving the given equations:

$$\text{If } 4c = 16, \text{ we get } c = \frac{16}{4} = 4. \text{ Thus, } c = 4.$$

$$\text{If } 4e + 8 = 20, \text{ we get } 4e = 12 \text{ or } e = \frac{12}{4} = 3. \text{ Thus, } e = 3.$$

$$\text{If } 2r - 3 = 7, \text{ we get } 2r = 10 \text{ or } r = \frac{10}{2} = 5, \text{ i.e., } r = 5.$$

$$\text{If } 3t + 8 = 29, \text{ we get } 3t = 29 - 8$$

$$\text{or } 3t = 21, \text{ or } t = \frac{21}{3}, \text{ or } t = 7$$

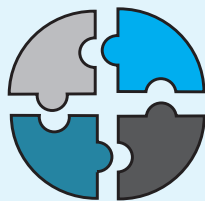
$$\text{If } 2s + 4 = 4s, \text{ we get } 4 = 4s - 2s$$

$$\text{or } 2s = 4 \text{ or } s = \frac{4}{2} \text{ or } s = 2.$$

Replacing the solutions by the corresponding letters we get

$$\frac{\boxed{s}}{2}, \frac{\boxed{e}}{3}, \frac{\boxed{c}}{4}, \frac{\boxed{r}}{5}, \frac{\boxed{e}}{\boxed{3}}, \frac{\boxed{t}}{7}$$

Application on Problem Solving Strategy



Example 18

Solve the following equation.

$$10 = 4 + 3(t + 2)$$

Solution :



Understand and Explore the Problem

- What do you know?
Solving an equation means to find value of the variable used in the equation.
Distributive property can be used to open the bracket of expression in RHS of the above equation.
Method of transposition can help in solving the equation
- To find value of 't' which satisfy the above equation.



Plan a Strategy

- What are the most appropriate steps to solve this equation?
First we should remove all the brackets appearing in the equation.
Solve and simplify the expression on one side of equation and then use method of transposition to collect terms with variable on one side and without variable on the other side of equation.



Solve

- Step 1 : $10 = 4 + 3(t + 2)$ [open the brackets]
- Step 2 : $10 = 4 + 3t + 6$ [simplify RHS]
- Step 3 : $10 = 10 + 3t$ [collect terms without variable on one side]
- Step 4 : $10 - 10 = 3t$
- Step 5 : $0 = 3t$
- Step 6 : $t = \frac{0}{3}$
i.e., $t = 0$



Revise

Solution of an equation can always be checked by substituting the value of variable and confirming whether LHS is equal to RHS or not

$$\text{LHS} = 10$$

$$\text{RHS} = 4 + 3(t + 2)$$

Substituting ' $t = 0$ '

$$= 4 + 3(0 + 2)$$

$$= 4 + 6$$

$$= 10 = \text{LHS}$$

Hence, $\text{LHS} = \text{RHS}$

Thus, ' $t = 0$ ' is the correct answer.

Think and Discuss

1. **Can** variable ' t ' take any other value also for same equation?
2. **Can** more equations have solution as ' $t = 0$ '?

(C) Exercise

In the Questions 1 to 18, there are four options out of which, one is correct. Choose the correct one.

1. The solution of the equation $ax + b = 0$ is

(a) $\frac{a}{b}$ (b) $-b$ (c) $-\frac{b}{a}$ (d) $\frac{b}{a}$

2. If a and b are positive integers, then the solution of the equation $ax = b$ will always be a

- (a) positive number (b) negative number
(c) 1 (d) 0

3. Which of the following is not allowed in a given equation?

- (a) Adding the same number to both sides of the equation.
(b) Subtracting the same number from both sides of the equation.

- (c) Multiplying both sides of the equation by the same non-zero number.
- (d) Dividing both sides of the equation by the same number.
4. The solution of which of the following equations is neither a fraction nor an integer?
- (a) $2x + 6 = 0$ (b) $3x - 5 = 0$
- (c) $5x - 8 = x + 4$ (d) $4x + 7 = x + 2$
5. The equation which cannot be solved in integers is
- (a) $5y - 3 = -18$ (b) $3x - 9 = 0$
- (c) $3z + 8 = 3 + z$ (d) $9y + 8 = 4y - 7$
6. If $7x + 4 = 25$, then x is equal to
- (a) $\frac{29}{7}$ (b) $\frac{100}{7}$ (c) 2 (d) 3
7. The solution of the equation $3x + 7 = -20$ is
- (a) $\frac{17}{7}$ (b) -9 (c) 9 (d) $\frac{13}{3}$
8. The value of y for which the expressions $(y - 15)$ and $(2y + 1)$ become equal is
- (a) 0 (b) 16 (c) 8 (d) -16

SUBTRACTION PROPERTY OF EQUALITY

Words	Numbers	Algebra
You can subtract the same number from both sides of an equation, and the statement will still be true.	$\begin{array}{r} 4 + 7 = 11 \\ - 3 = -3 \\ \hline 4 + 4 = 8 \end{array}$	$\begin{array}{l} x = y \text{ implies} \\ x - z = y - z \end{array}$

9. If $k + 7 = 16$, then the value of $8k - 72$ is
- (a) 0 (b) 1 (c) 112 (d) 56
10. If $43m = 0.086$, then the value of m is
- (a) 0.002 (b) 0.02 (c) 0.2 (d) 2

11. x exceeds 3 by 7, can be represented as
 (a) $x + 3 = 2$ (b) $x + 7 = 3$ (c) $x - 3 = 7$ (d) $x - 7 = 3$
12. The equation having 5 as a solution is:
 (a) $4x + 1 = 2$ (b) $3 - x = 8$ (c) $x - 5 = 3$ (d) $3 + x = 8$
13. The equation having -3 as a solution is:
 (a) $x + 3 = 1$ (b) $8 + 2x = 3$ (c) $10 + 3x = 1$ (d) $2x + 1 = 3$
14. Which of the following equations can be formed starting with $x = 0$?
 (a) $2x + 1 = -1$ (b) $\frac{x}{2} + 5 = 7$ (c) $3x - 1 = -1$ (d) $3x - 1 = 1$
15. Which of the following equations cannot be formed using the equation $x = 7$?
 (a) $2x + 1 = 15$ (b) $7x - 1 = 50$ (c) $x - 3 = 4$ (d) $\frac{x}{7} - 1 = 0$
16. If $\frac{x}{2} = 3$, then the value of $3x + 2$ is
 (a) 20 (b) 11 (c) $\frac{13}{2}$ (d) 8
17. Which of the following numbers satisfy the equation $-6 + x = -12$?
 (a) 2 (b) 6 (c) -6 (d) -2
18. Shifting one term from one side of an equation to another side with a change of sign is known as
 (a) commutativity (b) transposition
 (c) distributivity (d) associativity

Build Understanding

One-step equations can be solved by applying a single inverse operation. To solve two-step equations, apply more than one inverse operation.

The order of operations for $2x + 5 = 7$ is to start with x , multiply by 2 and add 5. The result is 7.

Order of Operations: x multiply by 2 add 5 7

To solve the equation, inverse the steps. Start with 7, subtract 5, then divide by 2 to find x .

Solve the equation: 7 subtract 5 divide by 2 x

In Questions 19 to 48, fill in the blanks to make the statements true.

- 19.** The sum of two numbers is 60 and their difference is 30.
- If smaller number is x , the other number is _____. (use sum)
 - The difference of numbers in term of x is _____.
 - The equation formed is _____.
 - The solution of the equation is _____.
 - The numbers are _____ and _____.

MULTIPLICATION PROPERTY OF EQUALITY

Words	Numbers	Algebra
Multiply both sides of an equation by the same non-zero number, and the statement will still be true.	$2 \times 3 = 6$ $4 \times 2 \times 3 = 4 \times 6$ $8 \times 3 = 24$	$x = y$ implies $zx = zy$ ($z \neq 0$)

- 20.** Sum of two numbers is 81. One is twice the other.
- If smaller number is x , the other number is _____.
 - The equation formed is _____.
 - The solution of the equation is _____.
 - The numbers are _____ and _____.
- 21.** In a test Abha gets twice the marks as that of Palak. Two times Abha's marks and three times Palak's marks make 280.
- If Palak gets x marks, Abha gets _____ marks.
 - The equation formed is _____.
 - The solution of the equation is _____.
 - Marks obtained by Abha are _____.
- 22.** The length of a rectangle is two times its breadth. Its perimeter is 60 cm.
- If the breadth of rectangle is x cm, the length of the rectangle is _____.

- (b) Perimeter in terms of x is _____.
- (c) The equation formed is _____.
- (d) The solution of the equation is _____.

23. In a bag there are 5 and 2 rupee coins. If they are equal in number and their worth is ₹ 70, then

- (a) The worth of x coins of ₹ 5 each _____.
- (b) The worth of x coins of ₹ 2 each _____.
- (c) The equation formed is _____.
- (d) There are _____ 5 rupee coins and _____ 2 rupee coins.

Think and Discuss

1. **Describe** how you would solve $4(x - 2) = 16$.
2. **Express** how to check your solution to an equation.

24. In a Mathematics quiz, 30 prizes consisting of 1st and 2nd prizes only are to be given. 1st and 2nd prizes are worth ₹ 2000 and ₹ 1000, respectively. If the total prize money is ₹ 52,000 then show that:

- (a) If 1st prizes are x in number the number of 2nd prizes are _____.
- (b) The total value of prizes in terms of x are _____.
- (c) The equation formed is _____.
- (d) The solution of the equation is _____.
- (e) The number of 1st prizes are _____ and the number of 2nd prizes are _____.

25. If $z + 3 = 5$, then $z =$ _____.

26. _____ is the solution of the equation $3x - 2 = 7$.

27. _____ is the solution of $3x + 10 = 7$.

28. If $2x + 3 = 5$, then value of $3x + 2$ is _____.

29. In integers, $4x - 1 = 8$ has _____ solution.

DIVISION PROPERTY OF EQUALITY

Words	Numbers	Algebra
You can divide both sides of an equation by the same non-zero number, and the statement will still be true.	$4 \times 3 = 12$ $\frac{4 \times 3}{2} = \frac{12}{2}$ $\frac{12}{2} = 6$	$x = y$, implies $\frac{x}{z} = \frac{y}{z}, z \neq 0$

30. In natural numbers, $4x + 5 = -7$ has _____ solution.
31. In natural numbers, $x - 5 = -5$ has _____ solution.
32. In whole numbers, $x + 8 = 12 - 4$ has _____ solution.
33. If 5 is added to three times a number, it becomes the same as 7 is subtracted from four times the same number. This fact can be represented as _____.
34. $x + 7 = 10$ has the solution _____.
35. $x - 0 =$ _____ ; when $3x = 12$.
36. $x - 1 =$ _____ ; when $2x = 2$.
37. $x -$ _____ $= 15$; when $\frac{x}{2} = 6$.
38. The solution of the equation $x + 15 = 19$ is _____.
39. Finding the value of a variable in a linear equation that _____ the equation is called a _____ of the equation.
40. Any term of an equation may be transposed from one side of the equation to the other side of the equation by changing the _____ of the term.
41. If $\frac{9}{5}x = \frac{18}{5}$, then $x =$ _____.
42. If $3 - x = -4$, then $x =$ _____.
43. If $x - \frac{1}{2} = -\frac{1}{2}$, then $x =$ _____.

44. If $\frac{1}{6} - x = \frac{1}{6}$, then $x =$ _____.
45. If 10 less than a number is 65, then the number is _____.
46. If a number is increased by 20, it becomes 45. Then the number is _____.
47. If 84 exceeds another number by 12, then the other number is _____.
48. If $x - \frac{7}{8} = \frac{7}{8}$, then $x =$ _____.

Think and Discuss

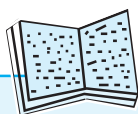
- Give** two words or phrases that can be used to express each operation: addition, subtraction, multiplication, and division.
- Express** $5 + 7n$ in words in at least two different ways.

In Questions 49 to 55, state whether the statements are True or False.

49. 5 is the solution of the equation $3x + 2 = 17$.
50. $\frac{9}{5}$ is the solution of the equation $4x - 1 = 8$.
51. $4x - 5 = 7$ does not have an integer as its solution.
52. One third of a number added to itself gives 10, can be represented as $\frac{x}{3} + 10 = x$.
53. $\frac{3}{2}$ is the solution of the equation $8x - 5 = 7$.
54. If $4x - 7 = 11$, then $x = 4$.
55. If 9 is the solution of variable x in the equation $\frac{5x - 7}{2} = y$, then the value of y is 28.

56. Match each of the entries in Column I with the appropriate entries in Column II.

Column I	Column II
(i) $x + 5 = 9$	(A) $-\frac{5}{3}$
(ii) $x - 7 = 4$	(B) $\frac{5}{3}$
(iii) $\frac{x}{12} = -5$	(C) 4
(iv) $5x = 30$	(D) 6
(v) The value of y which satisfies $3y = 5$	(E) 11
(vi) If $p = 2$, then the value of $\frac{1}{3}(1 - 3p)$	(F) -60
	(G) 3



Vocabulary

To become familiar with some of the vocabulary terms consider the following:

1. The word constant means “unchanging.” What do you think a **constant** in mathematics refers to?
2. The word **equation** looks like the word equal, which means “having the same value.” How do you think this meaning applies to an equation?
3. The word **inequality** begins with the prefix ‘in’ which means “not”, and has the same root as the word equation. Together, what do you think the prefix and root mean?
4. The word ‘vary’ which is the root of **variable** means “to change.” How do you think this applies to mathematics?

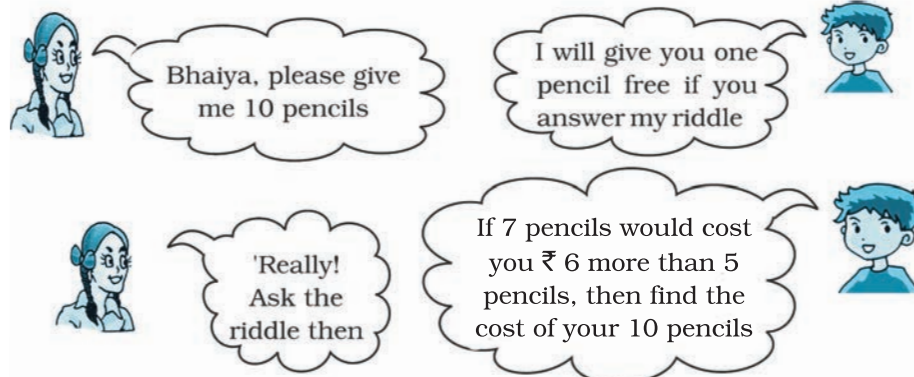
In Questions 57 to 67, express each of the given statements as an equation.

- 57.** 13 subtracted from twice of a number gives 3 .
- 58.** One-fifth of a number is 5 less than that number.
- 59.** A number is 7 more than one-third of itself.
- 60.** Six times a number is 10 more than the number.
- 61.** If 10 is subtracted from half of a number, the result is 4.
- 62.** Subtracting 5 from p , the result is 2.
- 63.** Five times a number increased by 7 is 27.
- 64.** Mohan is 3 years older than Sohan. The sum of their ages is 43 years.
- 65.** If 1 is subtracted from a number and the difference is multiplied by $\frac{1}{2}$, the result is 7.
- 66.** A number divided by 2 and then increased by 5 is 9.
- 67.** The sum of twice a number and 4 is 18.
- 68.** The age of Sohan Lal is four times that of his son Amit. If the difference of their ages is 27 years, find the age of Amit.
- 69.** A number exceeds the other number by 12. If their sum is 72, find the numbers.
- 70.** Seven times a number is 12 less than thirteen times the same number. Find the number.
- 71.** The interest received by Karim is ₹ 30 more than that of Ramesh. If the total interest received by them is ₹ 70, find the interest received by Ramesh.
- 72.** Subramaniam and Naidu donate some money in a Relief Fund. The amount paid by Naidu is ₹ 125 more than that of Subramaniam. If the total money paid by them is ₹ 975, find the amount of money donated by Subramaniam.

- 73.** In a school, the number of girls is 50 more than the number of boys. The total number of students is 1070. Find the number of girls.
- 74.** Two times a number increased by 5 equals 9. Find the number.
- 75.** 9 added to twice a number gives 13. Find the number.
- 76.** 1 subtracted from one-third of a number gives 1. Find the number.
- 77.** After 25 years, Rama will be 5 times as old as he is now. Find his present age.
- 78.** After 20 years, Manoj will be 5 times as old as he is now. Find his present age.
- 79.** My younger sister's age today is 3 times, what it will be 3 years from now minus 3 times what her age was 3 years ago. Find her present age.
- 80.** If 45 is added to half a number, the result is triple the number. Find the number.
- 81.** In a family, the consumption of wheat is 4 times that of rice. The total consumption of the two cereals is 80 kg. Find the quantities of rice and wheat consumed in the family.
- 82.** In a bag, the number of one rupee coins is three times the number of two rupees coins. If the worth of the coins is ₹ 120, find the number of 1 rupee coins.
- 83.** Anamika thought of a number. She multiplied it by 2, added 5 to the product and obtained 17 as the result. What is the number she had thought of ?
- 84.** One of the two numbers is twice the other. The sum of the numbers is 12. Find the numbers.
- 85.** The sum of three consecutive integers is 5 more than the smallest of the integers. Find the integers.
- 86.** A number when divided by 6 gives the quotient 6. What is the number?
- 87.** The perimeter of a rectangle is 40m. The length of the rectangle is 4 m less than 5 times its breadth. Find the length of the rectangle.

88. Each of the 2 equal sides of an isosceles triangle is twice as large as the third side. If the perimeter of the triangle is 30 cm, find the length of each side of the triangle.
89. The sum of two consecutive multiples of 2 is 18. Find the numbers.
90. Two complementary angles differ by 20° . Find the angles.
91. 150 has been divided into two parts such that twice the first part is equal to the second part. Find the parts.
92. In a class of 60 students, the number of girls is one third the number of boys. Find the number of girls and boys in the class.
93. Two-third of a number is greater than one-third of the number by 3. Find the number.
94. A number is as much greater than 27 as it is less than 73. Find the number.
95. A man travelled two fifth of his journey by train, one-third by bus, one-fourth by car and the remaining 3 km on foot. What is the length of his total journey?
96. Twice a number added to half of itself equals 24. Find the number.
97. Thrice a number decreased by 5 exceeds twice the number by 1. Find the number.
98. A girl is 28 years younger than her father. The sum of their ages is 50 years. Find the ages of the girl and her father.
99. The length of a rectangle is two times its width. The perimeter of the rectangle is 180 cm. Find the dimensions of the rectangle.
100. Look at this riddle?

If she answers the riddle correctly how ever will she pay for the pencils?



- 101.** In a certain examination, a total of 3768 students secured first division in the years 2006 and 2007. The number of first division in 2007 exceeded those in 2006 by 34. How many students got first division in 2006?
- 102.** Radha got ₹ 17,480 as her monthly salary and over-time. Her salary exceeds the over-time by ₹ 10,000. What is her monthly salary ?
- 103.** If one side of a square is represented by $18x - 20$ and the adjacent side is represented by $42 - 13x$, find the length of the side of the square.
- 104.** Follow the directions and correct the given incorrect equation, written in Roman numerals:

- (a) Remove two of these matchsticks to make a valid equation:

$$IX - VI = V$$

- (b) Move one matchstick to make the equation valid. Find two different solutions.

$$VI - IV = XI$$

- 105.** What does a duck do when it flies upside down? The answer to this riddle is hidden in the equation given below:

$$\text{If } i + 69 = 70, \text{ then } i = ? \quad \text{If } 8u = 6u + 8, \text{ then } u = ?$$

$$\text{If } 4a = -5a + 45, \text{ then } a = ? \quad \text{if } 4q + 5 = 17, \text{ then } q = ?$$

$$\text{If } -5t - 60 = -70, \text{ then } t = ? \quad \text{If } \frac{1}{4}s + 98 = 100, \text{ then } s = ?$$

$$\text{If } \frac{5}{3}p + 9 = 24, \text{ then } p = \underline{\hspace{2cm}}?$$

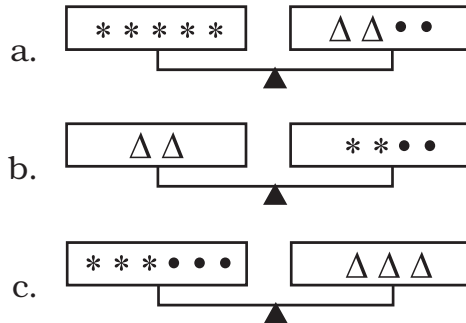
$$\text{If } 3c = c + 12, \text{ then } c = \underline{\hspace{2cm}}?$$

$$\text{If } 3(k + 1) = 24, \text{ then } k = \underline{\hspace{2cm}}?$$

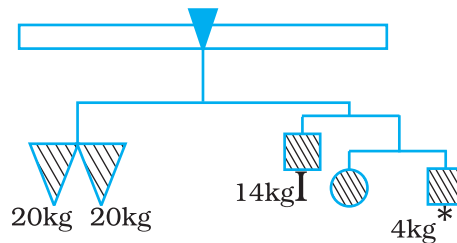
For riddle answer : substitute the number for the letter it equals

$$\begin{array}{cccccccccc} _ & _ & / & _ & _ & _ & _ & _ & / & _ & _ \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 4 & 9 \end{array}$$

- 106.** The three scales below are perfectly balanced if $\bullet = 3$. What are the values of Δ and $*$?



- 107.** The given figure represents a weighing balance. The weights of some objects in the balance are given. Find the weight of each square and the circle.



(D) Application

1. Crossword Number Puzzle

Get cracking on the following questions to fill the crossword puzzle as per mentioned clues of down and across. Clue number is written at the corner of boxes. Answers of Clue have to be filled up in their respective boxes.

- Down 1:** I spent one third of my sleeping time while dreaming. If I dreamt for 3 hours, then how long did I sleep?
- Down 2:** I ran around three sides of a square park whose perimeter is 200 m. How far am I from the starting point?
- Down 4:** I purchased three sarees and was left with ₹ 1000 out of my savings of ₹ 10000. How much is each saree worth?
- Down 8:** I have 4 coins worth 50 paise each and a few coins of ₹ 1 each. If I have ₹ 45 in total, how many coins of ₹ 1 do I possess?

Down 9: The unequal angle of an isosceles triangle measures 12° . How much is each of the remaining angles?

Down 11: For what value of y is $3(y - 1) + 7 = 40$?

Across 3: Out of 40 chocolates, Ram and I shared in the ratio 1 : 3. How many chocolates did Ram get?

Across 5: Sum of two consecutive numbers is 111. What is the smaller number of the two?

Across 7: Out of a flock of birds, half flew away while one got injured, if 244 remain, then how many did we begin with?

Across 10: If $2x + 7 = 1573$, then $x = ?$

Across 12: What value of z satisfies

$$\frac{2}{5}z + 8 = 58?$$

6					
1			10		4
	5	2			
	3		9		
8			7		
		11			
	12				

2. Crossword Puzzle

Fill the following crossword puzzle as per the mentioned clues of down and across. Clue's number is written at the corner of boxes. Answers of Clues have to fill up in their respective boxes.

Down 1: A mathematical statement with two expressions that have same value.

Down 2: The property that states $a(b + c) = ab + ac$

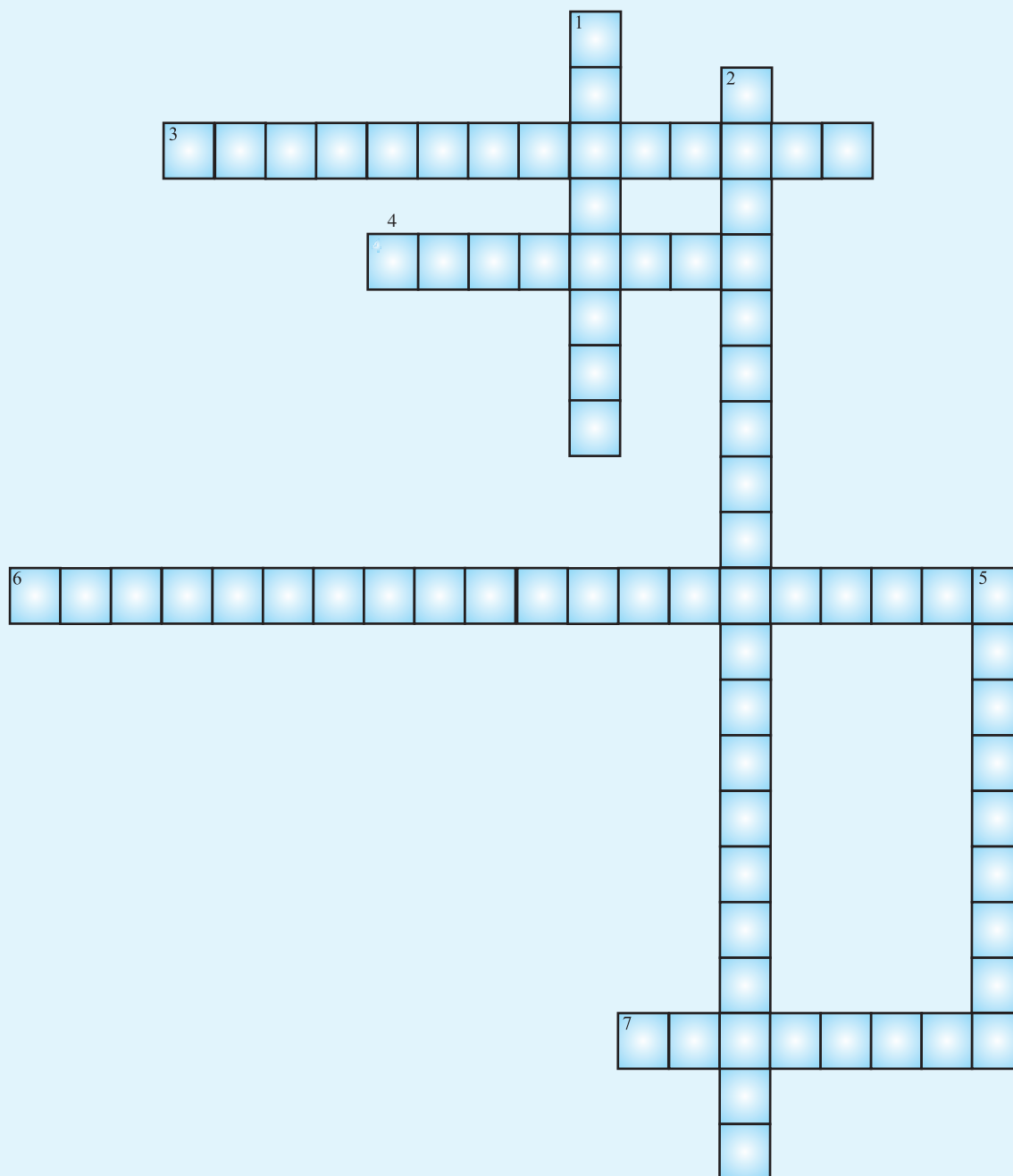
Down 5: An operation that undoes another operation.

Across 3: The expression which can be formed by performing mathematical operations on variables and constants.

Across 4: A number that does not change.

Across 6: A number that multiply the variable.

Across 7: A letter that represents an unknown number.



3.Game Time

There are nine identical looking pearl. Eight are real and one is fake. Using a balance scale that consists of two pans, you must find the fake pearl.



The real pearls weigh the same and the fake weighs less. Also, the scale can be used maximum twice.

Now find the Phony!

[**Hint:** Divide the pearls into three equal groups and then proceed for weighing.]

UNIT 5

LINES AND ANGLES

(A) Main Concepts and Results

- An angle is formed when two lines or rays or line segments meet or intersect.
- When the sum of the measures of two angles is 90° , the angles are called complementary angles. Each of them is called complement of the other.
- When the sum of the measures of two angles is 180° , the angles are called supplementary angles. Each of them is called supplement of the other.
- Two angles are called adjacent angles, if they have a common vertex and a common arm but no common interior points.
- A linear pair is a pair of adjacent angles whose non-common sides are opposite rays.
- When two lines intersect, the vertically opposite angles so formed are equal.
- When two lines are intersected by a transversal, eight angles are formed. These angles can be classified as 4 interior angles, 4 exterior angles, 4 pairs of corresponding angles, 2 pairs of alternate interior angles, 2 pairs of alternate exterior angles and two pairs of interior angles on the same side of the transversal.
- If two parallel lines are intersected by a transversal,
 - (i) each pair of corresponding angles is equal.
 - (ii) each pair of alternate interior angles is equal.
 - (iii) each pair of interior angles on the same side of the transversal is supplementary.
- Converses of the above results are also true.

(B) Solved Examples

In each of the Examples 1 to 4, there are four options, out of which one option is correct. Write the correct one.

Example 1: The angles between North and East and North and West are

- (a) complementary angles
- (b) supplementary angles
- (c) both acute angles
- (d) both obtuse angles

Solution: Correct answer is (b).

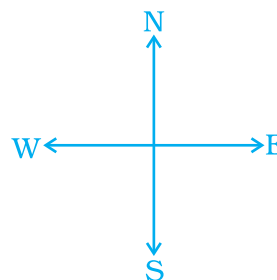


Fig. 5.1

Example 2: Which of the following pair of angles are supplementary?

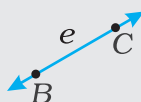
- (a) 48° , 42°
- (b) 60° , 60°
- (c) 75° , 105°
- (d) 179° , 2°

A point name a location.

$\bullet A$

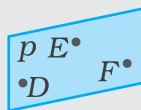
point A

A line is perfectly straight and extends for ever in both directions.



line e , or \overleftrightarrow{BC}

A plane is a perfectly flat surface that extends forever in all directions.



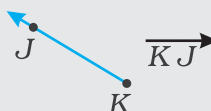
plane p , or plane DEF

A segment, or line segment, is the part of a line between two points.



\overline{GH}

A ray is part of a line that starts at one point and extends for ever in one direction.



\overrightarrow{KJ}

Solution: Correct answer is (c).

Example 3: In Fig. 5.2, a pair of corresponding angles is

- (a) $\angle 1, \angle 2$
- (b) $\angle 3, \angle 6$
- (c) $\angle 3, \angle 5$
- (d) $\angle 3, \angle 7$

Solution: Correct answer is (d).

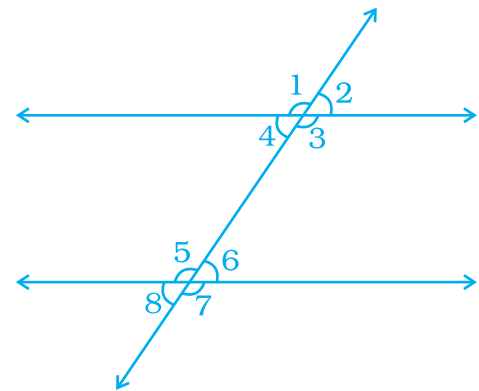


Fig. 5.2

Example 4: If two lines are intersected by a transversal, then the number of pairs of interior angles on the same side of the transversal is

- (a) 1
- (b) 2
- (c) 3
- (d) 4

Solution: Correct answer is (b).

An **angle** (\angle) is formed by two rays with a common endpoint called the *vertex* (plural, *vertices*). Angles can be measured in degrees, $m\angle 1$ means the measure of $\angle 1$. The angles can be named $\angle XYZ$, $\angle 1$, or $\angle Y$. The vertex must be the middle letter.

In Examples 5 to 7, fill in the blanks to make the statements true.

Example 5 : Two lines in a plane which never meet at any point are called _____.

Solution: parallel lines

Example 6: Angles of a linear pair are _____ as well as _____.

Solution: adjacent, supplementary

Example 7: Adjacent angles have a common vertex, a common _____ and no-common _____.

Solution: arm, interior points

In Examples 8 to 11, state whether the statements are True or False.

Example 8: Sum of two complementary angles is 180° .

Solution: False

Example 9: Sum of two supplementary angles is 180° .

Solution: True

Example 10: Sum of interior angles on the same side of a transversal with two parallel lines is 90° .

Solution: False

Example 11: Vertically opposite angles are equal.

Solution: True

Example 12: In Fig. 5.3, four line segments PQ, QR, RS and ST are making the letter W, $PQ \parallel RS$ and $QR \parallel ST$. If angle between PQ and QR is 39° , find the values of x and y .

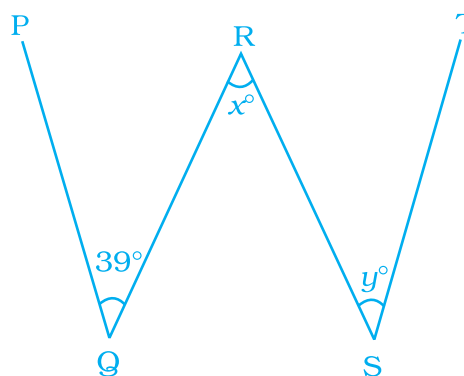


Fig. 5.3

Solution: Since $PQ \parallel RS$ and QR is transversal, so

$$x = 39^\circ \quad [\text{Alternate interior angles}]$$

Again $QR \parallel ST$ and RS is a transversal.

$$\text{Therefore, } y = x \quad [\text{Alternate interior angles}]$$

$$\text{or } y = 39^\circ$$

Example 13: In Fig. 5.4, are the angles 1 and 2 of the letter N forming a pair of adjacent angles? Give reasons.

Solution: No, $\angle 1$ and $\angle 2$ are not forming a pair of adjacent angles as they do not have a common vertex.

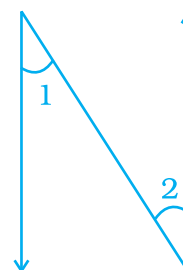


Fig. 5.4

Example 14: In Fig. 5.5, the points A, O and B are collinear. Ray OC \perp ray OD. Check whether

- (i) $\angle AOD$ and $\angle BOC$ are complementary,
 (ii) $\angle AOC$ and $\angle BOC$ are supplementary.

Solution:

Since points A, O and B are collinear (Given), therefore AB is a straight line.

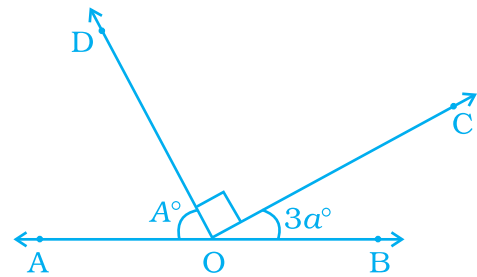


Fig. 5.5

- (i) As O is a point on the line AB, therefore $\angle AOD + \angle DOC + \angle BOC = 180^\circ$
 or, $\angle AOD + \angle BOC + 90^\circ = 180^\circ$
 or, $\angle AOD + \angle BOC = 90^\circ$
 So, $\angle AOD$ and $\angle BOC$ are complementary angles.
- (ii) Also, $\angle AOC$ and $\angle BOC$ are supplementary as $\angle AOC + \angle BOC = 180^\circ$

A **right angle** measures 90° . An **acute angle** measures greater than 0° and less than 90° . An **obtuse angle** measures greater than 90° and less than 180° . **Complementary angles** are two angles whose measures add to 90° . **Supplementary angles** are two angles whose measures add to 180° .

Example 15: In Fig. 5.6 $AB \parallel EF$, $ED \parallel CB$ and $\angle APE$ is 39° . Find $\angle CQF$.

Solution:

Since $ED \parallel BC$ and AB is a transversal, so

so $\angle QBP = \angle APE$
 [Corresponding angles]

or $\angle QBP = 39^\circ$

Now, $AB \parallel EF$ and BC is a transversal.

Therefore, $\angle FQB = \angle QBP$
 [Alternate interior angles]

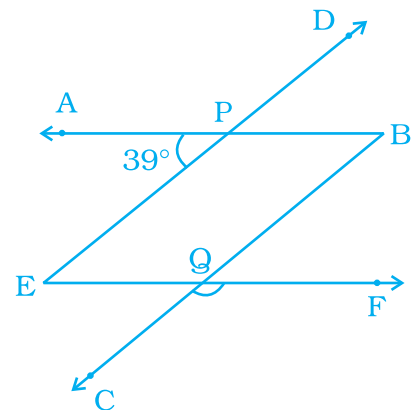


Fig. 5.6

$$\text{or } \angle FQB = 39^\circ$$

$$\text{Also, } \angle CQF + \angle FQB = 180^\circ \quad [\text{Linear pair}]$$

$$\text{So } \angle CQF + 39^\circ = 180^\circ$$

$$\text{or } \angle CQF = 180^\circ - 39^\circ$$

$$\text{or } \angle CQF = 141^\circ$$

Example 16: Out of a pair of complementary angles, one is two-third of the other. Find the angles.

Solution: Let one angle be x .

$$\text{So, other angle} = 90^\circ - x$$

$$\text{Thus, } \frac{2}{3} \times x = 90^\circ - x$$

$$\text{or } 2x = 270^\circ - 3x$$

$$\text{or } 2x + 3x = 270^\circ$$

$$\text{or } 5x = 270^\circ$$

$$\text{or } x = \frac{270^\circ}{5} = 54^\circ$$

$$\text{So, one angle} = 54^\circ \text{ and the other angle} = 90^\circ - 54^\circ = 36^\circ.$$

Congruent figures have the same size and same shape.

- Segments that have the same length are congruent.
- Angles that have the same measure are congruent.
- The symbol for congruence is \cong , which is read as “is congruent to.”

Example 17: In Fig. 5.7, CD intersects the line AB at F, $\angle CFB = 50^\circ$ and $\angle EFA = \angle AFD$. Find the measure of $\angle EFC$.

Solution: Let $\angle EFA = x$.

$$\text{Then } \angle AFD = x.$$

It is given that CD intersects line AB at F.

Therefore, $\angle CFB = \angle AFD$
(Vertically opposite angles)

$$\text{So, } x = 50^\circ$$

$$\text{But } \angle EFA = \angle AFD \text{ which gives } \angle EFA = 50^\circ$$

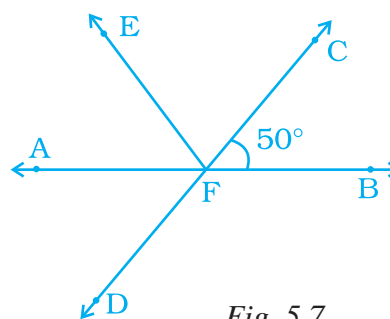


Fig. 5.7

Now $\angle CFB + \angle EFA + \angle EFC = 180^\circ$ [As AB is a straight line].

$$\text{or, } 50^\circ + 50^\circ + \angle EFC = 180^\circ$$

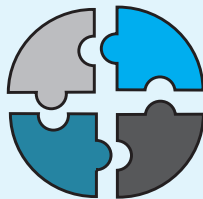
$$\text{or, } \angle EFC = 180^\circ - 100^\circ$$

$$\text{Thus, } \angle EFC = 80^\circ.$$

Think and Discuss

- Tell** which statements are correct: If $\angle X$ and $\angle Y$ are congruent,
 a. $\angle X = \angle Y$ b. $m\angle X = m\angle Y$ c. $\angle X \cong \angle Y$.
- Explain** why vertically opposite angles must always be congruent.

Application on Problem Solving Strategy



Example 18

In the given figure, find out which pair of lines are parallel.

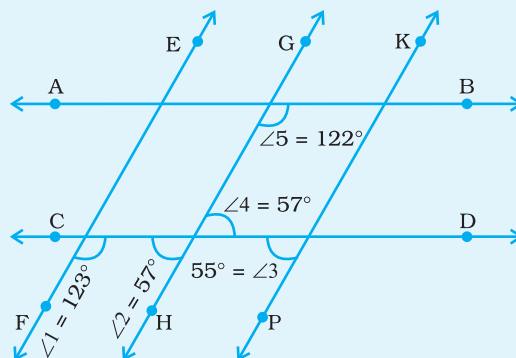


Fig. 5.8

Solution:



Understand and Explore the Problem

- What information is given in the question?

Lines AB and CD are intersecting three lines EF, GH and KP at distinct points forming angles $\angle 1 = 123^\circ$, $\angle 2 = 57^\circ$, $\angle 3 = 55^\circ$ and $\angle 5 = 122^\circ$.

- What are you trying to find?

We are trying to find

(a) $EF \parallel GH$ or not

- (b) $GH \parallel KP$ or not
- (c) $EF \parallel KP$ or not
- (d) $AB \parallel CD$ or not



Plan a Strategy

- (a) Since we want to find whether the lines are parallel or not, therefore recall the conditions when the lines are parallel.

The lines are parallel if it satisfies any one of the following,

- (1) when corresponding angles are equal
 - (2) when alternate interior angles are equal
 - (3) when the sum of interior angles on the same side of the transversal is 180° .
- (b) Find out what type of angles are formed by lines EF, GH, KP taking AB or CD as transversal.



Solve

- For lines EF and GH, taking CD as transversal, $\angle 1$ and $\angle 2$ are interior angles on the same side of the transversal. Therefore, we check whether the sum of $\angle 1$ and $\angle 2$ is 180° or not.

$$\angle 1 = 123^\circ, \angle 2 = 57^\circ, \angle 1 + \angle 2 = 123^\circ + 57^\circ = 180^\circ$$

Since the sum of interior \angle 's on the same side of the transversal is 180° , therefore $EF \parallel GH$.

- For lines GH and KP, taking CD as transversal, $\angle 2$ and $\angle 3$ are corresponding \angle 's. If these angles are equal, then lines are parallel.

$$\angle 2 = 57^\circ, \angle 3 = 55^\circ$$

$\angle 2 \neq \angle 3$. Since corresponding angles are not equal, therefore, GH is not parallel to KP.

- Similarly, for lines EF and KP, taking CD as transversal $\angle 1$ and $\angle 3$ are interior angles on the same side of the transversal.

$$\angle 1 = 123^\circ, \angle 3 = 55^\circ, \angle 1 + \angle 3 = 123^\circ + 55^\circ = 178^\circ. \text{ Since the sum is not equal to } 180^\circ,$$

therefore EF is not parallel to KP.

- For lines AB and CD, taking GH as a transversal $\angle 2 = \angle 4 = 57^\circ$ (vertically opp. \angle 's).

$\angle 5$ and $\angle 4$ are interior angles on the same side of the

transversal and $\angle 5 + \angle 4 = 122^\circ + 57^\circ = 179^\circ \neq 180^\circ$.
Therefore, AB is not parallel to CD.



Revise

- EF||GH, since sum of interior \angle 's on the same side of transversal is 180° .
- GH is not parallel to KP, since corresponding angles formed are not equal.
- EF is not parallel to KP, since the sum of interior \angle 's on the same side of the transversal is not equal to 180° .
- AB is not parallel to CD, since the sum of interior \angle 's on the same side of the transversal is not equal to 180° .

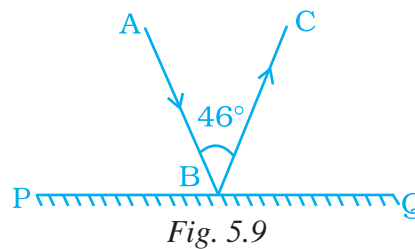
Think and Discuss

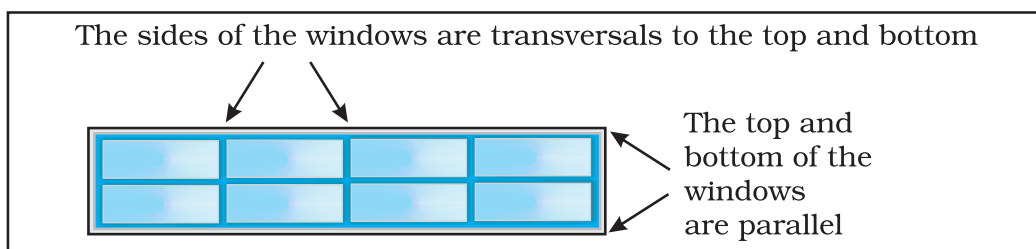
1. **Can** you find whether the lines EF, GH, KP, AB and CD are parallel or not by using other conditions of parallel lines?
2. **Discuss** with your classmates regarding their method towards this problem.

(C) Exercise

In questions 1 to 41, there are four options out of which one is correct. Write the correct one.

1. The angles between North and West and South and East are
(a) complementary (b) supplementary
(c) both are acute (d) both are obtuse
2. Angles between South and West and South and East are
(a) vertically opposite angles
(b) complementary angles
(c) making a linear pair
(d) adjacent but not supplementary
3. In Fig. 5.9, PQ is a mirror, AB is the incident ray and BC is the reflected ray. If $\angle ABC = 46^\circ$, then $\angle ABP$ is equal to
(a) 44° (b) 67°
(c) 13° (d) 62°





4. If the complement of an angle is 79° , then the angle will be of
 (a) 1° (b) 11° (c) 79° (d) 101°
5. Angles which are both supplementary and vertically opposite are
 (a) $95^\circ, 85^\circ$ (b) $90^\circ, 90^\circ$ (c) $100^\circ, 80^\circ$ (d) $45^\circ, 45^\circ$
6. The angle which makes a linear pair with an angle of 61° is of
 (a) 29° (b) 61° (c) 122° (d) 119°
7. The angles x and $90^\circ - x$ are
 (a) supplementary (b) complementary
 (c) vertically opposite (d) making a linear pair
8. The angles $x - 10^\circ$ and $190^\circ - x$ are
 (a) interior angles on the same side of the transversal
 (b) making a linear pair
 (c) complementary
 (d) supplementary
9. In Fig. 5.10, the value of x is
 (a) 110° (b) 46°
 (c) 64° (d) 150°
10. In Fig. 5.11, if $AB \parallel CD$, $\angle APQ = 50^\circ$ and $\angle PRD = 130^\circ$, then $\angle QPR$ is
 (a) 130° (b) 50°
 (c) 80° (d) 30°

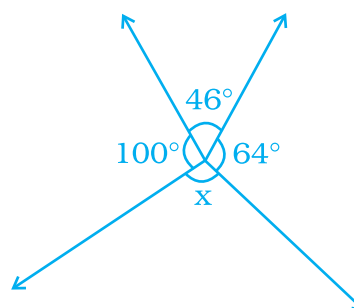


Fig. 5.10

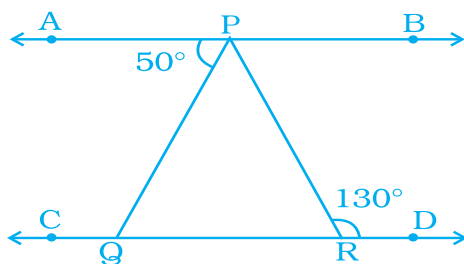


Fig. 5.11

11. In Fig. 5.12, lines l and m intersect each other at a point. Which of the following is false?

- (a) $\angle a = \angle b$ (b) $\angle d = \angle c$
(c) $\angle a + \angle d = 180^\circ$ (d) $\angle a = \angle d$

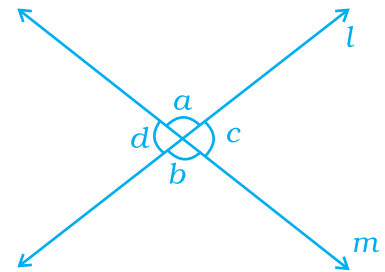


Fig. 5.12

12. If angle P and angle Q are supplementary and the measure of angle P is 60° , then the measure of angle Q is

- (a) 120° (b) 60°
(c) 30° (d) 20°

13. In Fig. 5.13, POR is a line. The value of a is

- (a) 40° (b) 45°
(c) 55° (d) 60°

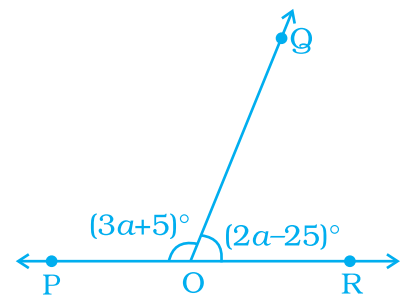


Fig. 5.13

14. In Fig. 5.14, POQ is a line. If $x = 30^\circ$, then $\angle QOR$ is

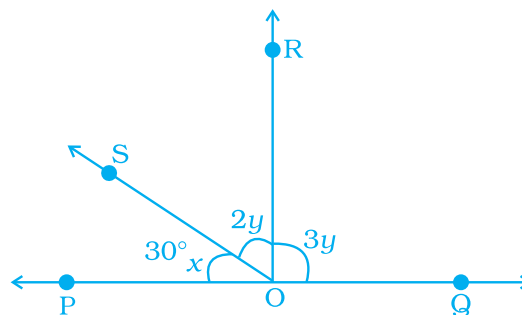


Fig. 5.14

- (a) 90° (b) 30°
(c) 150° (d) 60°

PROPERTIES OF TRANSVERALS TO PARALLEL LINES

If two parallel lines are intersected by a transversal, corresponding angles are congruent,

- alternate interior angles are congruent
- and alternate exterior angles are congruent.

If the transversal is perpendicular to the parallel lines, all of the angles formed are congruent to 90° angles.

15. The measure of an angle which is four times its supplement is

(a) 36° (b) 144°
(c) 16° (d) 64°

16. In Fig. 5.15, the value of y is

(a) 30° (b) 15°
(c) 20° (d) 22.5°

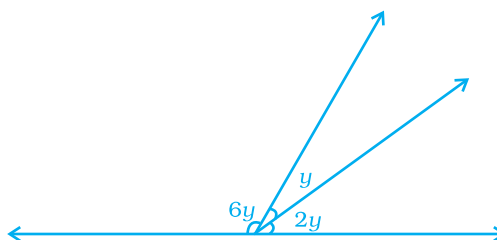


Fig. 5.15

17. In Fig. 5.16, $PA \parallel BC \parallel DT$ and $AB \parallel DC$. Then, the values of a and b are respectively.

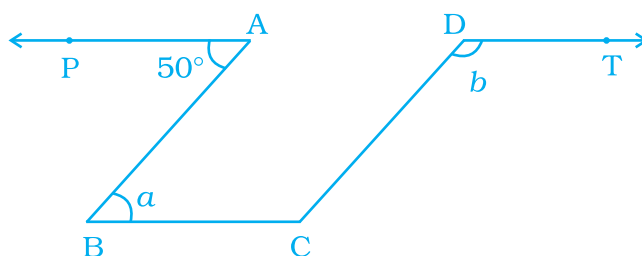


Fig. 5.16

(a) $60^\circ, 120^\circ$ (b) $50^\circ, 130^\circ$ (c) $70^\circ, 110^\circ$ (d) $80^\circ, 100^\circ$

18. The difference of two complementary angles is 30° . Then, the angles are

(a) $60^\circ, 30^\circ$ (b) $70^\circ, 40^\circ$
(c) $20^\circ, 50^\circ$ (d) $105^\circ, 75^\circ$

19. In Fig. 5.17, $PQ \parallel SR$ and $SP \parallel RQ$. Then, angles a and b are respectively

(a) $20^\circ, 50^\circ$ (b) $50^\circ, 20^\circ$
(c) $30^\circ, 50^\circ$ (d) $45^\circ, 35^\circ$

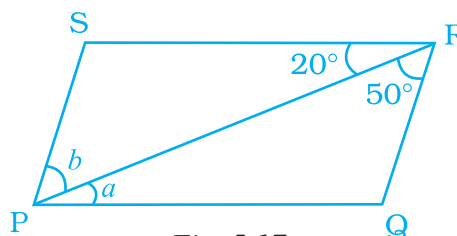


Fig. 5.17

20. In Fig. 5.18, a and b are

(a) alternate exterior angles
(b) corresponding angles
(c) alternate interior angles
(d) vertically opposite angles

21. If two supplementary angles are in the ratio $1 : 2$, then the bigger angle is

(a) 120° (b) 125°
(c) 110° (d) 90°

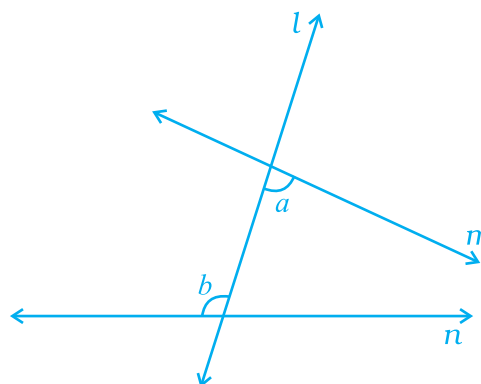


Fig. 5.18

22. In Fig. 5.19, $\angle ROS$ is a right angle and $\angle POR$ and $\angle QOS$ are in the ratio 1 : 5. Then, $\angle QOS$ measures

- (a) 150° (b) 75°
(c) 45° (d) 60°

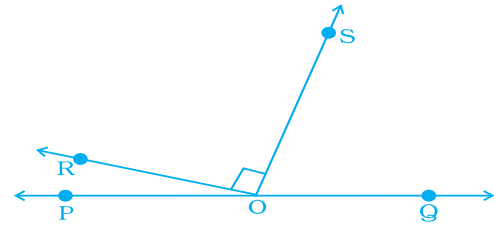


Fig. 5.19

23. Statements **a** and **b** are as given below:

- a** : If two lines intersect, then the vertically opposite angles are equal.
b : If a transversal intersects, two other lines, then the sum of two interior angles on the same side of the transversal is 180° .

Then

- (a) Both **a** and **b** are true (b) **a** is true and **b** is false
(c) **a** is false and **b** is true (d) both **a** and **b** are false

24. For Fig. 5.20, statements **p** and **q** are given below:

p : *a* and *b* are forming a linear pair.

q : *a* and *b* are forming a pair of adjacent angles.

Then,

- (a) both **p** and **q** are true
(b) **p** is true and **q** is false
(c) **p** is false and **q** is true
(d) both **p** and **q** are false

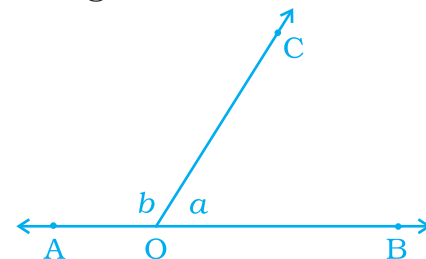
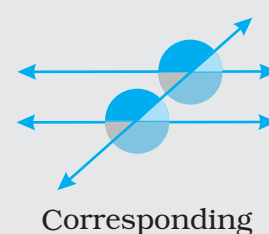
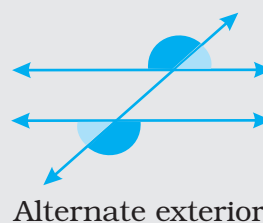
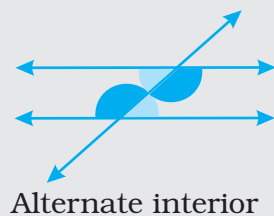


Fig. 5.20

A **transversal** is a line that intersects two or more lines that lie in the same plane in distinct points. Transversals to parallel lines form angles with special properties.



25. In Fig. 5.21, $\angle AOC$ and $\angle BOC$ form a pair of

- (a) vertically opposite angles
- (b) complementary angles
- (c) alternate interior angles
- (d) supplementary angles

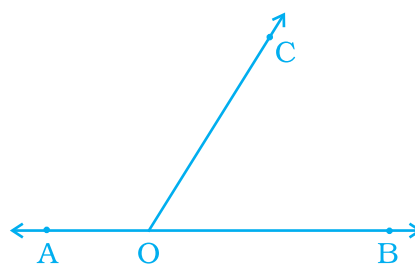


Fig. 5.21

26. In Fig. 5.22, the value of a is

- (a) 20°
- (b) 15°
- (c) 5°
- (d) 10°

27. In Fig. 5.23, if $QP \parallel SR$, the value of a is

- (a) 40°
- (b) 30°
- (c) 90°
- (d) 80°

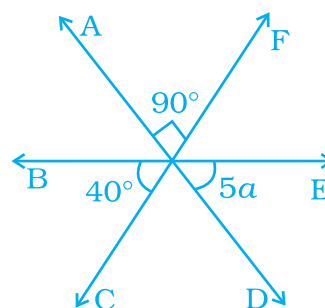


Fig. 5.22

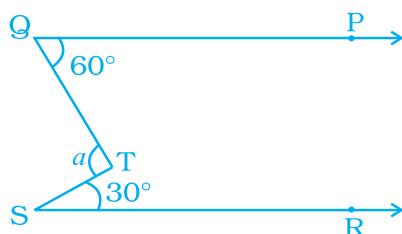


Fig. 5.23

28. In which of the following figures, a and b are forming a pair of adjacent angles?

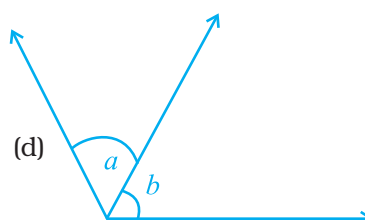
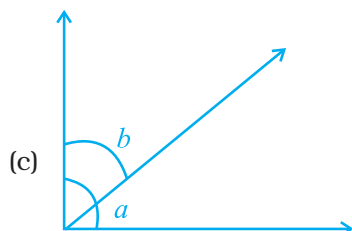
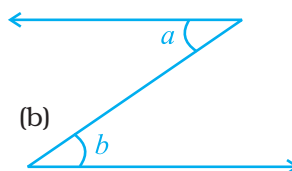
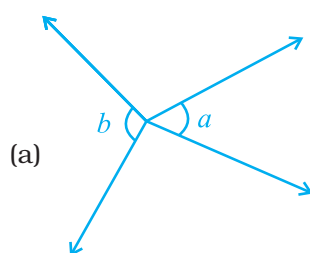


Fig. 5.24

Think and Discuss

- Tell** how many different angles would be formed by a transversal intersecting three parallel lines. How many different angle measures would there be?
- Explain** how a transversal could intersect two other lines so that corresponding angles are not congruent.

29. In a pair of adjacent angles, (i) vertex is always common, (ii) one arm is always common, and (iii) uncommon arms are always opposite rays

Then

- All (i), (ii) and (iii) are true
- (iii) is false
- (i) is false but (ii) and (iii) are true
- (ii) is false

30. In Fig. 5.25, lines PQ and ST intersect at O. If $\angle POR = 90^\circ$ and $x : y = 3 : 2$, then z is equal to

- 126°
- 144°
- 136°
- 154°

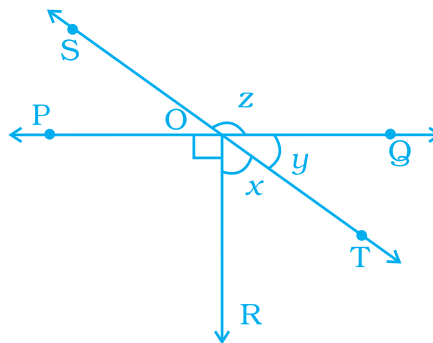
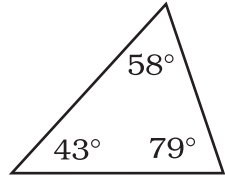
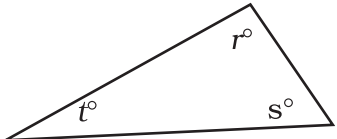


Fig. 5.25

TRIANGLE SUM THEOREM

Words	Numbers	Algebra
The angle measures of a triangle add to 180° .	 $43^\circ + 58^\circ + 79^\circ = 180^\circ$	 $r^\circ + s^\circ + t^\circ = 180^\circ$

- 31.** In Fig. 5.26, POQ is a line, then a is equal to

(a) 35° (b) 100°
(c) 80° (d) 135°

- 32.** Vertically opposite angles are always

(a) supplementary
(b) complementary
(c) adjacent
(d) equal

- 33.** In Fig. 5.27, $a = 40^\circ$. The value of b is

(a) 20° (b) 24°
(c) 36° (d) 120°

- 34.** If an angle is 60° less than two times of its supplement, then the greater angle is

(a) 100° (b) 80°
(c) 60° (d) 120°

- 35.** In Fig. 5.28, $PQ \parallel RS$.

If $\angle 1 = (2a + b)^\circ$ and $\angle 6 = (3a - b)^\circ$, then the measure of $\angle 2$ in terms of b is

(a) $(2 + b)^\circ$ (b) $(3 - b)^\circ$
(c) $(108 - b)^\circ$ (d) $(180 - b)^\circ$

- 36.** In Fig. 5.29, $PQ \parallel RS$ and $a : b = 3 : 2$.

Then, f is equal to

(a) 36° (b) 108°
(c) 72° (d) 144°

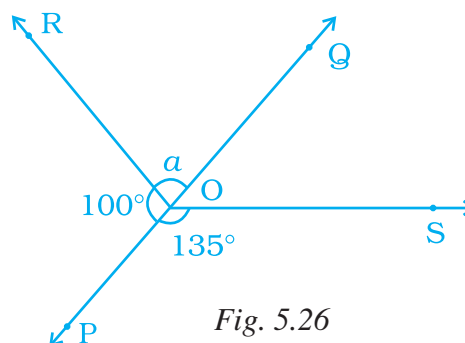


Fig. 5.26

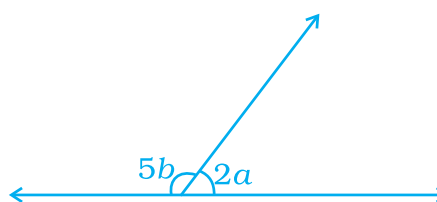


Fig. 5.27

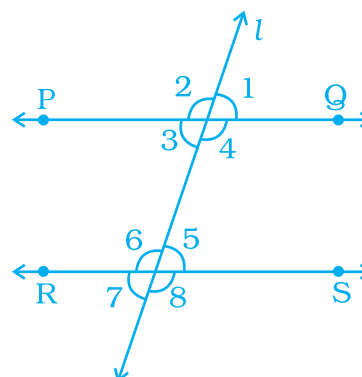


Fig. 5.28

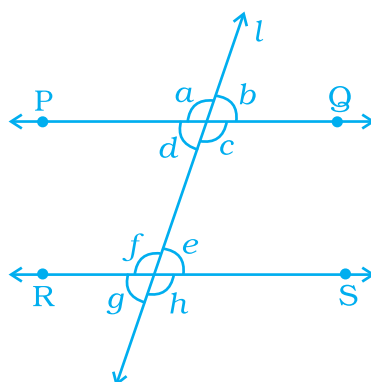


Fig. 5.29

An **acute triangle** has 3 acute angles. A **right triangle** has 1 right angle, An **obtuse triangle** has 1 obtuse angle.
An **equilateral triangle** has 3 congruent sides and 3 congruent angles. An **isosceles triangle** has at least 2 congruent sides and 2 congruent angles. A **scalene triangle** has no congruent sides and no congruent angles.

37. In Fig. 5.30, line l intersects two parallel lines PQ and RS . Then, which one of the following is not true?

- (a) $\angle 1 = \angle 3$ (b) $\angle 2 = \angle 4$
(c) $\angle 6 = \angle 7$ (d) $\angle 4 = \angle 8$

38. In Fig. 5.30, which one of the following is not true?

- (a) $\angle 1 + \angle 5 = 180^\circ$
(b) $\angle 2 + \angle 5 = 180^\circ$
(c) $\angle 3 + \angle 8 = 180^\circ$
(d) $\angle 2 + \angle 3 = 180^\circ$

39. In Fig. 5.30, which of the following is true?

- (a) $\angle 1 = \angle 5$ (b) $\angle 4 = \angle 8$ (c) $\angle 5 = \angle 8$ (d) $\angle 3 = \angle 7$

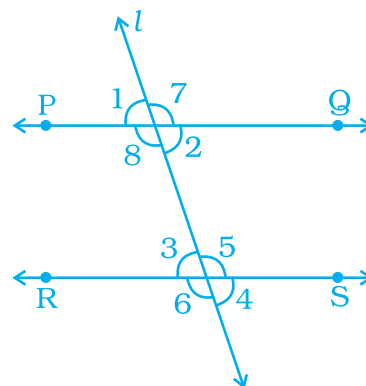


Fig. 5.30

Think and Discuss

- 1. Explain** whether a right triangle can be equilateral. Can it be isosceles? scalene?
- 2. Explain** whether a triangle can have two right angles. Can it have two obtuse angles?

40. In Fig. 5.31, $PQ \parallel ST$. Then, the value of $x + y$ is

- (a) 125° (b) 135°
(c) 145° (d) 120°

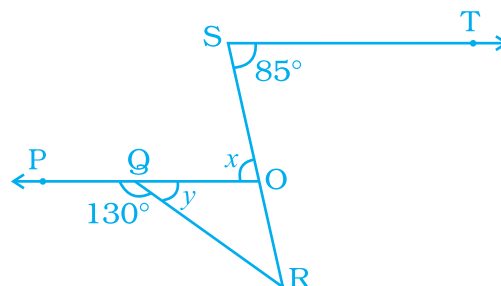


Fig. 5.31

41. In Fig. 5.32, if $PQ \parallel RS$ and $QR \parallel TS$, then the value a is
 (a) 95° (b) 90° (c) 85° (d) 75°

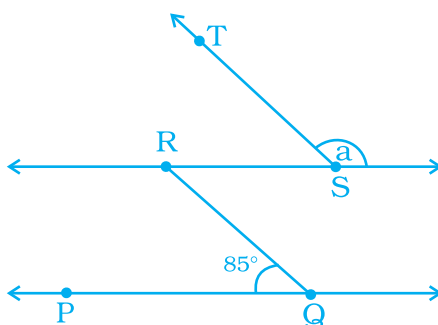


Fig. 5.32

In questions 42 to 56, fill in the blanks to make the statements true.

42. If sum of measures of two angles is 90° , then the angles are _____.
 43. If the sum of measures of two angles is 180° , then they are _____.
 44. A transversal intersects two or more than two lines at _____ points.

If a transversal intersects two parallel lines, then (Q. 45 to 48).

45. sum of interior angles on the same side of a transversal is _____.
 46. alternate interior angles have one common _____.
 47. corresponding angles are on the _____ side of the transversal.
 48. alternate interior angles are on the _____ side of the transversal.
 49. Two lines in a plane which do not meet at a point anywhere are called _____ lines.
 50. Two angles forming a _____ pair are supplementary.
 51. The supplement of an acute is always _____ angle.
 52. The supplement of a right angle is always _____ angle.
 53. The supplement of an obtuse angle is always _____ angle.
 54. In a pair of complementary angles, each angle cannot be more than _____ 90° .
 55. An angle is 45° . Its complementary angle will be _____.
 56. An angle which is half of its supplement is of _____.

In questions 57 to 71, state whether the statements are True or False.

57. Two right angles are complementary to each other.

58. One obtuse angle and one acute angle can make a pair of complementary angles.
59. Two supplementary angles are always obtuse angles.
60. Two right angles are always supplementary to each other.
61. One obtuse angle and one acute angle can make a pair of supplementary angles.
62. Both angles of a pair of supplementary angles can never be acute angles.
63. Two supplementary angles always form a linear pair.
64. Two angles making a linear pair are always supplementary.
65. Two angles making a linear pair are always adjacent angles.
66. Vertically opposite angles form a linear pair.
67. Interior angles on the same side of a transversal with two distinct parallel lines are complementary angles.
68. Vertically opposite angles are either both acute angles or both obtuse angles.
69. A linear pair may have two acute angles.
70. An angle is more than 45° . Its complementary angle must be less than 45° .
71. Two adjacent angles always form a linear pair.
72. Write down each pair of adjacent angles shown in the following figures:

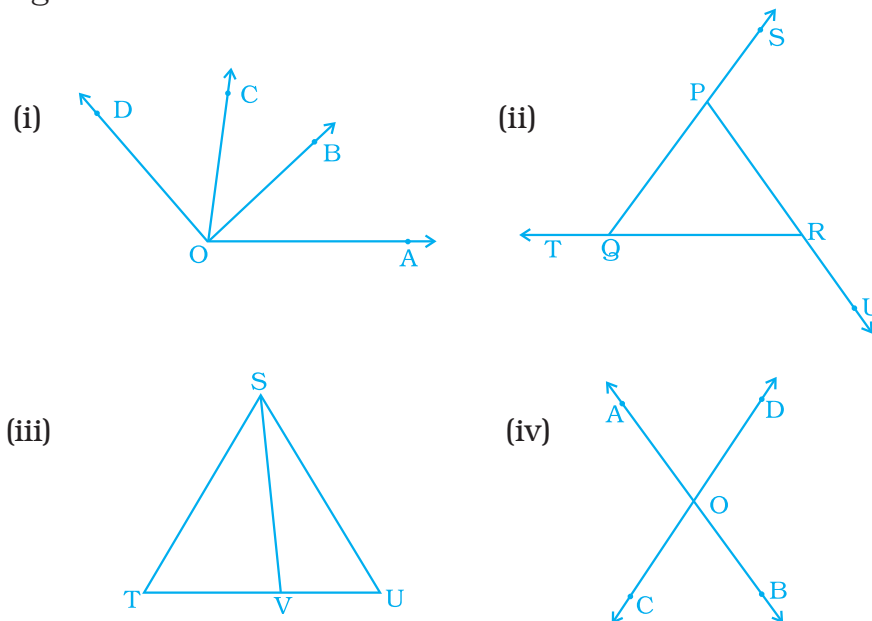


Fig. 5.33

- 73.** In each of the following figures, write, if any, (i) each pair of vertically opposite angles, and (ii) each linear pair.

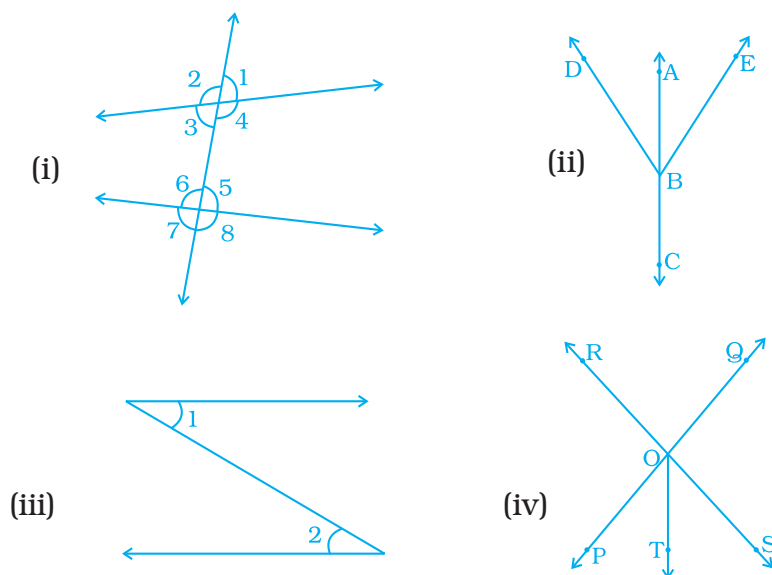


Fig. 5.34

- 74.** Name the pairs of supplementary angles in the following figures:

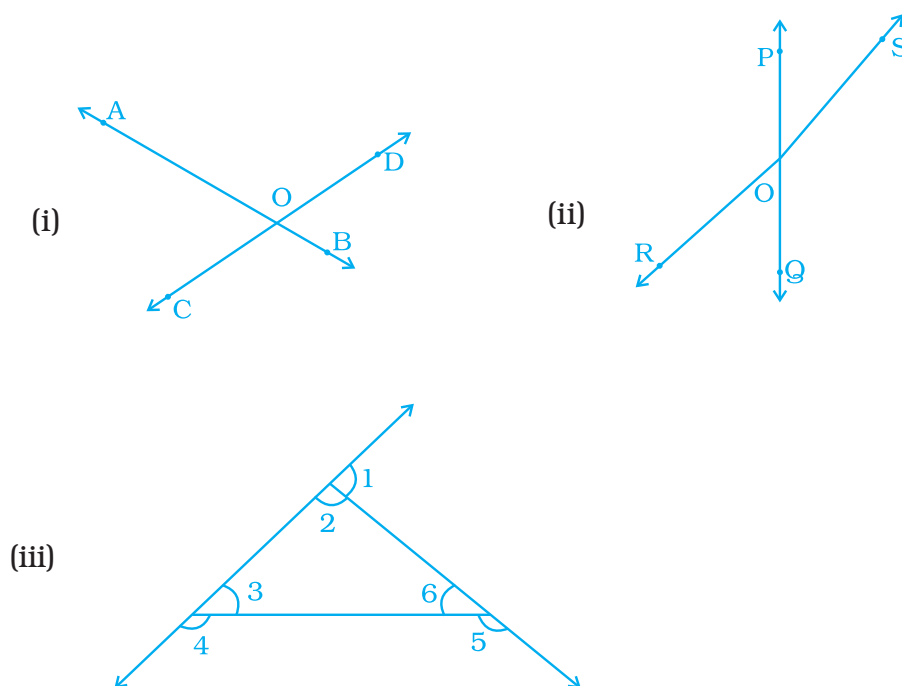


Fig. 5.35

75. In Fig. 5.36, $PQ \parallel RS$, $TR \parallel QU$ and $\angle PTR = 42^\circ$. Find $\angle QUR$.

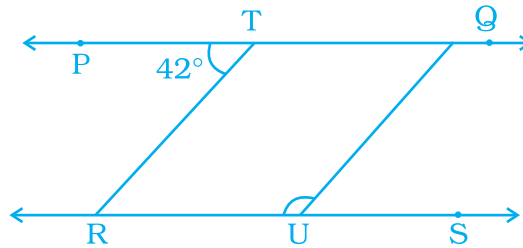
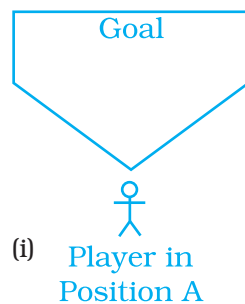
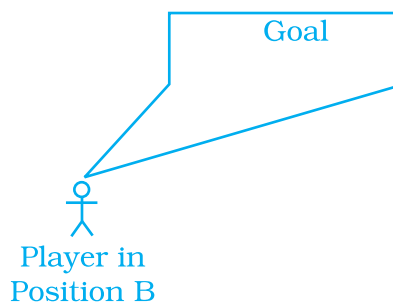


Fig. 5.36

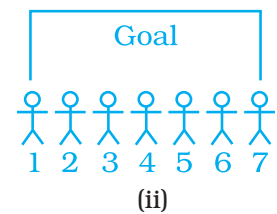
76. The drawings below (Fig. 5.37), show angles formed by the goalposts at different positions of a football player. The greater the angle, the better chance the player has of scoring a goal. For example, the player has a better chance of scoring a goal from Position A than from Position B.



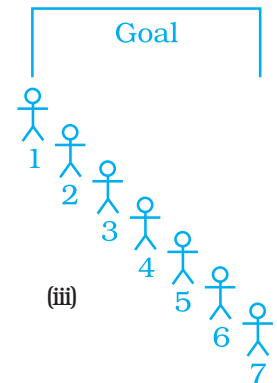
(i) Player in Position A



Player in Position B



(ii)



(iii)

Fig. 5.37

In Parts (a) and (b) given below it may help to trace the diagrams and draw and measure angles.

- Seven football players are practicing their kicks. They are lined up in a straight line in front of the goalpost [Fig.(ii)]. Which player has the best (the greatest) kicking angle?
 - Now the players are lined up as shown in Fig. (iii). Which player has the best kicking angle?
 - Estimate atleast two situations such that the angles formed by different positions of two players are complement to each other.
77. The sum of two vertically opposite angles is 166° . Find each of the angles.

78. In Fig. 5.38, $l \parallel m \parallel n$.

$\angle QPS = 35^\circ$ and $\angle QRT = 55^\circ$. Find $\angle PQR$.

79. In Fig. 5.39, P, Q and R are collinear points and $TQ \perp PR$,

Name; (a) pair of complementary angles

(b) two pairs of supplementary angles.

(c) four pairs of adjacent angles.

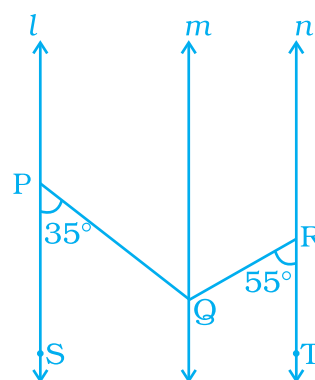


Fig. 5.38

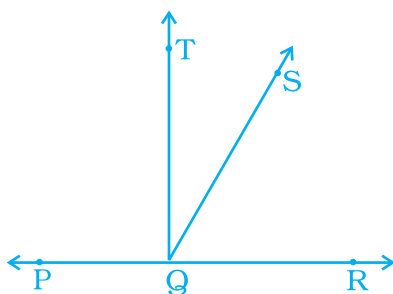


Fig. 5.39

80. In Fig. 5.40, $OR \perp OP$.

(i) Name all the pairs of adjacent angles.

(ii) Name all the pairs of complementary angles.

81. If two angles have a common vertex and their arms form opposite rays (Fig. 5.41), Then,

(a) how many angles are formed?

(b) how many types of angles are formed?

(c) write all the pairs of vertically opposite angles.

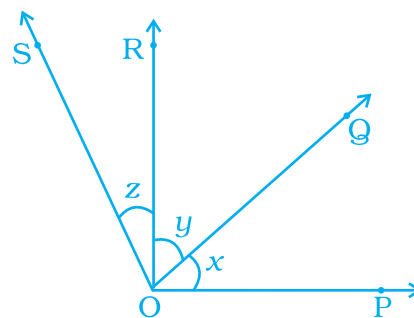


Fig. 5.40

82. In (Fig 5.42) are the following pairs of angles adjacent? Justify your answer.

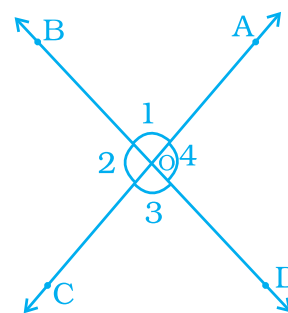


Fig. 5.41

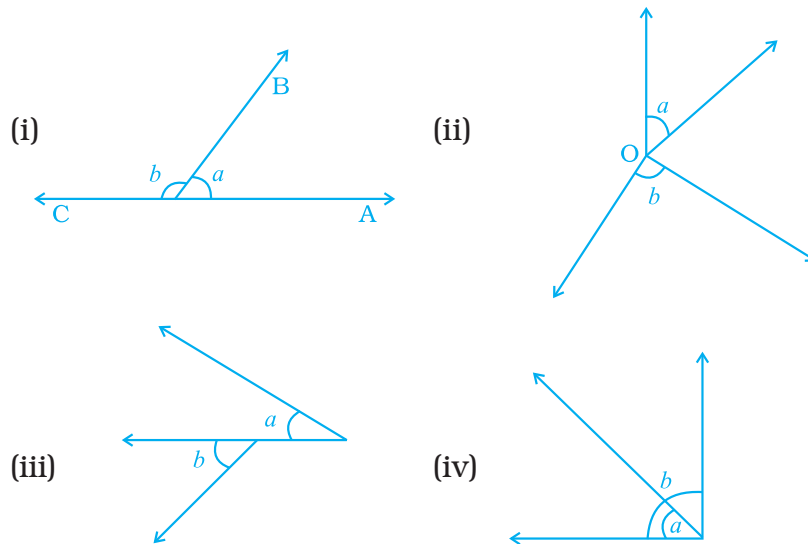


Fig. 5.42

83. In Fig. 5.43, write all the pairs of supplementary angles.

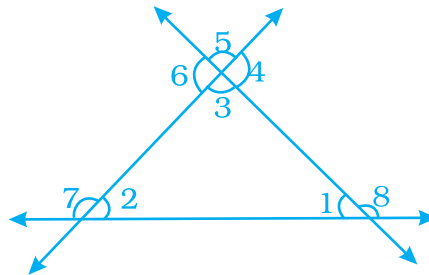


Fig. 5.43

- 84.** What is the type of other angle of a linear pair if
- one of its angle is acute?
 - one of its angles is obtuse?
 - one of its angles is right?
- 85.** Can two acute angles form a pair of supplementary angles? Give reason in support of your answer.
- 86.** Two lines AB and CD intersect at O (Fig. 5.44). Write all the pairs of adjacent angles by taking angles 1, 2, 3, and 4 only.

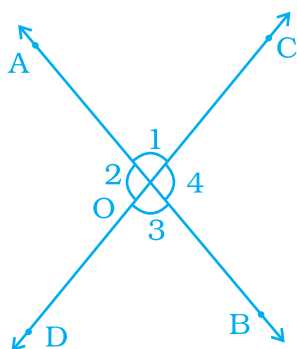
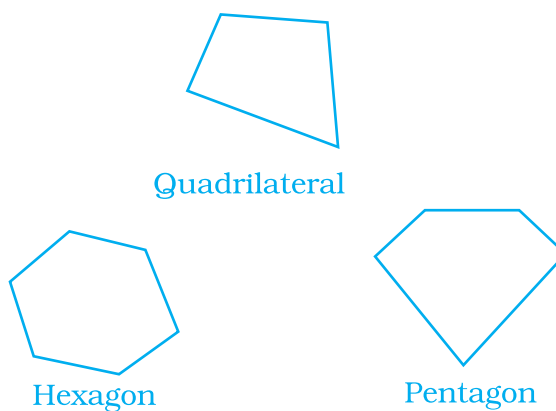


Fig. 5.44

Polygon	Number of Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
n -gon	n



87. If the complement of an angle is 62° , then find its supplement.
88. A road crosses a railway line at an angle of 30° as shown in Fig. 5.45. Find the values of a , b and c .

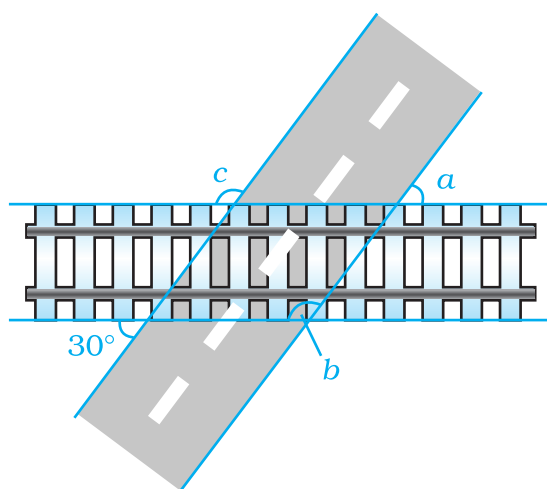


Fig. 5.45

89. The legs of a stool make an angle of 35° with the floor as shown in Fig. 5.46. Find the angles x and y .

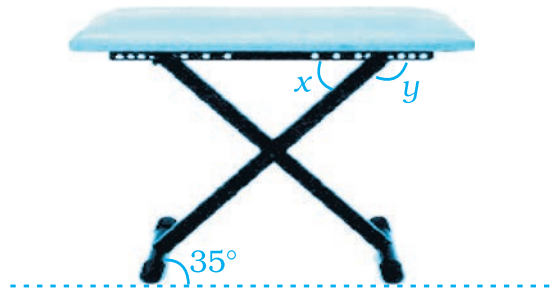


Fig. 5.46

90. Iron rods a, b, c, d, e and f are making a design in a bridge as shown in Fig. 5.47, in which $a \parallel b, c \parallel d, e \parallel f$. Find the marked angles between
- b and c
 - d and e
 - d and f
 - c and f

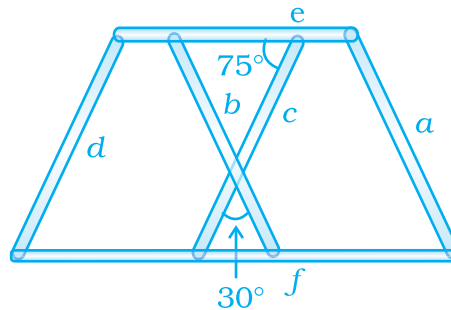


Fig. 5.47

91. Amisha makes a star with the help of line segments a, b, c, d, e and f , in which $a \parallel d, b \parallel e$ and $c \parallel f$. Chhaya marks an angle as 120° as shown in Fig. 5.48 and asks Amisha to find the $\angle x, \angle y$ and $\angle z$. Help Amisha in finding the angles.

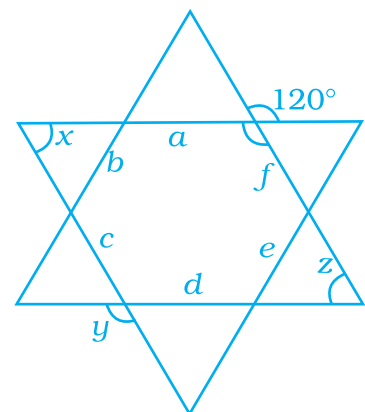


Fig. 5.48

92. In Fig. 5.49, $AB \parallel CD$, $AF \parallel ED$, $\angle AFC = 68^\circ$ and $\angle FED = 42^\circ$. Find $\angle EFD$.

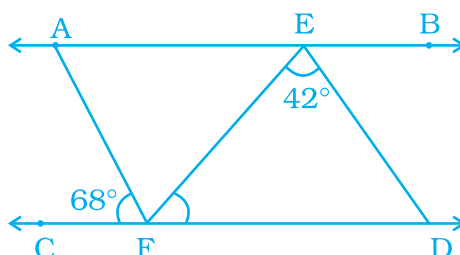


Fig. 5.49

93. In Fig. 5.50, OB is perpendicular to OA and $\angle BOC = 49^\circ$. Find $\angle AOD$.

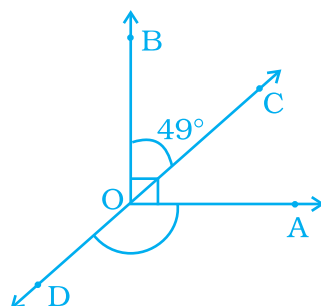


Fig. 5.50

94. Three lines AB, CD and EF intersect each other at O. If $\angle AOE = 30^\circ$ and $\angle DOB = 40^\circ$ (Fig. 5.51), find $\angle COF$.

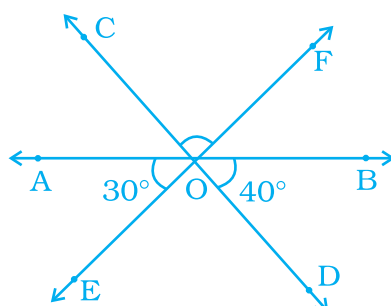


Fig. 5.51

95. Measures (in degrees) of two complementary angles are two consecutive even integers. Find the angles.
96. If a transversal intersects two parallel lines, and the difference of two interior angles on the same side of a transversal is 20° , find the angles.

97. Two angles are making a linear pair. If one of them is one-third of the other, find the angles.
98. Measures (in degrees) of two supplementary angles are consecutive odd integers. Find the angles.
99. In Fig. 5.52, $AE \parallel GF \parallel BD$, $AB \parallel CG \parallel DF$ and $\angle CHE = 120^\circ$. Find $\angle ABC$ and $\angle CDE$.

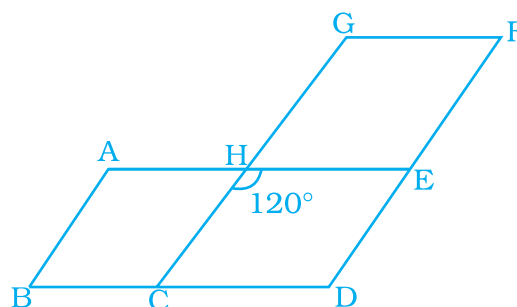


Fig. 5.52

100. In Fig. 5.53, find the value of $\angle BOC$, if points A, O and B are collinear.

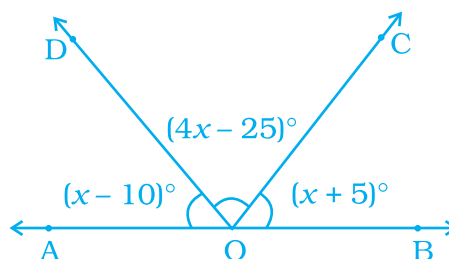


Fig. 5.53

101. In Fig. 5.54, if $l \parallel m$, find the values of a and b .

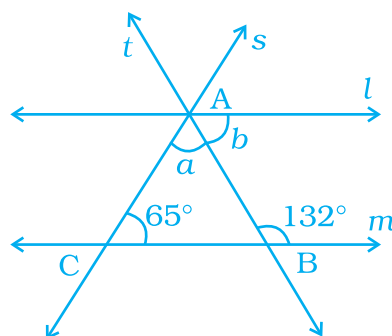


Fig. 5.54

- 102.** In Fig. 5.55, $l \parallel m$ and a line t intersects these lines at P and Q, respectively. Find the sum $2a + b$.

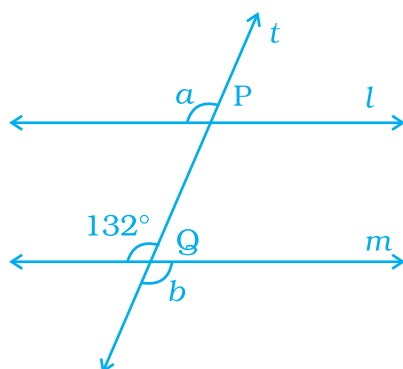


Fig. 5.55

- 103.** In Fig. 5.56, $QP \parallel RS$. Find the values of a and b .

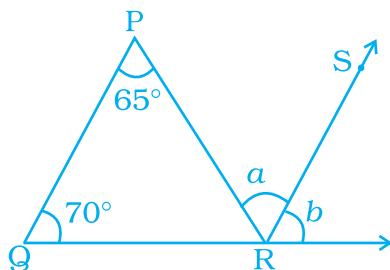


Fig. 5.56

- 104.** In Fig. 5.57, $PQ \parallel RT$. Find the value of $a + b$.

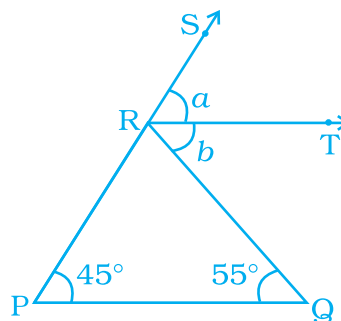


Fig. 5.57

- 105.** In Fig 5.58, PQ, RS and UT are parallel lines.

(i) If $c = 57^\circ$ and $a = \frac{c}{3}$, find the value of d .

(ii) If $c = 75^\circ$ and $a = \frac{2}{5}c$, find b .

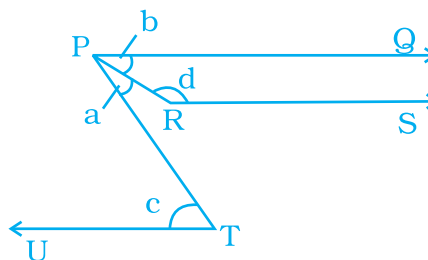


Fig. 5.58

106. In Fig. 5.59, $AB \parallel CD$. Find the reflex $\angle EFG$.

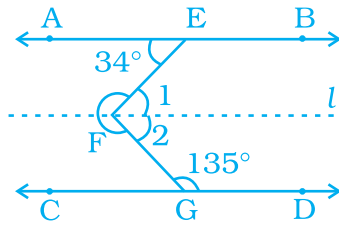
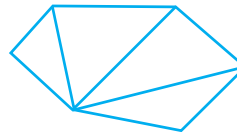


Fig. 5.59

Look for a pattern between the number of sides and the number of triangles.



Heptagon
7 sides
5 triangles



Hexagon
6 sides
4 triangles

107. In Fig. 5.60, two parallel lines l and m are cut by two transversals n and p . Find the values of x and y .

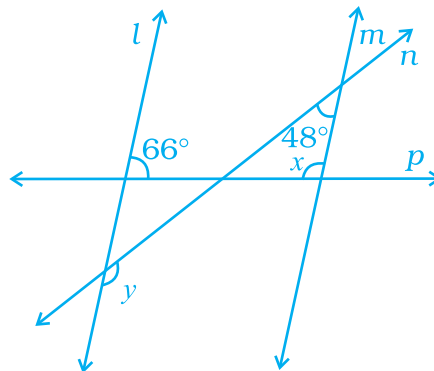


Fig. 5.60

108. In Fig. 5.61, l , m and n are parallel lines, and the lines p and q are also parallel. Find the values of a , b and c .

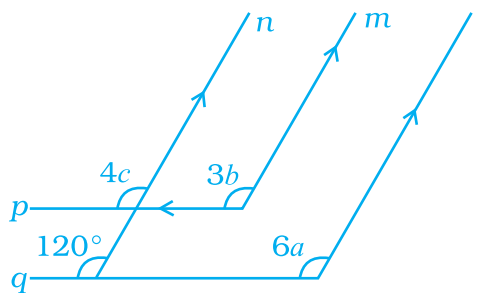


Fig. 5.61

109. In Fig. 5.62, state which pair of lines are parallel. Give reason.

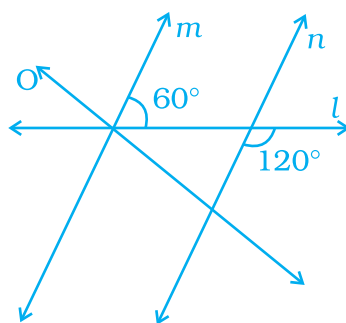


Fig. 5.62

110. In Fig. 5.63, examine whether the following pairs of lines are parallel or not:

(i) EF and GH

(ii) AB and CD

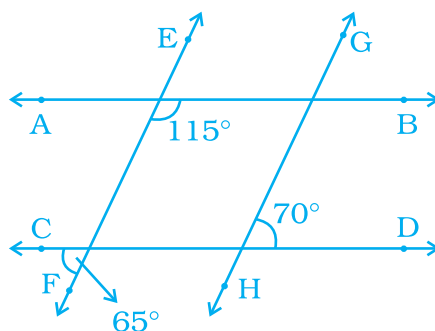


Fig. 5.63

111. In Fig. 5.64, find out which pair of lines are parallel:

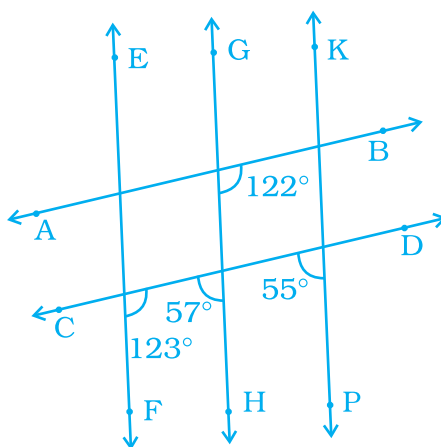


Fig. 5.64

112. In Fig. 5.65, show that

- (i) $AB \parallel CD$
- (ii) $EF \parallel GH$

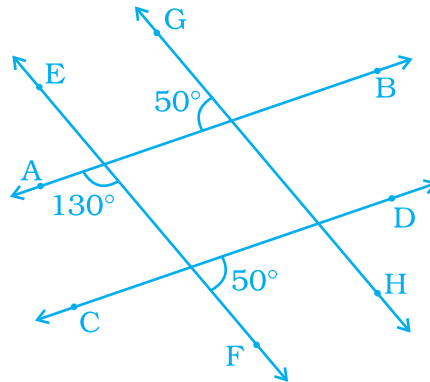


Fig. 5.65

113. In Fig. 5.66, two parallel lines l and m are cut by two transversals p and q . Determine the values of x and y .

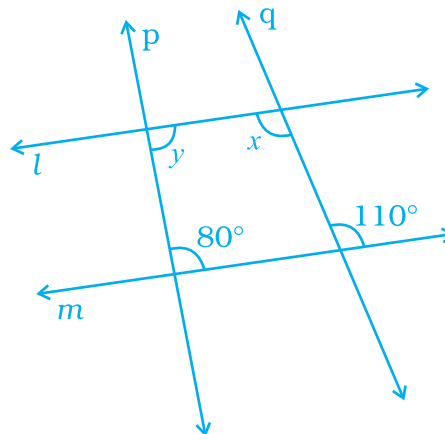


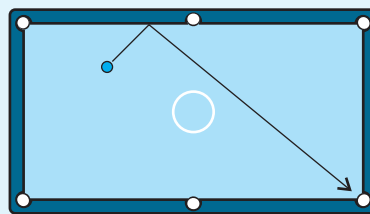
Fig. 5.66

(D) Applications

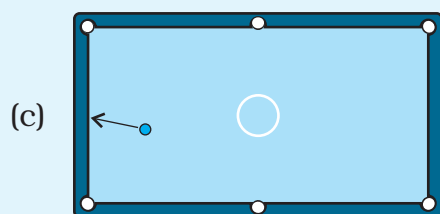
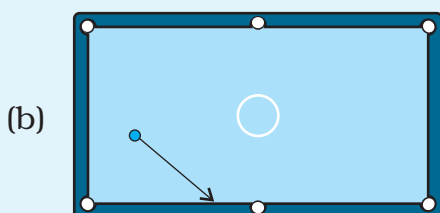
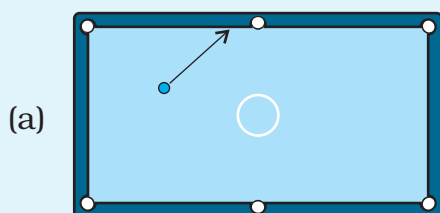
- The game pool belongs to billiard sports and generally played with a cue stick which is used to strike billiard balls, moving them around a cloth-covered billiards table with six pocket bounded by rubber cushions.

The angle at which a pool ball hits the side of a table has the same

measure as the angle at which it bounces off the side. This is shown in the drawing at the right. The marked angles have the same measure, and the arrow shows the ball's path.

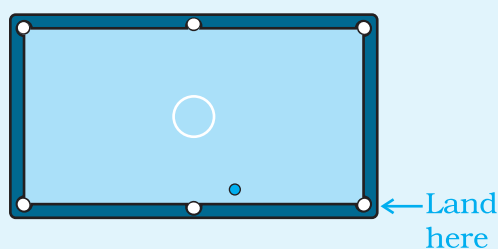


In Parts (a)–(c), trace the drawing. Then use your protractor to find the path the ball will take when it bounces off the side. Tell whether the ball will go into a pocket or hit another side. (Draw just one bounce.)



When light hits a mirror, it behaves in the same way as a pool ball hitting the side of a table. If light hits a mirror at an angle, it bounces off at the same angle. In physics, this law is often stated as “the angle of incidence = the angle of reflection.”

- (d) Try to trace this drawing. Draw a path for which the ball will bounce off a side and land in the lower-right pocket.



2. Crossword Puzzle

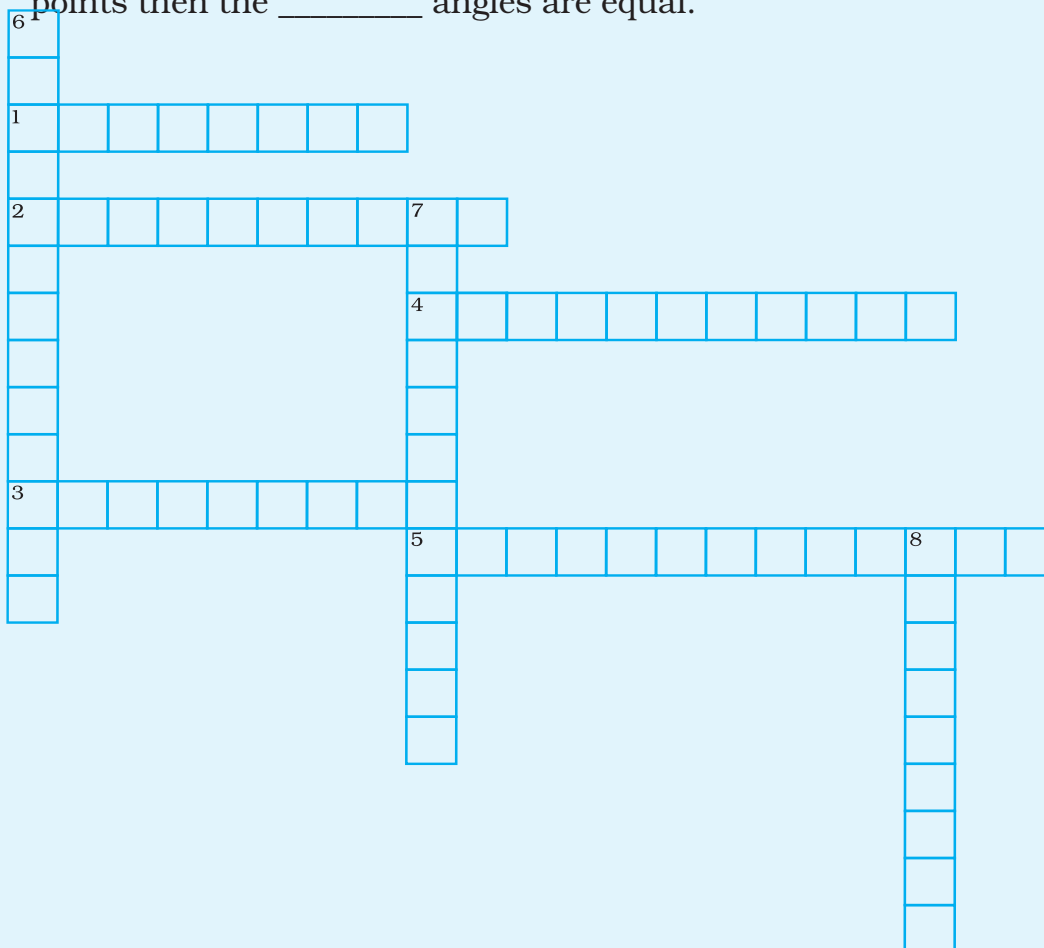
Fill the crossword puzzle with the help of following clues:

Across

- Two lines in a plane which do not intersect each other.
- A pair of adjacent angles having their non common arms opposite rays.
- A pair of angles having a common vertex, a common arm and their interiors do not overlap.
- The two lines are intersected by a line at distinct points.
- The sum of two angles is 90° .

Down

- Sum of two angles is 180° .
- The two lines in a plane intersect each other at one and only one point are called _____.
- When two parallel lines intersected by a transversal at two distinct points then the _____ angles are equal.



UNIT 6

TRIANGLES

(A) Main Concepts and Results

- The six elements of a triangle are its three angles and the three sides.
- The line segment joining a vertex of a triangle to the mid point of its opposite side is called a median of the triangle. A triangle has 3 medians.
- The perpendicular line segment from a vertex of a triangle to its opposite side is called an altitude of the triangle. A triangle has 3 altitudes.
- An exterior angle of a triangle is formed, when a side of a triangle is produced.
- The measure of any exterior angle of a triangle is equal to the sum of the measures of its two interior opposite angles.
- The sum of the three angles of a triangle is 180° .
- A triangle is said to be equilateral, if each of its sides has the same length.
- In an equilateral triangle, each angle has measure 60° .
- A triangle is said to be isosceles if at least two of its sides are of same length.
- The sum of the lengths of any two sides of a triangle is always greater than the length of the third side.
- The difference of the lengths of any two sides of a triangle is always smaller than the length of the third side.

- In a right-angled triangle, the side opposite to the right angle is called the hypotenuse and the other two sides are called its legs or arms.
- In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares on its legs.
- Two plane figures, say, F_1 and F_2 are said to be congruent, if the trace-copy of F_1 fits exactly on that of F_2 . We write this as $F_1 \cong F_2$.
- Two line segments, say \overline{AB} and \overline{CD} , are congruent, if they have equal lengths. We write this as $\overline{AB} \cong \overline{CD}$. However, it is common to write it as $\overline{AB} = \overline{CD}$.
- Two angles, say $\angle ABC$ and $\angle PQR$, are congruent, if their measures are equal. We write this as $\angle ABC \cong \angle PQR$ or as $m\angle ABC = m\angle PQR$ or simply as $\angle ABC = \angle PQR$.
- Under a given correspondence, two triangles are congruent, if the three sides of the one are equal to the three sides of the other (SSS).
- Under a given correspondence, two triangles are congruent if two sides and the angle included between them in one of the triangles are equal to the two sides and the angle included between them of the other triangle (SAS).
- Under a given correspondence, two triangles are congruent if two angles and the side included between them in one of the triangles are equal to the two angles and the side included between them of the other triangle (ASA).
- Under a given correspondence, two right-angled triangles are congruent if the hypotenuse and a leg (side) of one of the triangles are equal to the hypotenuse and one of the leg (side) of the other triangle (RHS).

(B) Solved Examples

In Examples 1 to 5, there are four options, out of which only one is correct. Write the correct one.

- Example 1:** In Fig. 6.1, side QR of a ΔPQR has been produced to the point S. If $\angle PRS = 115^\circ$ and $\angle P = 45^\circ$, then $\angle Q$ is equal to,
- (a) 70° (b) 105° (c) 51° (d) 80°

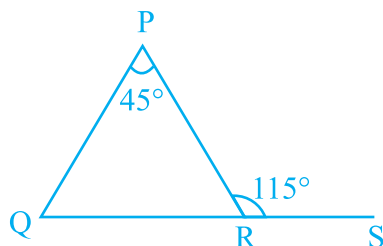


Fig. 6.1

Solution: Correct answer is (a).

Example 2: In an equilateral triangle ABC (Fig. 6.2), AD is an altitude. Then $4AD^2$ is equal to

- (a) $2BD^2$ (b) BC^2 (c) $3AB^2$ (d) $2DC^2$

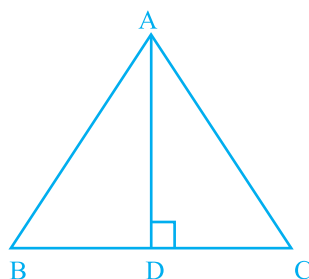


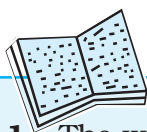
Fig. 6.2

Solution: Correct answer is (c).

Example 3: Which of the following cannot be the sides of a triangle?

- (a) 3 cm, 4 cm, 5 cm (b) 2 cm, 4 cm, 6 cm
(c) 2.5 cm, 3.5 cm, 4.5 cm (d) 2.3 cm, 6.4 cm, 5.2 cm

Solution: Correct answer is (b).



1. The word equilateral contains the roots *equi*, which means “equal,” and *lateral*, which means “of the side.” What do you suppose an equilateral is?
2. The Greek prefix *poly* means “many,” and the root *gon* means “angle.” What do you suppose a *polygon* is?

Example 4: Which one of the following is not a criterion for congruence of two triangles?

- (a) ASA (b) SSA (c) SAS (d) SSS

Solution: Correct answer is (b).

Example 5: In Fig. 6.3, PS is the bisector of $\angle P$ and $PQ = PR$. Then $\triangle PRS$ and $\triangle PQS$ are congruent by the criterion

- (a) AAA (b) SAS (c) ASA (d) both (b) and (c)

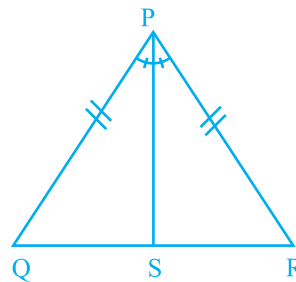


Fig. 6.3

Solution : Correct answer is (b).

In examples 6 to 9, fill in the blanks to make the statements true.

Example 6: The line segment joining a vertex of a triangle to the mid-point of its opposite side is called its _____.

Solution: median

Example 7: A triangle is said to be _____, if each one of its sides has the same length.

Solution: equilateral

Example 8: In Fig. 6.4, $\angle PRS = \angle QPR + \angle$ _____

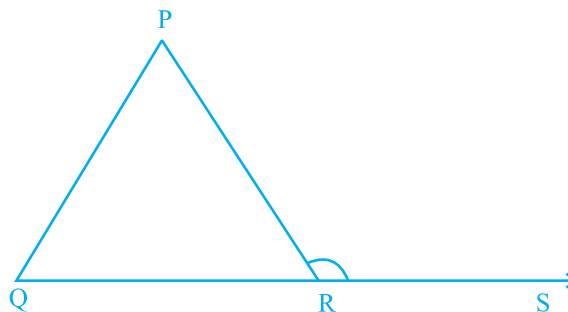


Fig. 6.4

Solution: PQR

Example 9: Let ABC and DEF be two triangles in which $AB = DE$, $BC = FD$ and $CA = EF$. The two triangles are congruent under the correspondence

$ABC \leftrightarrow$ _____

Solution: EDF

In Examples 10 to 12, state whether the statements are True or False.

Example 10: Sum of any two sides of a triangle is not less than the third side.

Solution: False

Example 11: The measure of any exterior angle of a triangle is equal to the sum of the measures of its two interior opposite angles.

Solution: True

Example 12: If in $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $\angle A = \angle D$ and $BC = EF$ then the two triangle ABC and DEF are congruent by SAS criterion.

Solution: False

Application on Problem Solving Strategy



Example 13
In Fig. 6.5, find x and y .

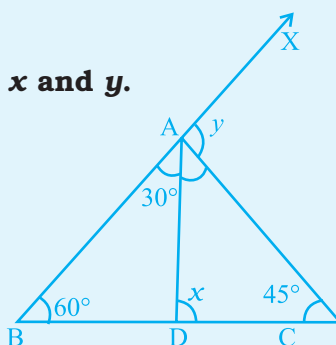


Fig. 6.5

Solution :



Understand and Explore the Problem

- What all are given?
 $\angle ABD = 60^\circ$, $\angle BAD = 30^\circ$ and $\angle ACD = 45^\circ$
- What are to be found?
 $\angle ADC$ and $\angle XAC$, which are respectively exterior angles for $\triangle ABD$ and $\triangle ABC$.



Plan a Strategy

- Find $\angle ADC$ using exterior angle property for $\triangle ABD$.
- Find y using exterior angle property for $\triangle ABC$.



Solve

- $x = \angle ADC = \angle DBA + \angle BAD$ (In $\triangle ABD$)
 $= 60^\circ + 30^\circ$
 $= 90^\circ$
- $y = \angle XAC = \angle ABC + \angle ACB$ (In $\triangle ABC$)
 $= 60^\circ + 45^\circ$
 $= 105^\circ$



Revise

- Verify your answer by using some other properties of triangle.
 In $\triangle ABD$, $\angle ADB = 180^\circ - (30^\circ + 60^\circ) = 90^\circ$ (Angle sum property of a triangle)

$$\begin{aligned} x &= \angle ADC = 180^\circ - \angle ADB \\ &= 180^\circ - 90^\circ = 90^\circ, \text{ Hence, } \angle ADC = 90^\circ \text{ verified.} \end{aligned}$$

$$\angle DAC = 180^\circ - (x + 45^\circ) = 180^\circ - 135^\circ = 45^\circ$$

At point A on \overrightarrow{BAX} , $30^\circ + \angle DAC + y = 180^\circ$

Hence for verifying value of y , $30^\circ + 45^\circ + y = 180^\circ$

$$\text{or } y = 180^\circ - 75^\circ = 105^\circ$$

Think and Discuss

1. If $AD = DC$? Why?
2. In given problem, can $\angle B$ be 85° instead of 60° ? If yes find the values of x and y in that case.
3. What type of triangle is $\triangle ADC$?

(C) Exercise

In each of the questions 1 to 49, four options are given, out of which only one is correct. Choose the correct one.

1. The sides of a triangle have lengths (in cm) 10, 6.5 and a , where a is a whole number. The minimum value that a can take is

(a) 6 (b) 5 (c) 3 (d) 4

2. Triangle DEF of Fig. 6.6 is a right triangle with $\angle E = 90^\circ$.

What type of angles are $\angle D$ and $\angle F$?

- (a) They are equal angles
(b) They form a pair of adjacent angles
(c) They are complementary angles
(d) They are supplementary angles

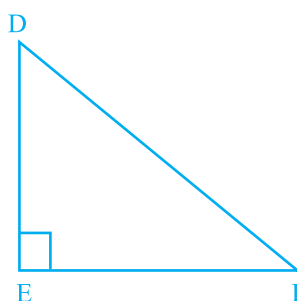


Fig. 6.6

CONGRUENT TRIANGLES

Diagram	Statement	Corresponding Angles	Corresponding Sides
	$\triangle ABC \cong \triangle DEF$	$\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$	$\overline{AB} \cong \overline{DE}$ $\overline{BC} \cong \overline{EF}$ $\overline{AC} \cong \overline{DF}$

3. In Fig. 6.7, $PQ = PS$. The value of x is

- (a) 35° (b) 45°
(c) 55° (d) 70°

4. In a right-angled triangle, the angles other than the right angle are

- (a) obtuse (b) right
(c) acute (d) straight

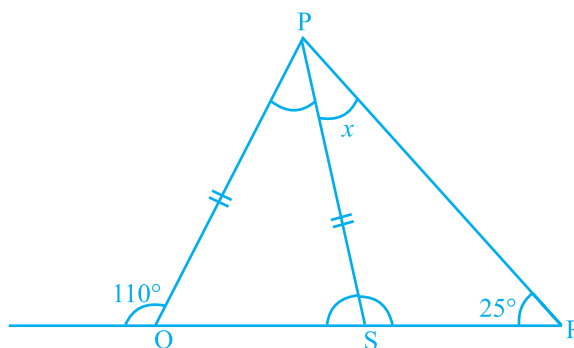


Fig. 6.7

5. In an isosceles triangle, one angle is 70° . The other two angles are of
(i) 55° and 55° (ii) 70° and 40° (iii) any measure

In the given option(s) which of the above statement(s) are true?

- (a) (i) only (b) (ii) only (c) (iii) only (d) (i) and (ii)

6. In a triangle, one angle is of 90° . Then

- (i) The other two angles are of 45° each
 (ii) In remaining two angles, one angle is 90° and other is 45°
 (iii) Remaining two angles are complementary

In the given option(s) which is true?

- (a) (i) only (b) (ii) only (c) (iii) only (d) (i) and (ii)

7. Lengths of sides of a triangle are 3 cm, 4 cm and 5 cm. The triangle is

- (a) Obtuse angled triangle (b) Acute-angled triangle
 (c) Right-angled triangle (d) An Isosceles right triangle

8. In Fig. 6.8, $PB = PD$. The value of x is

- (a) 85° (b) 90°
 (c) 25° (d) 35°

9. In $\triangle PQR$,

- (a) $PQ - QR > PR$
 (b) $PQ + QR < PR$
 (c) $PQ - QR < PR$
 (d) $PQ + PR < QR$

10. In $\triangle ABC$,

- (a) $AB + BC > AC$ (b) $AB + BC < AC$
 (c) $AB + AC < BC$ (d) $AC + BC < AB$

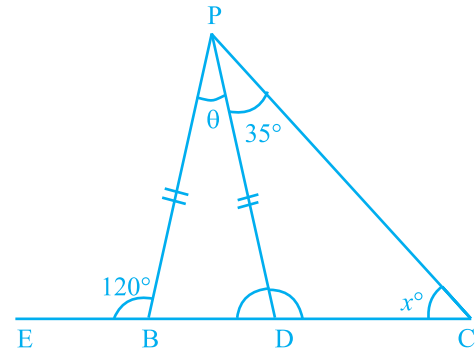


Fig. 6.8

Think and Discuss

1. **Explain** what it means for two polygons to be congruent.
2. **Tell** how to write a congruence statement for two triangles.

11. The top of a broken tree touches the ground at a distance of 12 m from its base. If the tree is broken at a height of 5 m from the ground then the actual height of the tree is
 (a) 25 m (b) 13 m (c) 18 m (d) 17 m
12. The triangle ABC formed by $AB = 5$ cm, $BC = 8$ cm, $AC = 4$ cm is
 (a) an isosceles triangle only (b) a scalene triangle only
 (c) an isosceles right triangle (d) scalene as well as a right triangle
13. Two trees 7 m and 4 m high stand upright on a ground. If their bases (roots) are 4 m apart, then the distance between their tops is
 (a) 3 m (b) 5 m (c) 4 m (d) 11 m
14. If in an isosceles triangle, each of the base angles is 40° , then the triangle is
 (a) Right-angled triangle (b) Acute angled triangle
 (c) Obtuse angled triangle (d) Isosceles right-angled triangle
15. If two angles of a triangle are 60° each, then the triangle is
 (a) Isosceles but not equilateral (b) Scalene
 (c) Equilateral (d) Right-angled
16. The perimeter of the rectangle whose length is 60 cm and a diagonal is 61 cm is
 (a) 120 cm (b) 122 cm (c) 71 cm (d) 142 cm
17. In $\triangle PQR$, if $PQ = QR$ and $\angle Q = 100^\circ$, then $\angle R$ is equal to
 (a) 40° (b) 80° (c) 120° (d) 50°
18. Which of the following statements is not correct?
 (a) The sum of any two sides of a triangle is greater than the third side
 (b) A triangle can have all its angles acute
 (c) A right-angled triangle cannot be equilateral
 (d) Difference of any two sides of a triangle is greater than the third side
19. In Fig. 6.9, $BC = CA$ and $\angle A = 40^\circ$. Then, $\angle ACD$ is equal to
 (a) 40° (b) 80°
 (c) 120° (d) 60°

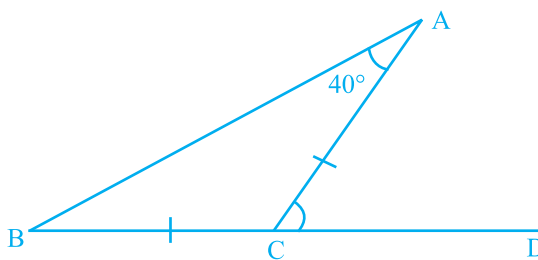


Fig. 6.9

- 20.** The length of two sides of a triangle are 7 cm and 9 cm. The length of the third side may lie between
- 1 cm and 10 cm
 - 2 cm and 8 cm
 - 3 cm and 16 cm
 - 1 cm and 16 cm

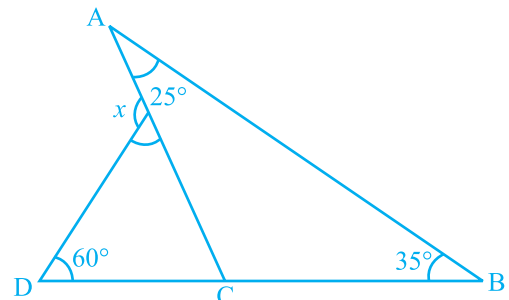


Fig. 6.10

- 21.** From Fig. 6.10, the value of x is
- 75°
 - 90°
 - 120°
 - 60°

- 22.** In Fig. 6.11, the value of $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F$ is
- 190°
 - 540°
 - 360°
 - 180°

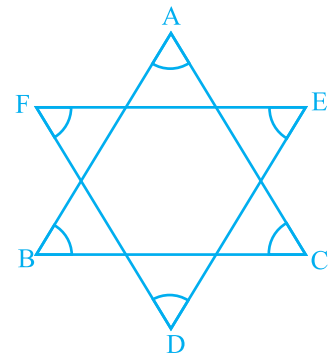


Fig. 6.11

- 23.** In Fig. 6.12, $PQ = PR$, $RS = RQ$ and $ST \parallel QR$. If the exterior angle RPQ is 140° , then the measure of angle TSR is
- 55°
 - 40°
 - 50°
 - 45°

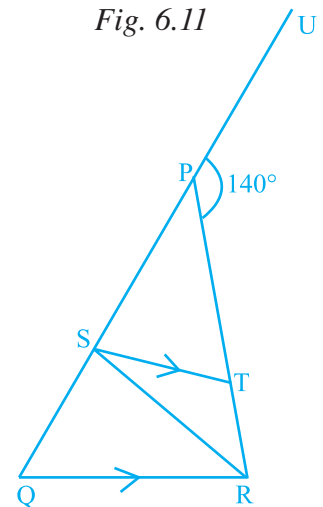


Fig. 6.12

- 24.** In Fig. 6.13, $\angle BAC = 90^\circ$, $AD \perp BC$ and $\angle BAD = 50^\circ$, then $\angle ACD$ is
- 50°
 - 40°
 - 70°
 - 60°

- 25.** If one angle of a triangle is equal to the sum of the other two angles, the triangle is
- obtuse
 - acute
 - right
 - equilateral

- 26.** If the exterior angle of a triangle is 130° and its interior opposite angles are equal, then measure of each interior opposite angle is
- 55°
 - 65°
 - 50°
 - 60°

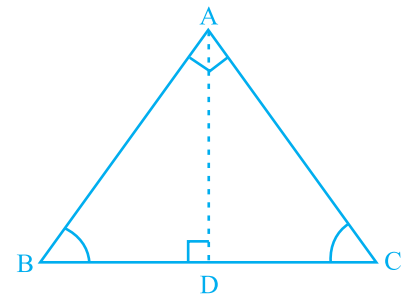


Fig. 6.13

- 27.** If one of the angles of a triangle is 110° , then the angle between the bisectors of the other two angles is
 (a) 70° (b) 110° (c) 35° (d) 145°
- 28.** In $\triangle ABC$, AD is the bisector of $\angle A$ meeting BC at D, $CF \perp AB$ and E is the mid-point of AC. Then median of the triangle is
 (a) AD (b) BE
 (c) FC (d) DE
- 29.** In $\triangle PQR$, if $\angle P = 60^\circ$, and $\angle Q = 40^\circ$, then the exterior angle formed by producing QR is equal to
 (a) 60° (b) 120° (c) 100° (d) 80°
- 30.** Which of the following triplets cannot be the angles of a triangle?
 (a) $67^\circ, 51^\circ, 62^\circ$ (b) $70^\circ, 83^\circ, 27^\circ$
 (c) $90^\circ, 70^\circ, 20^\circ$ (d) $40^\circ, 132^\circ, 18^\circ$
- 31.** Which of the following can be the length of the third side of a triangle whose two sides measure 18 cm and 14 cm?
 (a) 4 cm (b) 3 cm (c) 5 cm (d) 32 cm
- 32.** How many altitudes does a triangle have?
 (a) 1 (b) 3 (c) 6 (d) 9
- 33.** If we join a vertex to a point on opposite side which divides that side in the ratio 1:1, then what is the special name of that line segment?
 (a) Median (b) Angle bisector
 (c) Altitude (d) Hypotenuse
- 34.** The measures of $\angle x$ and $\angle y$ in Fig. 6.14 are respectively
 (a) $30^\circ, 60^\circ$ (b) $40^\circ, 40^\circ$
 (c) $70^\circ, 70^\circ$ (d) $70^\circ, 60^\circ$
- 35.** If length of two sides of a triangle are 6 cm and 10 cm, then the length of the third side can be
 (a) 3 cm (b) 4 cm (c) 2 cm (d) 6 cm
- 36.** In a right-angled triangle ABC, if angle B = 90° , BC = 3 cm and AC = 5 cm, then the length of side AB is
 (a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

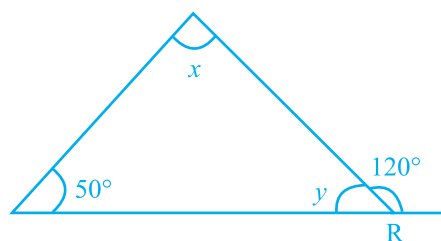


Fig. 6.14

- 37.** In a right-angled triangle ABC, if angle B = 90°, then which of the following is true?
- (a) $AB^2 = BC^2 + AC^2$ (b) $AC^2 = AB^2 + BC^2$
 (c) $AB = BC + AC$ (d) $AC = AB + BC$
- 38.** Which of the following figures will have its altitude outside the triangle?

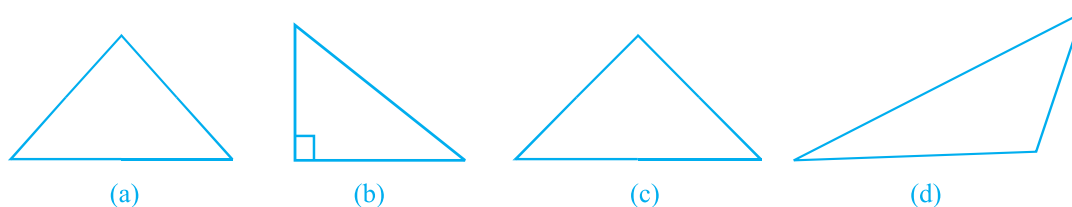
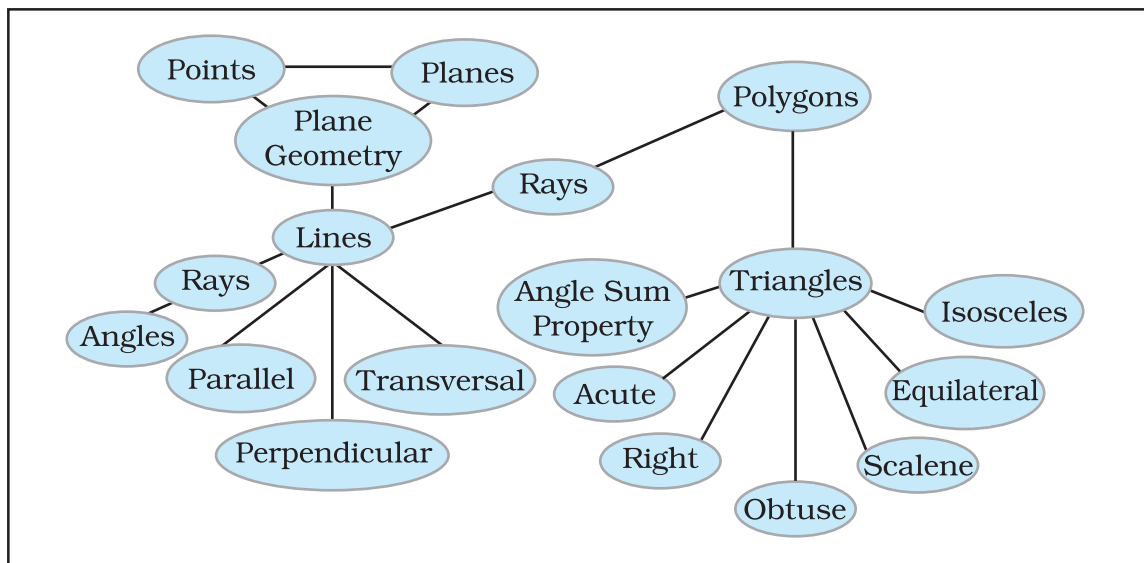


Fig. 6.15



- 39.** In Fig. 6.16, if $AB \parallel CD$, then

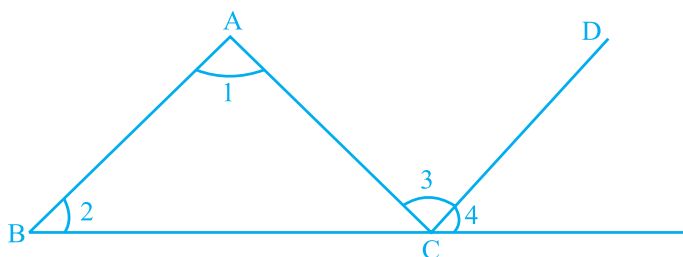


Fig. 6.16

- (a) $\angle 2 = \angle 3$ (b) $\angle 1 = \angle 4$
 (c) $\angle 4 = \angle 1 + \angle 2$ (d) $\angle 1 + \angle 2 = \angle 3 + \angle 4$

- 40.** In $\triangle ABC$, $\angle A = 100^\circ$, AD bisects $\angle A$ and $AD \perp BC$. Then, $\angle B$ is equal to
 (a) 80° (b) 20° (c) 40° (d) 30°
- 41.** In $\triangle ABC$, $\angle A = 50^\circ$, $\angle B = 70^\circ$ and bisector of $\angle C$ meets AB in D (Fig. 6.17). Measure of $\angle ADC$ is.

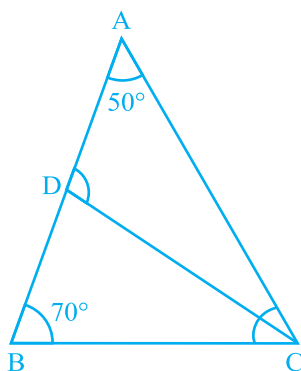


Fig. 6.17

- (a) 50° (b) 100° (c) 30° (d) 70°
- 42.** If for $\triangle ABC$ and $\triangle DEF$, the correspondence $CAB \leftrightarrow EDF$ gives a congruence, then which of the following is not true?
 (a) $AC = DE$ (b) $AB = EF$
 (c) $\angle A = \angle D$ (d) $\angle C = \angle E$

- 43.** In Fig. 6.18, M is the mid-point of both AC and BD . Then

- (a) $\angle 1 = \angle 2$ (b) $\angle 1 = \angle 4$
 (c) $\angle 2 = \angle 4$ (d) $\angle 1 = \angle 3$

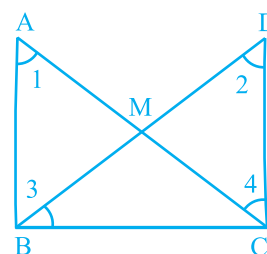


Fig. 6.18

- 44.** If D is the mid-point of the side BC in $\triangle ABC$ where $AB = AC$, then $\angle ADC$ is
 (a) 60° (b) 45°
 (c) 120° (d) 90°

45. Two triangles are congruent, if two angles and the side included between them in one of the triangles are equal to the two angles and the side included between them of the other triangle. This is known as the

- (a) RHS congruence criterion
- (b) ASA congruence criterion
- (c) SAS congruence criterion
- (d) AAA congruence criterion

46. By which congruency criterion, the two triangles in Fig. 6.19 are congruent?

- (a) RHS
- (b) ASA
- (c) SSS
- (d) SAS

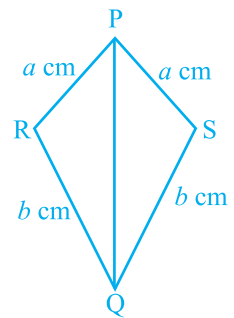


Fig. 6.19

47. By which of the following criterion two triangles cannot be proved congruent?

- (a) AAA
- (b) SSS
- (c) SAS
- (d) ASA

48. If $\triangle PQR$ is congruent to $\triangle STU$ (Fig. 6.20), then what is the length of TU?

- (a) 5 cm
- (b) 6 cm
- (c) 7 cm
- (d) cannot be determined

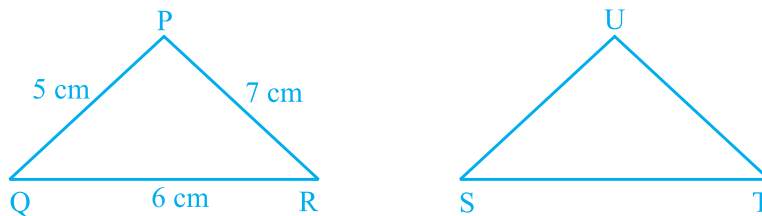


Fig. 6.20

49. If $\triangle ABC$ and $\triangle DBC$ are on the same base BC, $AB = DC$ and $AC = DB$ (Fig. 6.21), then which of the following gives a congruence relationship?

- (a) $\triangle ABC \cong \triangle DBC$
- (b) $\triangle ABC \cong \triangle CBD$
- (c) $\triangle ABC \cong \triangle DCB$
- (d) $\triangle ABC \cong \triangle BCD$

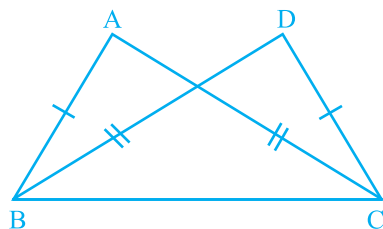


Fig. 6.21

In questions 50 to 69, fill in the blanks to make the statements true.

50. The _____ triangle always has altitude outside itself.
51. The sum of an exterior angle of a triangle and its adjacent angle is always _____.
52. The longest side of a right angled triangle is called its _____.
53. Median is also called _____ in an equilateral triangle.
54. Measures of each of the angles of an equilateral triangle is _____.
55. In an isosceles triangle, two angles are always _____.
56. In an isosceles triangle, angles opposite to equal sides are _____.
57. If one angle of a triangle is equal to the sum of other two, then the measure of that angle is _____.
58. Every triangle has at least _____ acute angle (s).
59. Two line segments are congruent, if they are of _____ lengths.
60. Two angles are said to be _____, if they have equal measures.
61. Two rectangles are congruent, if they have same _____ and _____.
62. Two squares are congruent, if they have same _____.
63. If $\triangle PQR$ and $\triangle XYZ$ are congruent under the correspondence $QPR \leftrightarrow XYZ$, then

(i) $\angle R =$ _____	(ii) $QR =$ _____
(iii) $\angle P =$ _____	(iv) $QP =$ _____
(v) $\angle Q =$ _____	(vi) $RP =$ _____

64. In Fig. 6.22, $\triangle PQR \cong \triangle$ _____

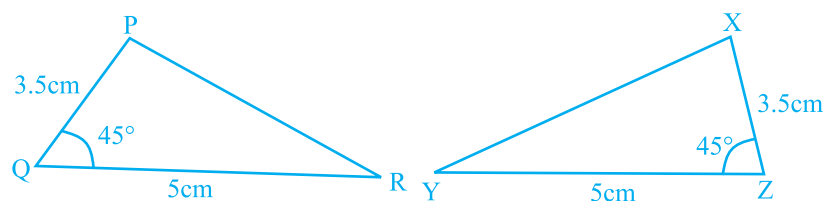


Fig. 6.22

65. In Fig. 6.23, $\triangle PQR \cong \triangle$ _____

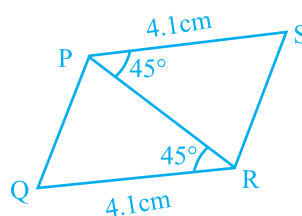


Fig. 6.23

66. In Fig. 6.24, \triangle _____ $\cong \triangle PQR$

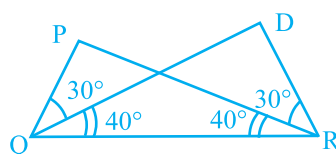


Fig. 6.24

67. In Fig. 6.25, $\triangle ARO \cong \triangle$ _____

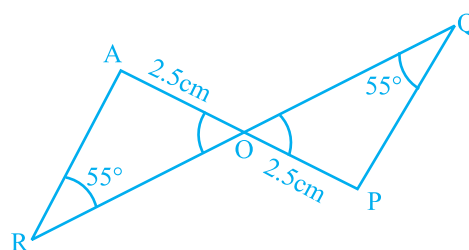


Fig. 6.25

68. In Fig. 6.26, $AB = AD$ and $\angle BAC = \angle DAC$. Then

(i) \triangle _____ $\cong \triangle ABC$.

(ii) $BC =$ _____.

(iii) $\angle BCA = \underline{\hspace{2cm}}$.

(iv) Line segment AC bisects $\underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$.

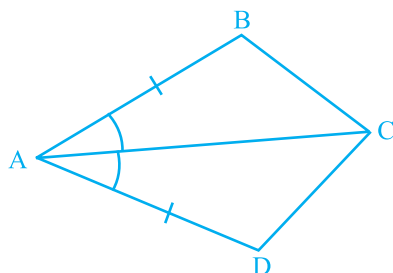


Fig. 6.26

69. In Fig. 6.27,

(i) $\angle TPQ = \angle \underline{\hspace{1cm}} + \angle \underline{\hspace{1cm}}$

(ii) $\angle UQR = \angle \underline{\hspace{1cm}} + \angle \underline{\hspace{1cm}}$

(iii) $\angle PRS = \angle \underline{\hspace{1cm}} + \angle \underline{\hspace{1cm}}$

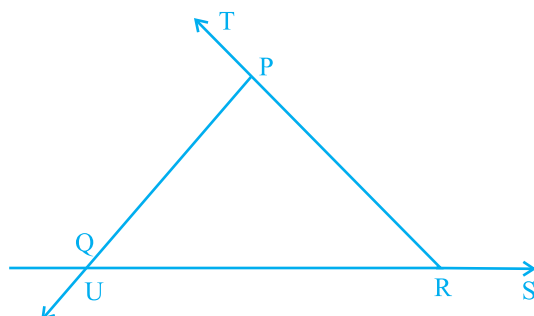


Fig. 6.27

In questions 70 to 106 state whether the statements are True or False.

70. In a triangle, sum of squares of two sides is equal to the square of the third side.
71. Sum of two sides of a triangle is greater than or equal to the third side.
72. The difference between the lengths of any two sides of a triangle is smaller than the length of third side.
73. In $\triangle ABC$, $AB = 3.5$ cm, $AC = 5$ cm, $BC = 6$ cm and in $\triangle PQR$, $PR = 3.5$ cm, $PQ = 5$ cm, $RQ = 6$ cm. Then $\triangle ABC \cong \triangle PQR$.

74. Sum of any two angles of a triangle is always greater than the third angle.
75. The sum of the measures of three angles of a triangle is greater than 180° .
76. It is possible to have a right-angled equilateral triangle.
77. If M is the mid-point of a line segment AB, then we can say that AM and MB are congruent.
78. It is possible to have a triangle in which two of the angles are right angles.
79. It is possible to have a triangle in which two of the angles are obtuse.
80. It is possible to have a triangle in which two angles are acute.
81. It is possible to have a triangle in which each angle is less than 60° .
82. It is possible to have a triangle in which each angle is greater than 60° .
83. It is possible to have a triangle in which each angle is equal to 60° .
84. A right-angled triangle may have all sides equal.
85. If two angles of a triangle are equal, the third angle is also equal to each of the other two angles.
86. In Fig. 6.28, two triangles are congruent by RHS.
87. The congruent figures super impose each other completely.
88. A one rupee coin is congruent to a five rupee coin.
89. The top and bottom faces of a kaleidoscope are congruent.
90. Two acute angles are congruent.
91. Two right angles are congruent.
92. Two figures are congruent, if they have the same shape.
93. If the areas of two squares is same, they are congruent.
94. If the areas of two rectangles are same, they are congruent.
95. If the areas of two circles are the same, they are congruent.
96. Two squares having same perimeter are congruent.
97. Two circles having same circumference are congruent.

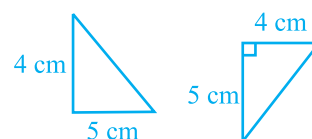


Fig. 6.28

- 98.** If three angles of two triangles are equal, triangles are congruent.
- 99.** If two legs of a right triangle are equal to two legs of another right triangle, then the right triangles are congruent.
- 100.** If two sides and one angle of a triangle are equal to the two sides and angle of another triangle, then the two triangles are congruent.
- 101.** If two triangles are congruent, then the corresponding angles are equal.
- 102.** If two angles and a side of a triangle are equal to two angles and a side of another triangle, then the triangles are congruent.
- 103.** If the hypotenuse of one right triangle is equal to the hypotenuse of another right triangle, then the triangles are congruent.
- 104.** If hypotenuse and an acute angle of one right triangle are equal to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent.
- 105.** AAS congruence criterion is same as ASA congruence criterion.
- 106.** In Fig. 6.29, $AD \perp BC$ and AD is the bisector of angle BAC . Then, $\triangle ABD \cong \triangle ACD$ by RHS.

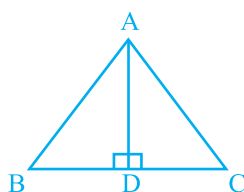


Fig. 6.29

- 107.** The measure of three angles of a triangle are in the ratio $5 : 3 : 1$. Find the measures of these angles.
- 108.** In Fig. 6.30, find the value of x .

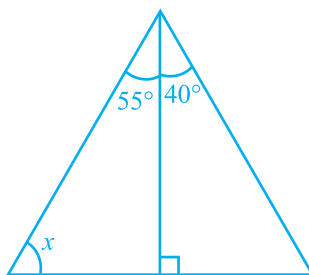


Fig. 6.30

- 109.** In Fig. 6.31(i) and (ii), find the values of a , b and c .

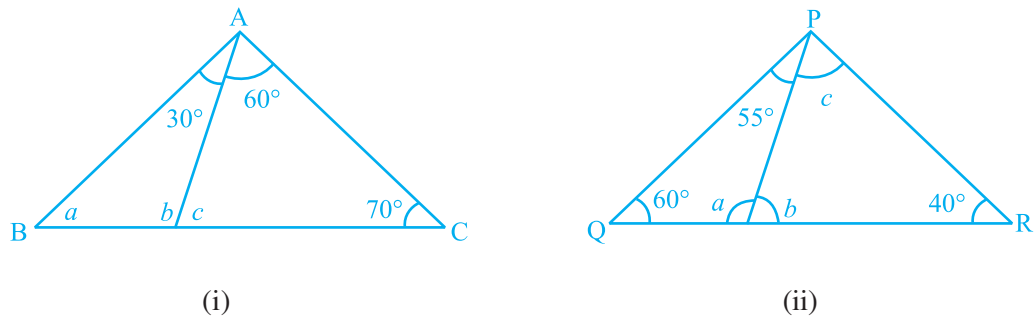


Fig. 6.31

- 110.** In triangle XYZ, the measure of angle X is 30° greater than the measure of angle Y and angle Z is a right angle. Find the measure of $\angle Y$.
- 111.** In a triangle ABC, the measure of angle A is 40° less than the measure of angle B and 50° less than that of angle C. Find the measure of $\angle A$.
- 112.** I have three sides. One of my angle measures 15° . Another has a measure of 60° . What kind of a polygon am I? If I am a triangle, then what kind of triangle am I?
- 113.** Jiya walks 6 km due east and then 8 km due north. How far is she from her starting place?
- 114.** Jayanti takes shortest route to her home by walking diagonally across a rectangular park. The park measures 60 metres \times 80 metres. How much shorter is the route across the park than the route around its edges?



Understand the Problem

- If you write a problem in your own words, you may understand it better. Before writing a problem in your own words, you may need to read it over several times – perhaps aloud, so you can hear yourself say the words.
- Once you have written the problem in your own words, you may want to make sure you included all of the necessary information to solve the problem.

- 115.** In $\triangle PQR$ of Fig. 6.32, $PQ = PR$. Find the measures of $\angle Q$ and $\angle R$.

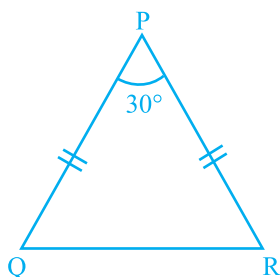


Fig. 6.32

- 116.** In Fig. 6.33, find the measures of $\angle x$ and $\angle y$.

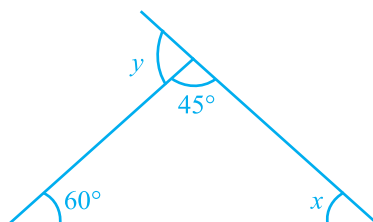


Fig. 6.33

- 117.** In Fig. 6.34, find the measures of $\angle PON$ and $\angle NPO$.

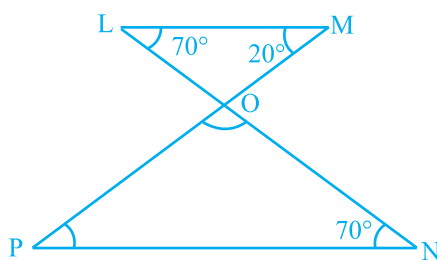


Fig. 6.34

- 118.** In Fig. 6.35, $QP \parallel RT$. Find the values of x and y .

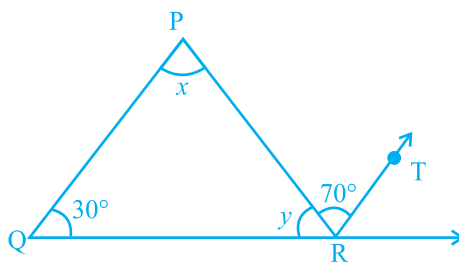


Fig. 6.35

- 119.** Find the measure of $\angle A$ in Fig. 6.36.

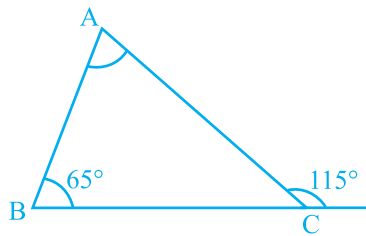


Fig. 6.36

- 120.** In a right-angled triangle if an angle measures 35° , then find the measure of the third angle.
- 121.** Each of the two equal angles of an isosceles triangle is four times the third angle. Find the angles of the triangle.
- 122.** The angles of a triangle are in the ratio $2 : 3 : 5$. Find the angles.
- 123.** If the sides of a triangle are produced in an order, show that the sum of the exterior angles so formed is 360° .
- 124.** In $\triangle ABC$, if $\angle A = \angle C$, and exterior angle $ABX = 140^\circ$, then find the angles of the triangle.

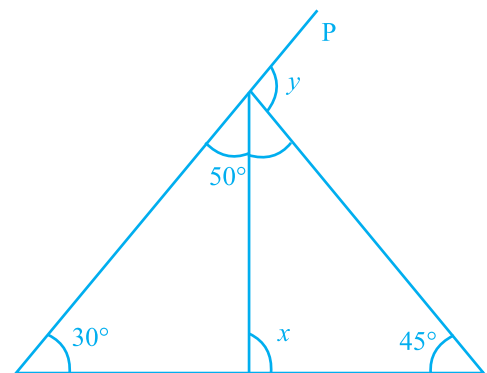


Fig. 6.37

- 125.** Find the values of x and y in Fig. 6.37.



Plan a Strategy

- Concept maps are visual tools for organising information. A concept map shows how key concepts are related and can help you summarise and analyse information in lessons or chapters.

Create a Concept Map

- Give your concept map a title.
- Identify the main idea of your concept map.
- List the key concepts.
- Link the concepts to show the relationships between the concepts and the main idea.

- 126.** Find the value of x in Fig. 6.38.

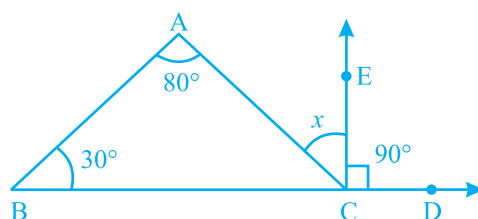


Fig. 6.38

- 127.** The angles of a triangle are arranged in descending order of their magnitudes. If the difference between two consecutive angles is 10° , find the three angles.
- 128.** In $\triangle ABC$, $DE \parallel BC$ (Fig. 6.39). Find the values of x , y and z .

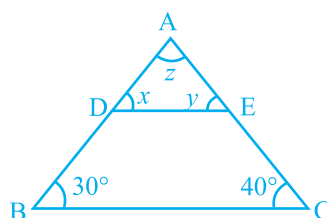


Fig. 6.39

- 129.** In Fig. 6.40, find the values of x , y and z .

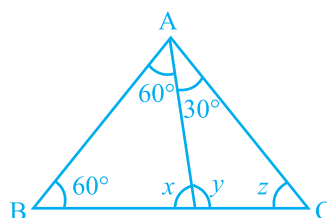


Fig. 6.40

- 130.** If one angle of a triangle is 60° and the other two angles are in the ratio 1 : 2, find the angles.
- 131.** In $\triangle PQR$, if $3\angle P = 4\angle Q = 6\angle R$, calculate the angles of the triangle.
- 132.** In $\triangle DEF$, $\angle D = 60^\circ$, $\angle E = 70^\circ$ and the bisectors of $\angle E$ and $\angle F$ meet at O. Find (i) $\angle F$ (ii) $\angle EOF$.
- 133.** In Fig. 6.41, $\triangle PQR$ is right-angled at P. U and T are the points on line QRF. If $QP \parallel ST$ and $US \parallel RP$, find $\angle S$.

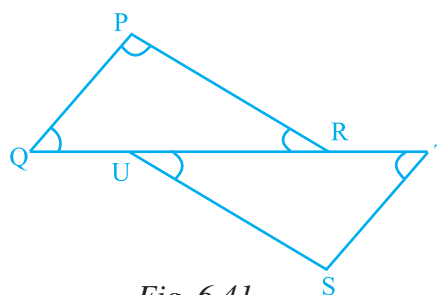
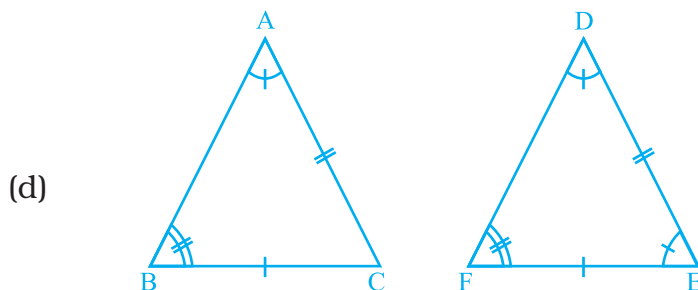
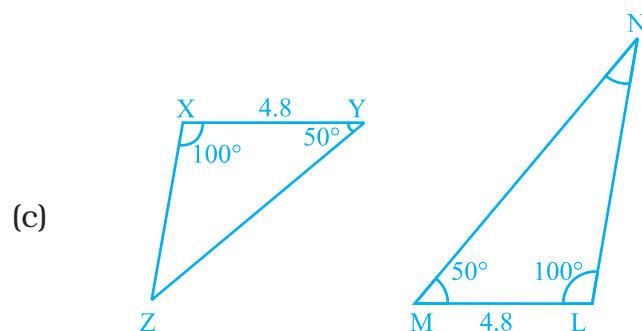
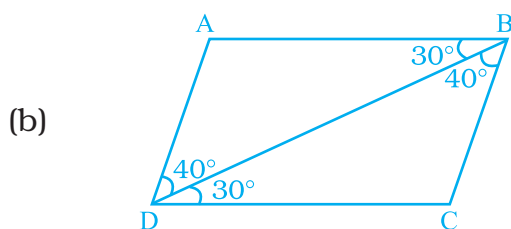
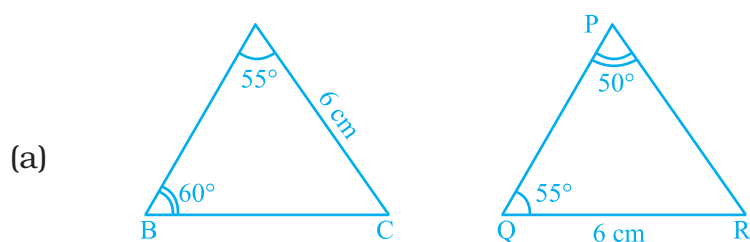


Fig. 6.41

- 134.** In each of the given pairs of triangles of Fig. 6.42, applying only ASA congruence criterion, determine which triangles are congruent. Also, write the congruent triangles in symbolic form.



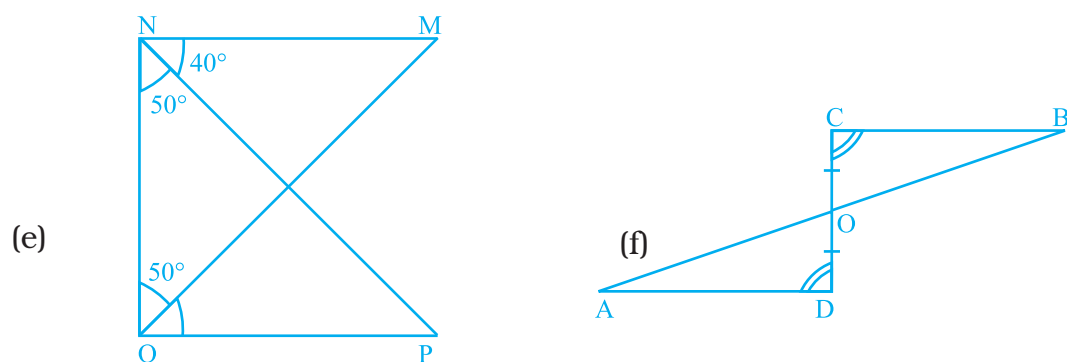
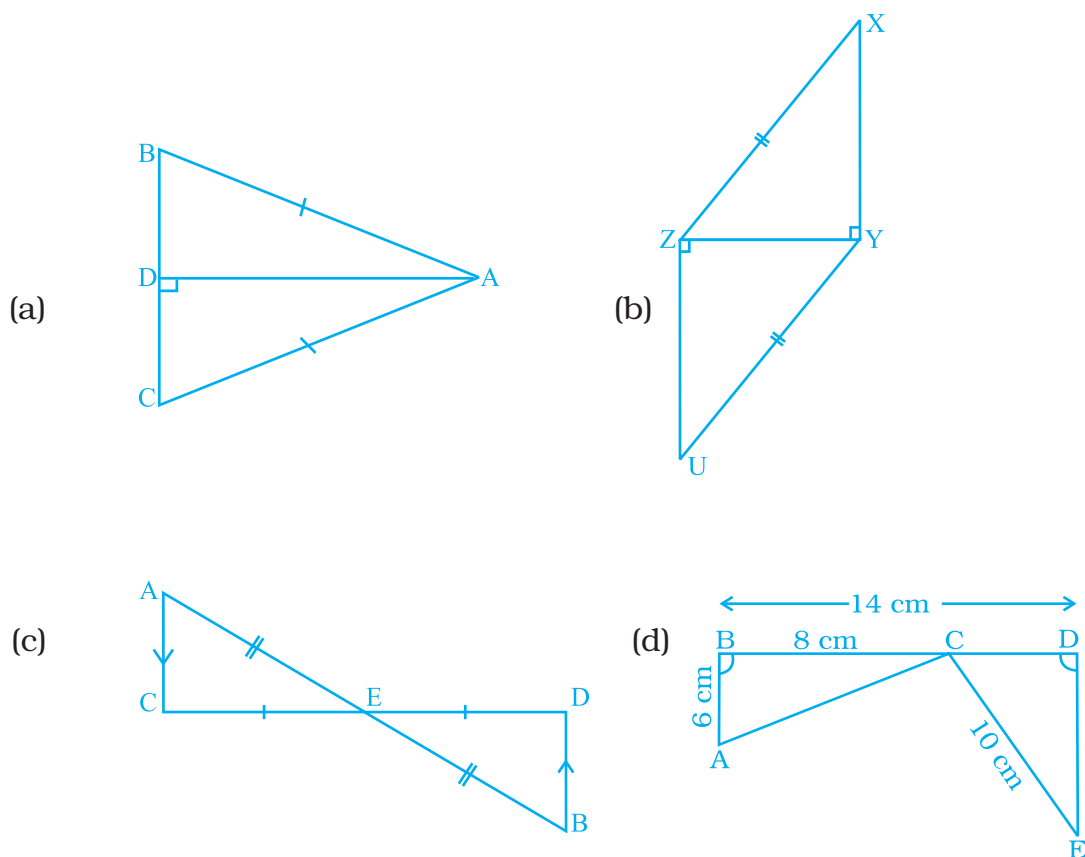


Fig. 6.42

- 135.** In each of the given pairs of triangles of Fig. 6.43, using only RHS congruence criterion, determine which pairs of triangles are congruent. In case of congruence, write the result in symbolic form:



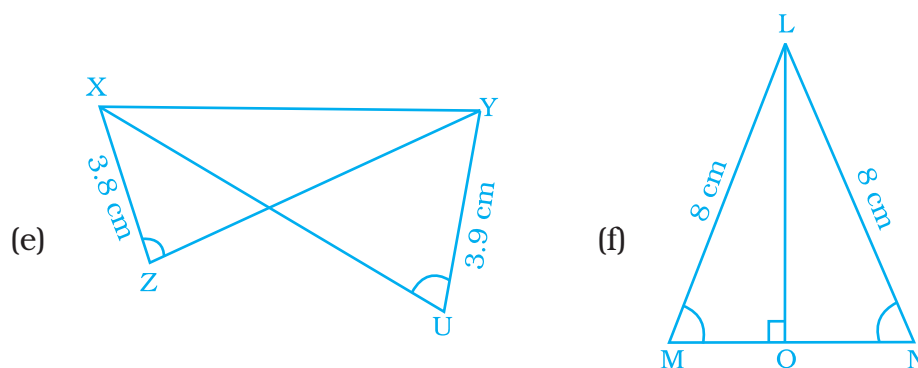


Fig. 6.43

- 136.** In Fig. 6.44, if $RP = RQ$, find the value of x .

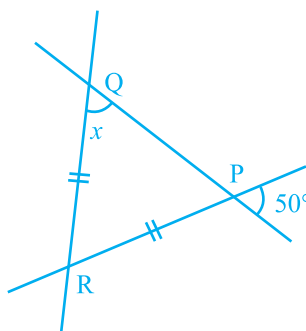


Fig. 6.44

- 137.** In Fig. 6.45, if $ST = SU$, then find the values of x and y .

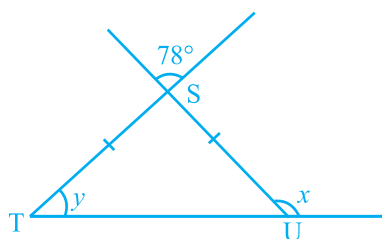


Fig. 6.45

- 138.** Check whether the following measures (in cm) can be the sides of a right-angled triangle or not.
1.5, 3.6, 3.9
- 139.** Height of a pole is 8 m. Find the length of rope tied with its top from a point on the ground at a distance of 6 m from its bottom.

- 140.** In Fig. 6.46, if y is five times x , find the value of z .

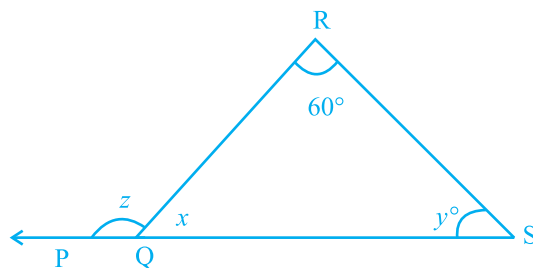
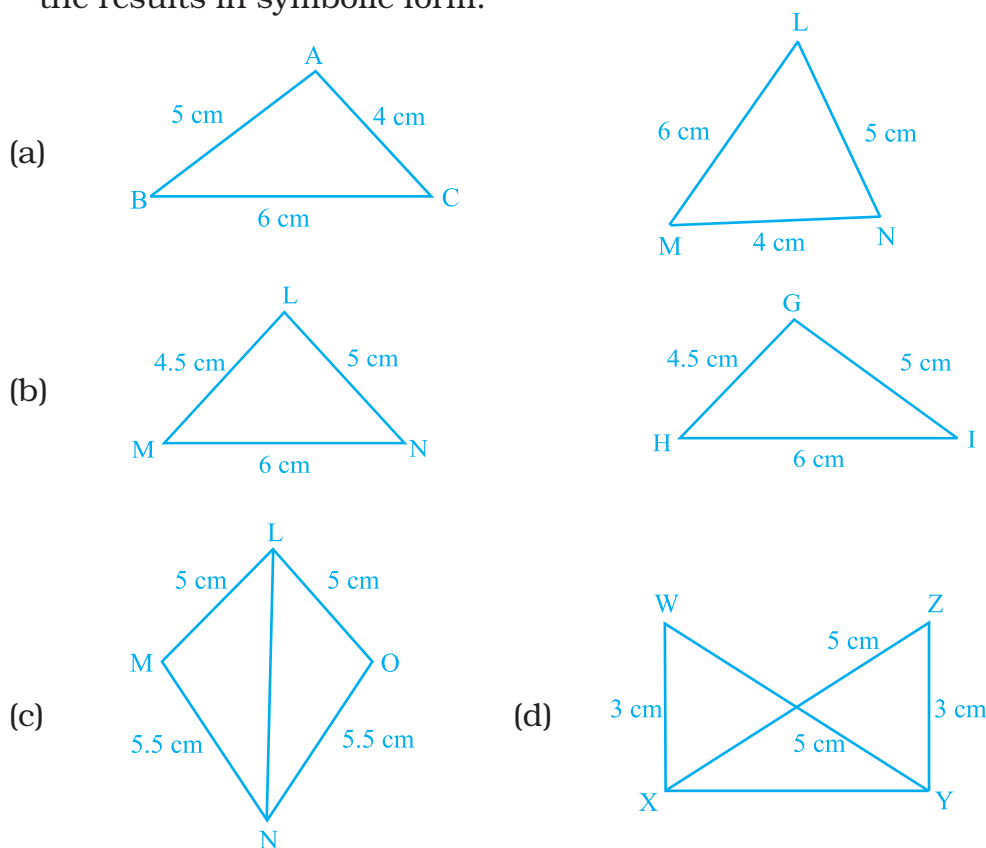


Fig. 6.46

- 141.** The lengths of two sides of an isosceles triangle are 9 cm and 20 cm. What is the perimeter of the triangle? Give reason.
- 142.** Without drawing the triangles write all six pairs of equal measures in each of the following pairs of congruent triangles.
- (a) $\triangle STU \cong \triangle DEF$ (b) $\triangle ABC \cong \triangle LMN$
 (c) $\triangle YZX \cong \triangle PQR$ (d) $\triangle XYZ \cong \triangle MLN$
- 143.** In the following pairs of triangles of Fig. 6.47, the lengths of the sides are indicated along the sides. By applying SSS congruence criterion, determine which triangles are congruent. If congruent, write the results in symbolic form.



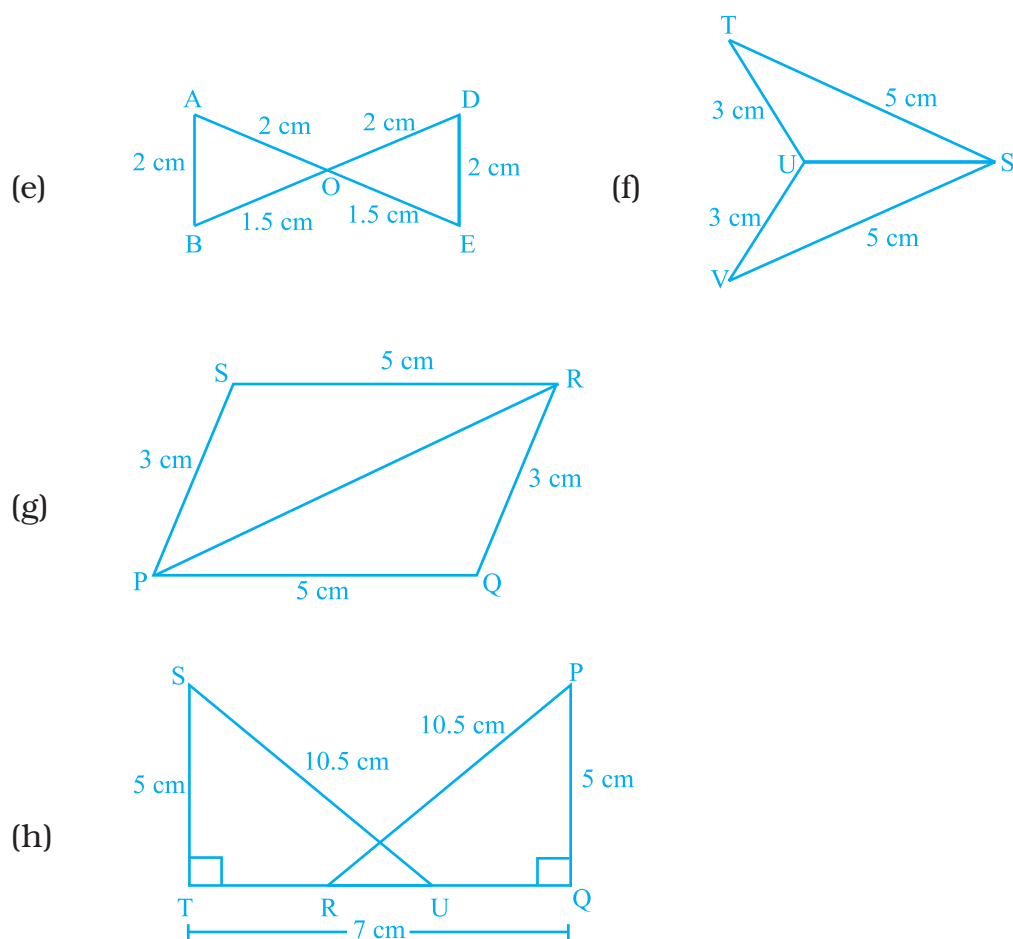


Fig. 6.47

144. ABC is an isosceles triangle with $AB = AC$ and D is the mid-point of base BC (Fig. 6.48).

- State three pairs of equal parts in the triangles ABD and ACD.
- Is $\triangle ABD \cong \triangle ACD$. If so why?

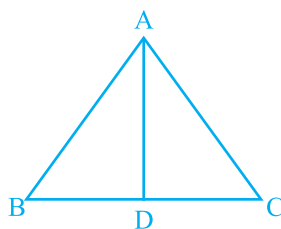


Fig. 6.48

145. In Fig. 6.49, it is given that $LM = ON$ and $NL = MO$

- (a) State the three pairs of equal parts in the triangles NOM and MLN .
- (b) Is $\triangle NOM \cong \triangle MLN$. Give reason?

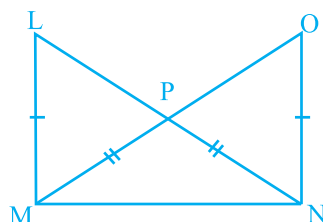


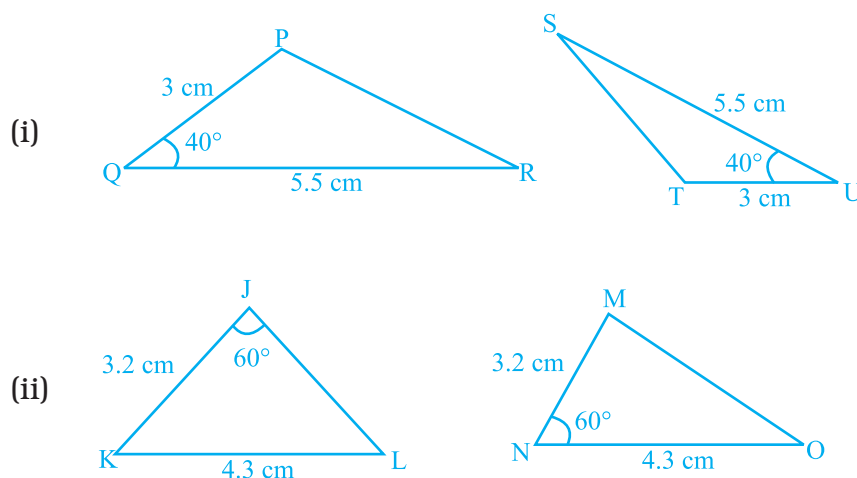
Fig. 6.49

146. Triangles DEF and LMN are both isosceles with $DE = DF$ and $LM = LN$, respectively. If $DE = LM$ and $EF = MN$, then, are the two triangles congruent? Which condition do you use?

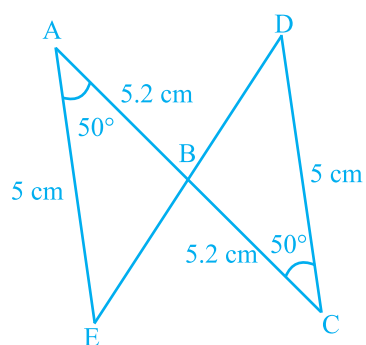
If $\angle E = 40^\circ$, what is the measure of $\angle N$?

147. If $\triangle PQR$ and $\triangle SQR$ are both isosceles triangle on a common base QR such that P and S lie on the same side of QR . Are triangles PSQ and PSR congruent? Which condition do you use?

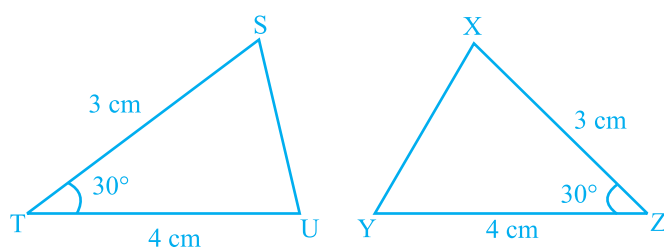
148. In Fig. 6.50, which pairs of triangles are congruent by SAS congruence criterion (condition)? If congruent, write the congruence of the two triangles in symbolic form.



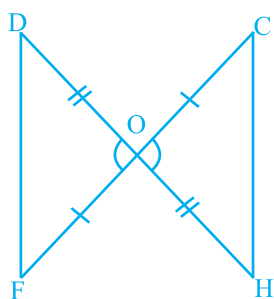
(iii)



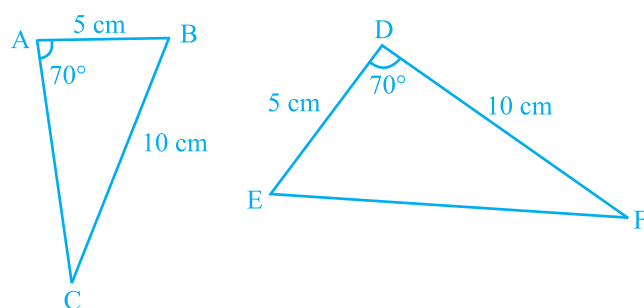
(iv)



(v)



(vi)



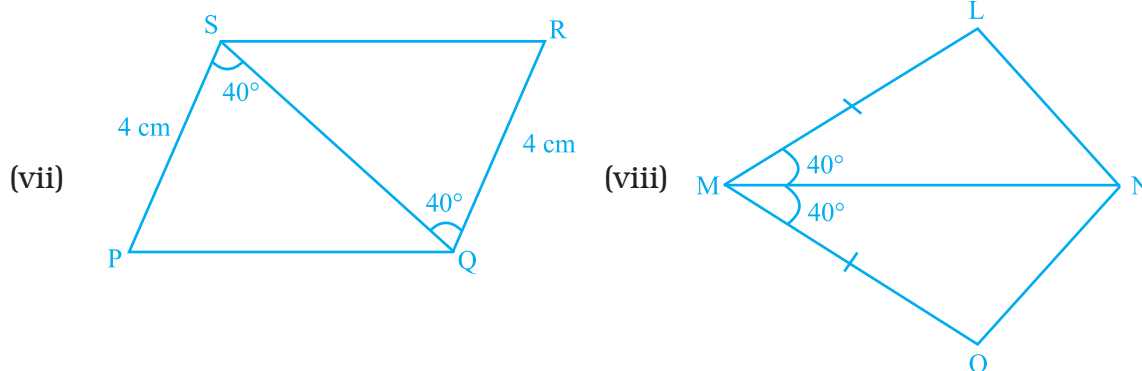


Fig. 6.50

149. State which of the following pairs of triangles are congruent. If yes, write them in symbolic form (you may draw a rough figure).

- (a) $\triangle PQR$: $PQ = 3.5$ cm, $QR = 4.0$ cm, $\angle Q = 60^\circ$
 $\triangle STU$: $ST = 3.5$ cm, $TU = 4$ cm, $\angle T = 60^\circ$
- (b) $\triangle ABC$: $AB = 4.8$ cm, $\angle A = 90^\circ$, $AC = 6.8$ cm
 $\triangle XYZ$: $YZ = 6.8$ cm, $\angle X = 90^\circ$, $ZX = 4.8$ cm

150. In Fig. 6.51, $PQ = PS$ and $\angle 1 = \angle 2$.

- (i) Is $\triangle PQR \cong \triangle PSR$? Give reasons.
 (ii) Is $QR = SR$? Give reasons.

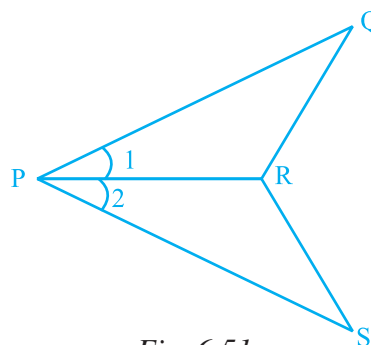


Fig. 6.51

151. In Fig. 6.52, $DE = IH$, $EG = FI$ and $\angle E = \angle I$. Is $\triangle DEF \cong \triangle HIG$? If yes, by which congruence criterion?

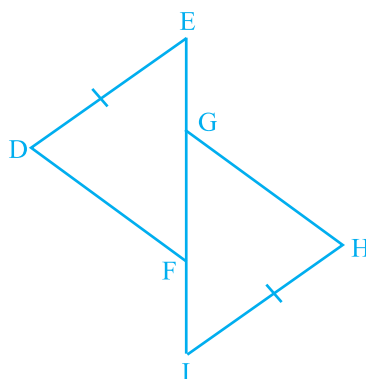


Fig. 6.52

152. In Fig. 6.53, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$.

- (i) Is $\triangle ADC \cong \triangle ABC$? Why?
- (ii) Show that $AD = AB$ and $CD = CB$.

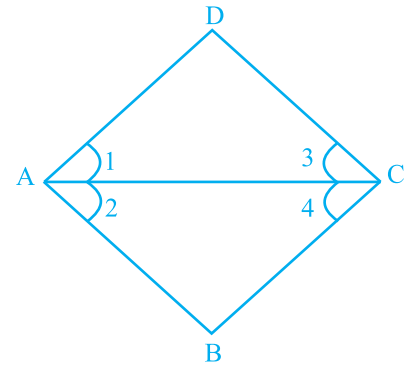


Fig. 6.53

153. Observe Fig. 6.54 and state the three pairs of equal parts in triangles ABC and DCB.

- (i) Is $\triangle ABC \cong \triangle DCB$? Why?
- (ii) Is $AB = DC$? Why?
- (iii) Is $AC = DB$? Why?

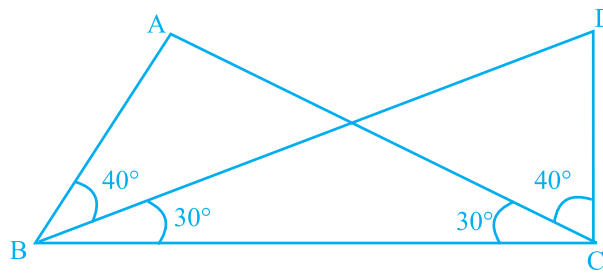


Fig. 6.54

154. In Fig. 6.55, $QS \perp PR$, $RT \perp PQ$ and $QS = RT$.

- (i) Is $\triangle QSR \cong \triangle RTQ$? Give reasons.
- (ii) Is $\angle PQR = \angle PRQ$? Give reasons.

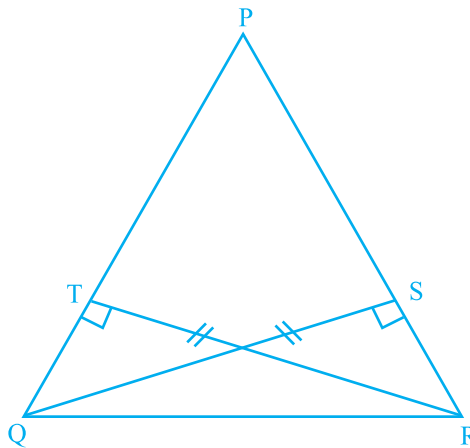


Fig. 6.55

155. Points A and B are on the opposite edges of a pond as shown in Fig. 6.56. To find the distance between the two points, the surveyor makes a right-angled triangle as shown. Find the distance AB.

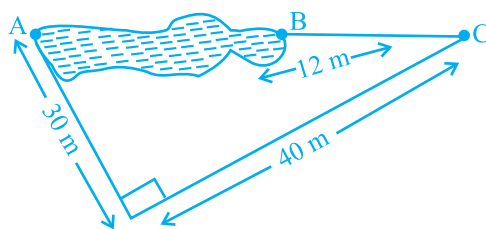


Fig. 6.56

- 156.** Two poles of 10 m and 15 m stand upright on a plane ground. If the distance between the tops is 13 m, find the distance between their feet.
- 157.** The foot of a ladder is 6 m away from its wall and its top reaches a window 8 m above the ground, (a) Find the length of the ladder. (b) If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its top reach?
- 158.** In Fig. 6.57, state the three pairs of equal parts in $\triangle ABC$ and $\triangle EOD$. Is $\triangle ABC \cong \triangle EOD$? Why?

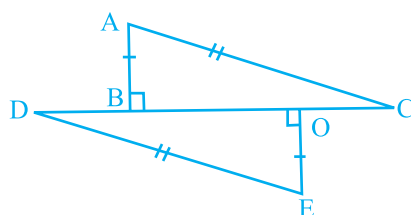


Fig. 6.57

(D) Applications

1. Draw an equilateral triangle of side 6 cm, an isosceles triangle of base 3 cm and equal sides 6 cm each and a scalene triangle of sides 3 cm, 6 cm and 7 cm. Now draw a median and an altitude in each triangle from the top vertex, measure and tabulate the lengths of all the medians and altitude's of respective triangles. What can you conclude from this activity (This activity can also be done by paper folding)?
2. Draw two triangles which have a pair of corresponding sides equal but are not congruent.
3. Draw two triangles which have two pairs of corresponding sides equal but are not congruent.

4. Draw two triangles, which have one pair of corresponding angles equal and one pair of corresponding sides equal but are not congruent.
5. Draw two triangles which have three pairs of corresponding angles equal but are not congruent.

Solve the given cross number/word and then fill up the given boxes in activities 6 and 7. Clues are given below for across as well as downward fillings. For across and downward clue numbers are written at the corner of boxes. Answers of clues have to fill up in their respective boxes.

Cross Number Puzzle 6

Across

- (a) If 6, 8, m are the sides of a right triangle, then the value of m is _____.
- (b) In $\triangle ABC$, AC is the longest side, then what can be the measure of angle B (in degree), if the three angles of triangle are 120° , 40° , 20° ?
- (c) In a right-angled triangle, one acute angle measures twice the other angle, then the smaller angle shall measure _____.
- (d) If three angles in $\triangle ABC$ are in the ratio $2 : 3 : 5$, then measure of $\angle B$ is _____.
- (e) Length of third side of a triangle whose two sides are 5 cm and 6 cm, must be less than _____.
- (f) The perimeter of $\triangle ABC$ in Fig. 6.58 is _____.

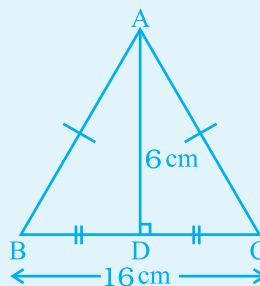


Fig. 6.58

Down

- (a) In an isosceles triangle if one of the equal angles measures 35° , then the third angle is _____.
- (b) In Fig. 6.59, the value of x is _____.

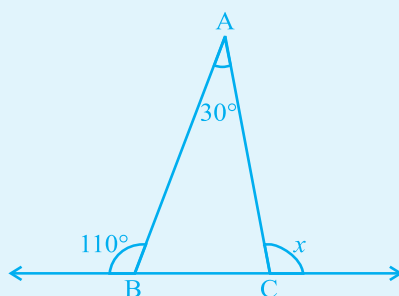
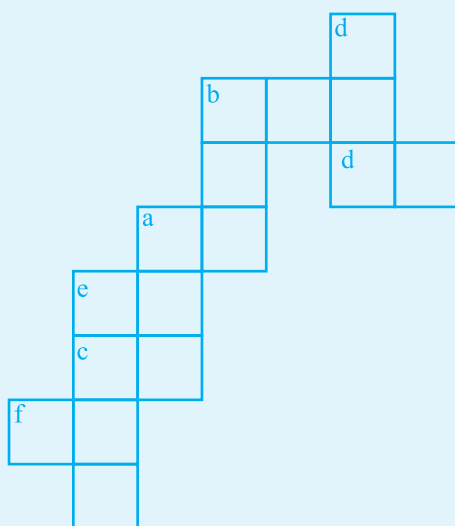


Fig. 6.59

- (c) The sum of the angles in a quadrilateral is _____.
- (d) In $\triangle ABC$, $\angle B = 80^\circ$, $\angle A = 30^\circ$, the bisectors of $\angle B$ and $\angle C$ meet at O. The measure of $\angle BOC$ is _____.



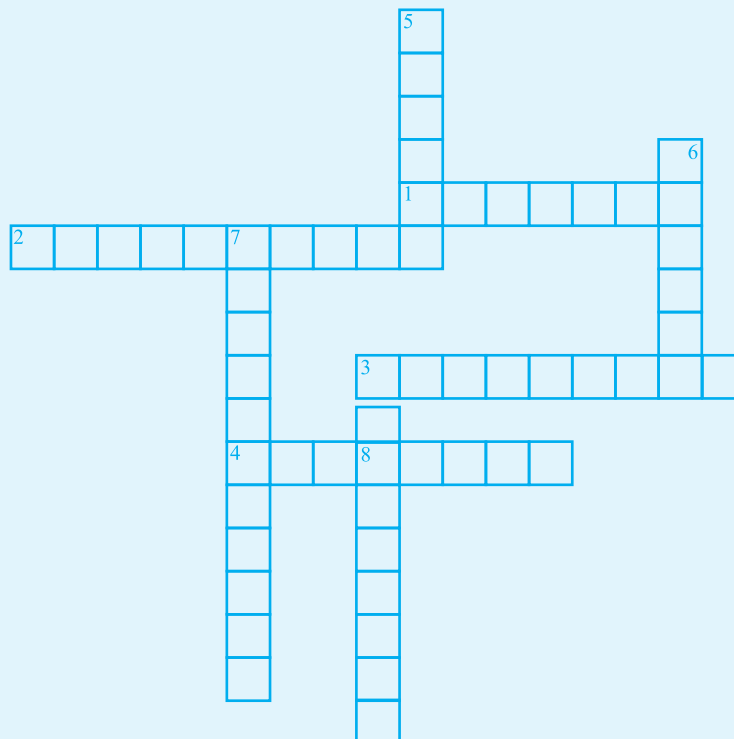
Cross Word Puzzle 7

Across

1. A triangle with all its sides unequal.
2. The longest side of a right-angled triangle.
3. Two squares having same side lengths.
4. Line segment drawn from a vertex of a triangle perpendicular to its opposite side.

Down

5. A type of triangle in which altitude falls outside the triangle.
6. A line segment joining vertex with the mid-point of the opposite side.
7. A regular triangle.
8. In a parallelogram, the line segment that divides it into two congruent triangles.



UNIT 7

COMPARING QUANTITIES

(A) Main Concepts and Results

- To compare two quantities, their units must be the same.
- Two ratios can be compared by converting them into like fractions. If the two fractions are equal, we say that the two given ratios are equivalent.
- If two ratios are equivalent (or equal), then the involved four quantities are said to be in proportion.
- One of the ways of comparing quantities is percentage. Per cent is derived from Latin word 'per centum' meaning 'per hundred'.
- Percent is represented by the symbol % and means hundredth too.
- Fractions can be converted into percentages and vice-versa.
- Decimals can also be converted into percentages and vice-versa.
- The buying price of any item is known as its cost price. It is written in short as CP.
- The price at which an item is sold, is known as its selling price or in short SP.
- If $CP < SP$, then a profit is made and $\text{Profit} = SP - CP$.
- If $CP = SP$, there is no profit or loss.
- If $CP > SP$, then a loss is made and $\text{Loss} = CP - SP$.
- Profit per cent = $\frac{\text{Profit}}{CP} \times 100$
- Loss per cent = $\frac{\text{Loss}}{CP} \times 100$

- 'Principal' P, means the borrowed money.
- The extra money paid by borrower for using borrowed money for given time is called 'Interest' I.
- The period for which the money is borrowed is called 'Time Period' T.
- To determine Interest to be paid, we have 'Rate of Interest'.
- Rate of Interest is generally given in per cent per year.
- On a principal of ₹ P at R % rate of interest per year, the interest

(simple) I paid for T years is given by $I = \frac{P \times R \times T}{100}$.

- The total money paid alongwith interest or principal P is called amount (A). Thus $A = P + I$.

(B) Solved Examples

In Examples 1 to 3, there are four options, out of which one is correct. Choose the correct one.

Example 1: The ratio of the heights 1.50 m and 75 cm of two persons can be written as

- (a) 1 : 50 (b) 1 : 5 (c) 2 : 1 (d) 1 : 2

Solution: Correct answer is (c).

CROSS PRODUCTS

Cross products in proportions are equal. If the ratios are not in proportion, the cross products are not equal.

Proportions	Not Proportions
$\begin{array}{c} \frac{6}{8} = \frac{9}{12} \\ 6 \cdot 12 = 8 \cdot 9 \\ 72 = 72 \end{array}$	$\begin{array}{c} \frac{5}{2} = \frac{15}{6} \\ 5 \cdot 6 = 2 \cdot 15 \\ 30 = 30 \end{array}$
$\begin{array}{c} \frac{1}{6} = \frac{2}{7} \\ 1 \cdot 7 \neq 6 \cdot 2 \\ 7 \neq 12 \end{array}$	$\begin{array}{c} \frac{5}{12} = \frac{2}{5} \\ 5 \cdot 5 \neq 12 \cdot 2 \\ 25 \neq 24 \end{array}$

Example 2: Out of 50 children in a class, 20 are boys. Then the percentage of girls is

- (a) 60 (b) 30 (c) 50 (d) $66\frac{2}{3}$

Solution: Correct answer is (a).

Example 3: The interest on ₹ 5000 at the rate of 15% per annum for one month is

- (a) ₹ 750 (b) ₹ 75 (c) ₹ 625 (d) ₹ 62.50

Solution: Correct answer is (d).

In Examples 4 and 5, fill in the blanks to make the statements true.

Example 4: If two ratios are equivalent, then the four quantities are said to be in _____.

Solution: Proportion

Example 5: 40% of 250 km is _____.

Solution: 100 km.

Think and Discuss

1. **Describe** how two ratios can form a proportion.
2. **Give** three ratios equivalent to 12 : 24.
3. **Explain** why the ratios 2 : 4 and 6 : 10 do not form a proportion.
4. **Give an example** of two ratios that are proportional and have numerators with different signs.

In Examples 6 and 7, state whether the statements are True or False.

Example 6: If 25% of a journey is 800 km, the total distance of the journey is 3000 km.

Solution: False

Example 7: 0.05 is equivalent to 5%.

Solution: True

Example 8: Suhana sells a sofa set for ₹ 9600 making a profit of 20%. What is the C.P. of the sofa set?

Solution: Let the CP be ₹ 100
 Profit (20%) = ₹ 20
 Therefore, SP = ₹ (100 + 20) = ₹ 120
 If SP is ₹ 120, CP = ₹ 100

$$\text{If SP is ₹ 9600, CP} = \frac{100}{120} \times 9600$$

$$= ₹ 8000$$

An alternate method to solve the same example is:

Profit = 20% of CP SP = CP + Profit

So, 9600 = CP + 20% of CP

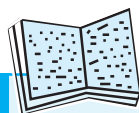
$$= \text{CP} + \frac{20}{100} \times \text{CP}$$

$$= \left(1 + \frac{1}{5}\right) \text{CP}$$

$$= \frac{6}{5} \text{CP}$$

$$\text{Therefore, } 9600 \times \frac{5}{6} = \text{CP}$$

$$\text{or} \quad \text{CP} = ₹ 8000$$



Vocabulary

1. The word cross can mean “to intersect,” forming an “X” shape. Since a product is the result of multiplying, what do you suppose you multiply to find the **cross products** of two fractions?
2. The word indirect means “not direct”. What do you think it means to find the length of something using **indirect measurement** ?
3. A **ratio** compares two quantities using a particular operation. Knowing what you do about rational numbers, which operation do you think you use in a ratio?

Example 9: John borrowed ₹ 75000 from his friend and after one year returned ₹ 80000 to his friend. Find the interest.

Solution: Principal = ₹ 75000
 Amount = ₹ 80000
 Interest = Amount – Principal
 = ₹ 80000 – ₹ 75000
 = ₹ 5000

Per cent	Decimal	Fraction
5%	0.05	$\frac{1}{20}$
10%	0.1	$\frac{1}{10}$
25%	0.25	$\frac{1}{4}$
33.3%	0.3	$\frac{1}{3}$

Example 10: If Meenakshee pays an interest of ₹ 1500 for 4 years on a sum of ₹ 2500, find the rate of interest per annum(p.a.)

Solution: P = ₹ 2500, T = 4 years, I = ₹ 1500
 R = ?

Now,
$$I = \frac{P \times R \times T}{100}$$

Therefore,
$$1500 = \frac{2500 \times R \times 4}{100}$$

$$R = \frac{1500 \times 100}{2500 \times 4} = 15$$

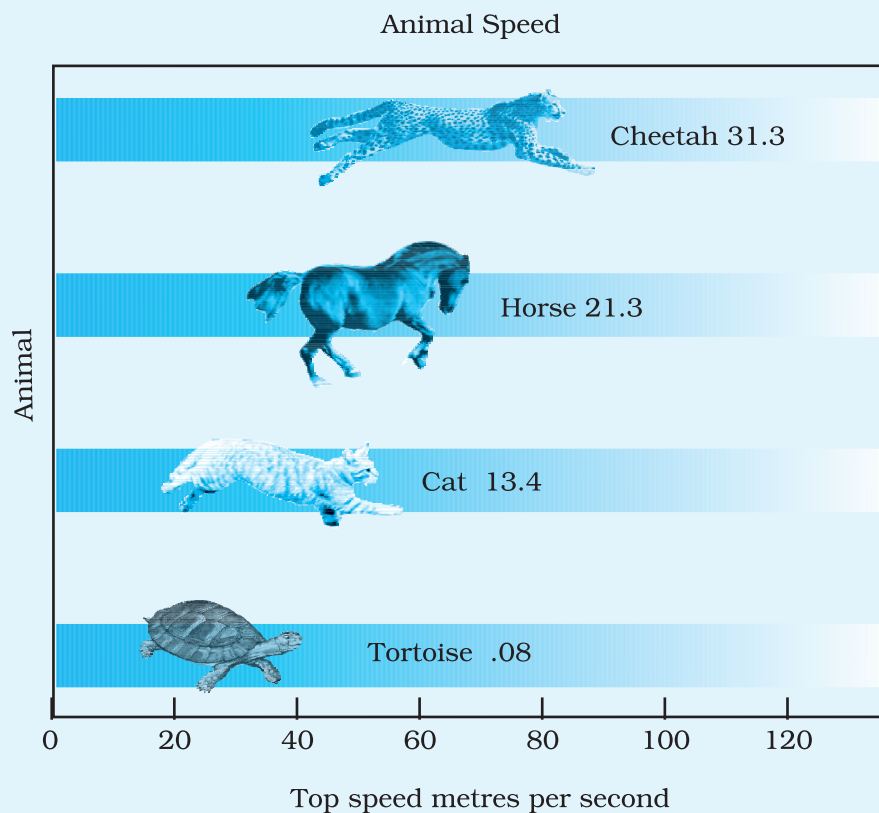
So, the rate of interest is 15%.

Application on Problem Solving Strategy



Example 11

Refer to the graphic. If a cheetah and tortoise travel at their top speeds for 1 minute; how much farther does the cheetah travel?



Solution:



Understand and Explore the Problem

- **What do you know?**

We know the top speeds for a Cheetah and a Tortoise in m/sec.

- **What are you trying to find?**

We need to find the difference in the distances travelled by Cheetah and the tortoise in 1 minute.



Plan a Strategy

- Begin by determining the distance travelled by each animal in 1 minute.
- 1 min = 60 seconds.
- Multiply each top speed (m/s) by 60.
- Subtract to find the difference of the distances travelled by two animals.



Solve

- $31.3 \times 60 = 1878 \text{ m}$ (Distance Cheetah travels in 1 minute)
- $.08 \times 60 = 4.8 \text{ m}$ (Distance tortoise travels in 1 minute)
- $1878 \text{ m} - 4.8 \text{ m} = 1873.2 \text{ m}$ (Distance travelled by Cheetah farther than tortoise in one minute).



Revise

- **Working backward**

$$\text{Speed of Cheetah} = \frac{D}{T} = \frac{1878 \text{ m}}{60 \text{ s}} = 31.3 \text{ m/s}$$

$$\text{Speed of Tortoise} = \frac{D}{T} = \frac{4.8 \text{ m}}{60 \text{ s}} = 0.08 \text{ m/s}$$

Hence, our answer is correct.

Think and Discuss

1. Find the ratio of speeds Cheetah and Tortoise in m/s with the given data.
2. Discuss with your friends to estimate the top speeds of other animals and verify it by searching the available data in other books.

(C) Exercise

In questions 1 to 23, there are four options, out of which one is correct. write the correct one.

1. 20% of 700 m is
(a) 560 m (b) 70 m (c) 210 m (d) 140 m
2. Gayatri's income is ₹ 1,60,000 per year. She pays 15% of this as house rent and 10% of the remainder on her child's education. The money left with her is
(a) ₹136000 (b) ₹120000 (c) ₹122400 (d) ₹14000
3. The ratio of Fatima's income to her savings is 4 : 1. The percentage of money saved by her is :
(a) 20% (b) 25% (c) 40% (d) 80%
4. 0.07 is equal to
(a) 70% (b) 7% (c) 0.7% (d) 0.07%
5. In a scout camp, 40% of the scouts were from Gujarat State and 20% of these were from Ahmedabad. The percentage of scouts in the camp from Ahmedabad is:
(a) 25 (b) 32.5 (c) 8 (d) 50
6. What percent of ₹ 4500 is ₹ 9000?
(a) 200 (b) $\frac{1}{2}$ (c) 2 (d) 50

Ratio	Decimal	Per cent
$\frac{3}{10} = \frac{30}{100}$	0.30	30 %
$\frac{1}{2} = \frac{50}{100}$	0.50	50 %
$\frac{3}{4} = \frac{75}{100}$	0.75	75 %

7. 5.2 is equal to

- (a) 52% (b) 5.2% (c) 520% (d) 0.52%

Think and Discuss

1. **Determine** the ratios that are nearly equivalent to each of the following per cents : 23%, 53%, 65%, 12% and 76%.
2. **Describe** how to find 35% of a number when you know 10% of the number.

8. The ratio 3 : 8 is equal to

- (a) 3.75% (b) 37.5% (c) 0.375% (d) 267%

9. 225% is equal to

- (a) 9 : 4 (b) 4 : 9 (c) 3 : 2 (d) 2 : 3

10. A bicycle is purchased for ₹ 1800 and is sold at a profit of 12%. Its selling price is

- (a) ₹ 1584 (b) ₹ 2016 (c) ₹ 1788 (d) ₹ 1812

11. A cricket bat was purchased for ₹ 800 and was sold for ₹ 1600. Then profit earned is

- (a) 100% (b) 64% (c) 50% (d) 60%

12. A farmer bought a buffalo for ₹ 44000 and a cow for ₹ 18000. He sold the buffalo at a loss of 5% but made a profit of 10% on the cow. The net result of the transaction is

- (a) loss of ₹ 200 (b) profit of ₹ 400
(c) loss of ₹ 400 (d) profit of ₹ 200

13. If Mohan's income is 25% more than Raman's income, then Raman's income is less than Mohan's income by

- (a) 25% (b) 80% (c) 20% (d) 75%

14. The interest on ₹ 30000 for 3 years at the rate of 15% per annum is

- (a) ₹ 4500 (b) ₹ 9000 (c) ₹ 18000 (d) ₹ 13500

- 15.** Amount received on ₹ 3000 for 2 years at the rate of 11% per annum is
 (a) ₹ 2340 (b) ₹ 3660
 (c) ₹ 4320 (d) ₹ 3330
- 16.** Interest on ₹ 12000 for 1 month at the rate of 10 % per annum is
 (a) ₹ 1200 (b) ₹ 600
 (c) ₹ 100 (d) ₹ 12100
- 17.** Rajni and Mohini deposited ₹ 3000 and ₹ 4000 in a company at the rate of 10% per annum for 3 years and $2\frac{1}{2}$ years respectively. The difference of the amounts received by them will be
 (a) ₹ 100 (b) ₹ 1000
 (c) ₹ 900 (d) ₹ 1100
- 18.** If 90% of x is 315 km, then the value of x is
 (a) 325 km (b) 350 km
 (c) 405 km (d) 340 km
- 19.** On selling an article for ₹ 329, a dealer lost 6%. The cost price of the article is
 (a) ₹ 310.37 (b) ₹ 348.74 (c) ₹ 335 (d) ₹ 350
- 20.** $\frac{25\% \text{ of } 50\% \text{ of } 100\%}{25 \times 50}$ is equal to
 (a) 1.1% (b) 0.1% (c) 0.01% (d) 1%
- 21.** The sum which will earn a simple interest of ₹ 126 in 2 years at 14% per annum is
 (a) ₹ 394 (b) ₹ 395 (c) ₹ 450 (d) ₹ 540

Mental Math

You may find it helpful to equivalent per cents, decimals and fractions

$$12\frac{1}{2}\% = 0.125 = \frac{1}{8}$$

$$20\% = 0.2 = \frac{1}{5}$$

$$25\% = 0.25 = \frac{1}{4}$$

$$33\frac{1}{3}\% \approx 0.33 = \frac{1}{3}$$

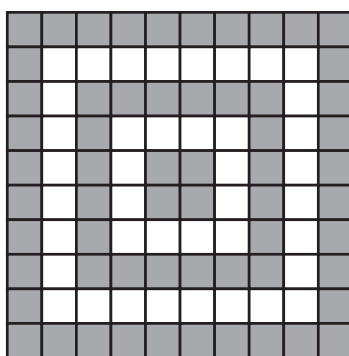
$$40\% = 0.4 = \frac{2}{5}$$

$$50\% = 0.5 = \frac{1}{2}$$

$$60\% = 0.6 = \frac{3}{5}$$

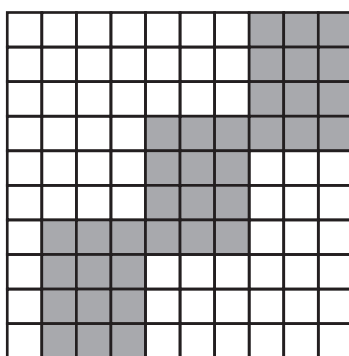
$$80\% = 0.8 = \frac{4}{5}$$

22. The per cent that represents the unshaded region in the figure.



- (a) 75% (b) 50% (c) 40% (d) 60%

23. The per cent that represents the shaded region in the figure is



- (a) 36% (b) 64% (c) 27% (d) 48%

In each of the questions 24 to 59, fill in the blanks to make the statements true.

24. $2 : 3 = \underline{\hspace{1cm}} \%$

Per cent Problem	Equation	Proportion
Finding the per cent of a number	$15\% \text{ of } 120 = n$	$\frac{15}{100} = \frac{n}{120}$
Finding the per cent of one number with another number	$p\% \text{ of } 120 = 18$	$\frac{p}{100} = \frac{18}{120}$
Finding a number when the per cent is known	$15\% \text{ of } n = 18$	$\frac{15}{100} = \frac{18}{n}$

25. $18\frac{3}{4}\% = \underline{\hspace{2cm}} : \underline{\hspace{2cm}}$

26. 30% of ₹ 360 = $\underline{\hspace{2cm}}$.

27. 120 % of 50 km = $\underline{\hspace{2cm}}$.

28. 2.5 = $\underline{\hspace{2cm}}\%$

29. $\frac{8}{5} = \underline{\hspace{2cm}}\%$

30. A $\underline{\hspace{2cm}}$ with its denominator 100 is called a per cent.

31. 15 kg is $\underline{\hspace{2cm}}\%$ of 50 kg.

32. Weight of Nikhil increased from 60 kg to 66 kg. Then, the increase in weight is $\underline{\hspace{2cm}}\%$.

33. In a class of 50 students, 8 % were absent on one day. The number of students present on that day was $\underline{\hspace{2cm}}$.

34. Savitri obtained 440 marks out of 500 in an examination. She secured $\underline{\hspace{2cm}}\%$ marks in the examination.

35. Out of a total deposit of ₹ 1500 in her bank account, Abida withdrew 40% of the deposit. Now the balance in her account is $\underline{\hspace{2cm}}$.

36. $\underline{\hspace{2cm}}$ is 50% more than 60.

37. John sells a bat for ₹ 75 and suffers a loss of ₹ 8. The cost price of the bat is $\underline{\hspace{2cm}}$.

38. If the price of sugar is decreased by 20%, then the new price of 3kg sugar originally costing ₹ 120 will be $\underline{\hspace{2cm}}$.

39. Mohini bought a cow for ₹ 9000 and sold it at a loss of ₹ 900. The selling price of the cow is $\underline{\hspace{2cm}}$.

Check Understanding

50% of a number is half of that number.

100% of a number is that number.

200% of a number is twice that number.

What is 200% of 5?

What is 300% of 5?

40. Devangi buys a chair for ₹ 700 and sells it for ₹ 750. She earns a profit of _____ % in the transaction.
41. Sonal bought a bed sheet for ₹ 400 and sold it for ₹ 440. Her _____ % is _____.
42. Nasim bought a pen for ₹ 60 and sold it for ₹ 54. His _____ % is _____.
43. Aahuti purchased a house for ₹ 50,59,700 and spent ₹ 40300 on its repairs. To make a profit of 5%, she should sell the house for ₹ _____.
44. If 20 lemons are bought for ₹ 10 and sold at 5 for three rupees, then _____ in the transaction is _____ %.
45. Narain bought 120 oranges at ₹ 4 each. He sold 60 % of the oranges at ₹ 5 each and the remaining at ₹ 3.50 each. His _____ is _____ %.
46. A fruit seller purchased 20 kg of apples at ₹ 50 per kg. Out of these, 5% of the apples were found to be rotten. If he sells the remaining apples at ₹ 60 per kg, then his _____ is _____ %.
47. Interest on ₹ 3000 at 10% per annum for a period of 3 years is _____.
48. Amount obtained by depositing ₹ 20,000 at 8 % per annum for six months is _____.
49. Interest on ₹ 12500 at 18% per annum for a period of 2 years and 4 months is _____.
50. 25 ml is _____ per cent of 5 litres.
51. If A is increased by 20%, it equals B. If B is decreased by 50%, it equals C. Then _____ % of A is equal to C.

Think and Discuss

1. **Explain** whether a 150% increase or a 150% decrease is possible.
2. **Compare** finding a 20% increase for finding 120% of a number.

52. Interest = $\frac{P \times R \times T}{100}$, where

T is _____

R% is _____ and

P is _____.

53. The difference of interest for 2 years and 3 years on a sum of ₹ 2100 at 8% per annum is _____.

54. To convert a fraction into a per cent, we _____ it by 100.

55. To convert a decimal into a per cent, we shift the decimal point two places to the _____.

56. The _____ of interest on a sum of ₹ 2000 at the rate of 6% per annum for $1\frac{1}{2}$ years and 2 years is ₹ 420.

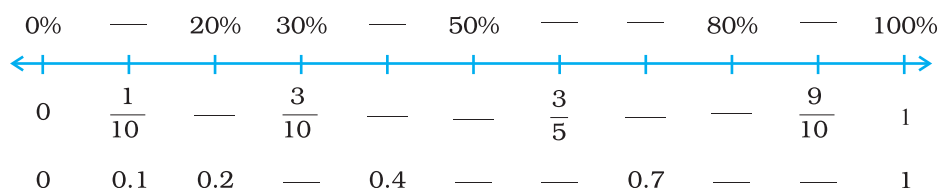
57. When converted into percentage, the value of 6.5 is _____ than 100%.

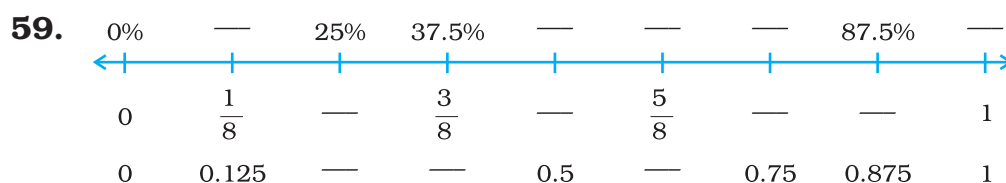
Think and Discuss

1. **Give an example** of a real-world situation in which you would use (1) decimals (2) fractions and (3) per cents.
2. **Show** ₹ 25 as a part of a 100 rupee note in terms of (1) a reduced fraction (2) a per cent and (3) a decimal. Which is most common?
3. **Explain** how you can find a fraction, decimal, or per cent when you have only one form of a number.

In questions 58 and 59, copy each number line. Fill in the blanks so that each mark on the number line is labelled with a per cent, a fraction and a decimal. Write all fractions in lowest terms.

58.





Look in any magazine or newspaper and you're likely to see numbers written as per cents. Listen to any sporting event, and you'll probably hear statistics reported using per cents. Per cents are everywhere. But what are per cents? The word per cent means for each 100, so a per cent like 50% is the same amount as the fraction $\frac{50}{100}$ (or $\frac{1}{2}$) or the decimal 0.50 (or 0.5). Fractions, decimals, and per cents can be used interchangeably to represent parts of a whole quantity.

Often, the word percent is used in connection with the per cent of some quantity. For example, you might hear, "Only 3% of voters voted in the last election." No matter what the quantity, 100% of a quantity always means all of it, and 50% always means half of it. The amount indicated by a certain per cent changes as the size of the quantity changes. For example, 50% of 10 dogs is 5 dogs, but 50% of 100 dogs is 50 dogs.

In questions 60 to 79, state whether the statements are True or False.

60. $\frac{2}{3} = 66\frac{2}{3}\%$.

61. When an improper fraction is converted into percentage then the answer can also be less than 100.

62. 8 hours is 50% of 4 days.

63. The interest on ₹ 350 at 5% per annum for 73 days is ₹ 35.

64. The simple interest on a sum of ₹ P for T years at R% per annum is given by the formula: Simple Interest = $\frac{T \times P \times R}{100}$.

65. $75\% = \frac{4}{3}$.

66. 12% of 120 is 100.

67. If Ankita obtains 336 marks out of 600, then percentage of marks obtained by her is 33.6.

68. 0.018 is equivalent to 8%.

69. 50% of ₹ 50 is ₹ 25.

70. 250 cm is 4% of 1 km.

71. Out of 600 students of a school, 126 go for a picnic. The percentage of students that did not go for the picnic is 75.

72. By selling a book for ₹ 50, a shopkeeper suffers a loss of 10%. The cost price of the book is ₹ 60.

73. If a chair is bought for ₹ 2000 and is sold at a gain of 10%, then selling price of the chair is ₹ 2010.

74. If a bicycle was bought for ₹ 650 and sold for ₹ 585, then the percentage of profit is 10.

75. Sushma sold her watch for ₹ 3320 at a gain of ₹ 320. For earning a gain of 10% she should have sold the watch for ₹ 3300.

76. Interest on ₹ 1200 for $1\frac{1}{2}$ years at the rate of 15% per annum is ₹ 180.

77. Amount received after depositing ₹ 800 for a period of 3 years at the rate of 12% per annum is ₹ 896.

78. ₹ 6400 were lent to Feroz and Rashmi at 15% per annum for $3\frac{1}{2}$ and 5 years respectively. The difference in the interest paid by them is ₹ 150.

79. A vendor purchased 720 lemons at ₹ 120 per hundred. 10% of the lemons were found rotten which he sold at ₹ 50 per hundred. If he sells the remaining lemons at ₹ 125 per hundred, then his profit will be 16%.

Build Understanding

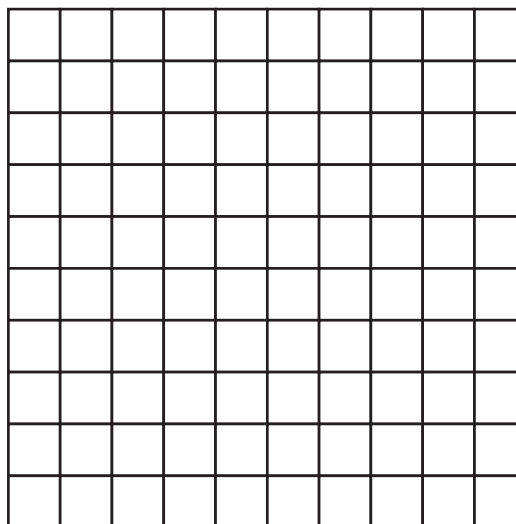
You can use an equation to find the 'percent of a number', 'find what per cent one number is of another', and 'find a number when a per cent of it is known'. Translate the phrase into an equation. Then solve for the unknown. "Is" means equal. "Of" means multiply.

80. Find the value of x if

- (i) 8% of ₹ x is ₹ 100 (ii) 32% of x kg is 400 kg
 (iii) 35% of ₹ x is ₹ 280 (v) 45% of marks x is 405.

81. Imagine that a 10×10 grid has value 300 and that this value is divided evenly among the small squares. In other words, each small square is worth 3. Use a new grid for each part of this problem, and label each grid “Value : 300.”

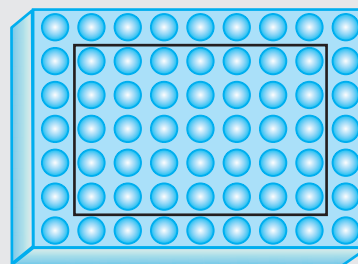
- (a) Shade 25% of the grid. What is 25% of 300? Compare the two answers.
 (b) What is the value of 25 squares?
 (c) Shade 17% of the grid. What is 17% of 300? Compare the two answers.
 (d) What is the value of $\frac{1}{10}$ of the grid?



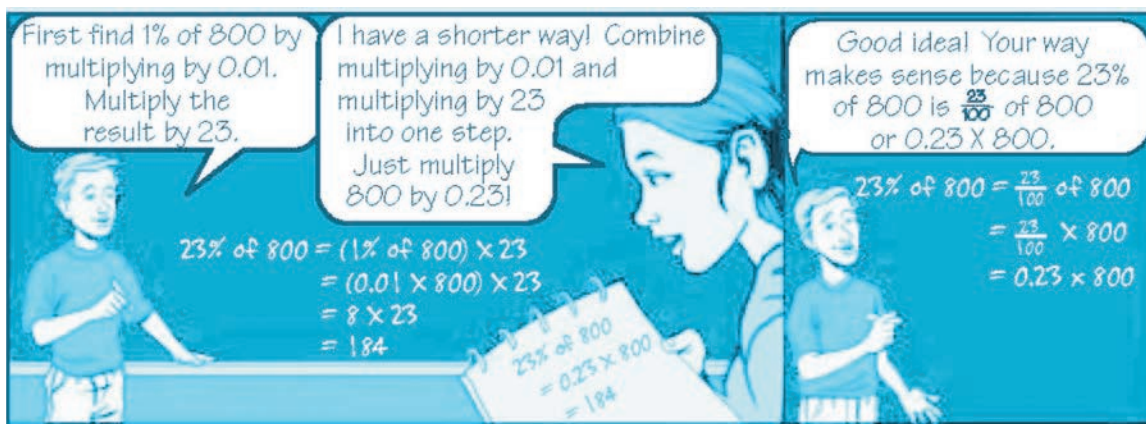
82. Express $\frac{1}{6}$ as a per cent.

The figure shown is a geoboard with a rectangle outlined using a rubberband.

- What is the area of the rectangle?
- Draw a similar figure whose area is 50% larger than this figure.
- Draw a similar figure whose area is 25% larger than this figure.
- Suppose that the figure shown is 75% of another figure, what would the other figure look like?



83. Express $\frac{9}{40}$ as a per cent.
84. Express $\frac{1}{100}$ as a per cent.
85. Express 80% as fraction in its lowest term.
86. Express $33\frac{1}{3}\%$ as a ratio in the lowest term.
87. Express $16\frac{2}{3}\%$ as a ratio in the lowest form.
88. Express 150% as a ratio in the lowest form.
89. Sachin and Sanjana are calculating 23% of 800.



Now calculate 52% of 700 using both the ways described above. Which way do you find easier?



Revise

- Is your answer reasonable?

After you solve a word problem, ask yourself if your answer makes sense. You can round the numbers in the problem and estimate to find a reasonable answer. It may also help to write your answer in sentence form.

90. Write 0.089 as a per cent.
91. Write 1.56 as a per cent.
92. What is 15% of 20?
93. What is 800% of 800?
94. What is 100% of 500?
95. What per cent of 1 hour is 30 minutes?
96. What per cent of 1 day is 1 minute?
97. What per cent of 1 km is 1000 metres?
98. Find out 8% of 25 kg.
99. What percent of ₹ 80 is ₹ 100?
100. 45% of the population of a town are men and 40% are women. What is the percentage of children?
101. The strength of a school is 2000. If 40 % of the students are girls then how many boys are there in the school?
102. Chalk contains 10% calcium, 3% carbon and 12% oxygen. Find the amount of carbon and calcium (in grams) in $2\frac{1}{2}$ kg of chalk.
103. 800 kg of mortar consists of 55% sand, 33% cement and rest lime. What is the mass of lime in mortar?
104. In a furniture shop, 24 tables were bought at the rate of ₹ 450 per table. The shopkeeper sold 16 of them at the rate of ₹ 600 per table and the remaining at the rate of 400 per table. Find her gain or loss percent.
105. Medha deposited 20% of her money in a bank. After spending 20% of the remainder, she has ₹ 4800 left with her. How much did she originally have?
106. The cost of a flower vase got increased by 12%. If the current cost is ₹ 896, what was its original cost?
107. Radhika borrowed ₹ 12000 from her friends. Out of which ₹ 4000 were borrowed at 18% and the remaining at 15% rate of interest per annum. What is the total interest after 3 years?

Think and Discuss

1. **Show** why 5% of a number is less than $\frac{1}{10}$ of the number.
2. **Demonstrate** two ways to find 70% of a number.
3. **Name** fractions in simplest form that are the same as 40% and as 250%.

108. A man travelled 60 km by car and 240 km by train. Find what per cent of total journey did he travel by car and what per cent by train?
109. By selling a chair for ₹ 1440, a shopkeeper loses 10%. At what price did he buy it?
110. Dhruvika invested money for a period from May 2006 to April 2008 at rate of 12% per annum. If interest received by her is ₹ 1620, find the money invested.
111. A person wanted to sell a scooter at a loss of 25%. But at the last moment he changed his mind and sold the scooter at a loss of 20%. If the difference in the two SP's is ₹ 4000, then find the CP of the scooter.
112. The population of a village is 8000. Out of these, 80% are literate and of these literate people, 40% are women. Find the ratio of the number of literate women to the total population.
113. In an entertainment programme, 250 tickets of ₹ 400 and 500 tickets of ₹ 100 were sold. If the entertainment tax is 40% on ticket of ₹ 400 and 20% on ticket of ₹ 100, find how much entertainment tax was collected from the programme.
114. Bhavya earns ₹ 50,000 per month and spends 80% of it. Due to pay revision, her monthly income increases by 20% but due to price rise, she has to spend 20% more. Find her new savings.

115. In an examination, there are three papers each of 100 marks. A candidate obtained 53 marks in the first and 75 marks in the second paper. How many marks must the candidate obtain in the third paper to get an overall of 70 per cent marks?

116. Health Application

A doctor reports blood pressure in millimetres of mercury (mm Hg) as a ratio of systolic blood pressure to diastolic blood pressure (such as 140 over 80). Systolic pressure is measured when the heart beats, and diastolic pressure is measured when it rests. Refer to the table of blood pressure ranges for adults.

Blood Pressure Ranges			
	Normal	Prehypertension	Hypertension (Very High)
Systolic	Under 120 mm Hg	120-139 mm Hg	140 mm Hg and above
Diastolic	Under 80 mm Hg	80-89 mm Hg	90 mm Hg and above

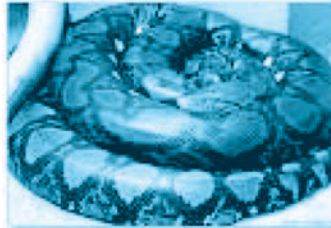
Manohar is a healthy 37 years old man whose blood pressure is in the normal category.

- Calculate an approximate ratio of systolic to diastolic blood pressures in the normal range.
- If Manohar's systolic blood pressure is 102 mm Hg, use the ratio from part (a) to predict his diastolic blood pressure.
- Calculate ratio of average systolic to average diastolic blood pressure in the prehypertension category.

Think and Discuss

- Compare** finding a number when a per cent of the number is known to finding the per cent of one number with that of another number.
- Explain** whether a number is greater than or less than 36, if 22 per cent of the number is 36.

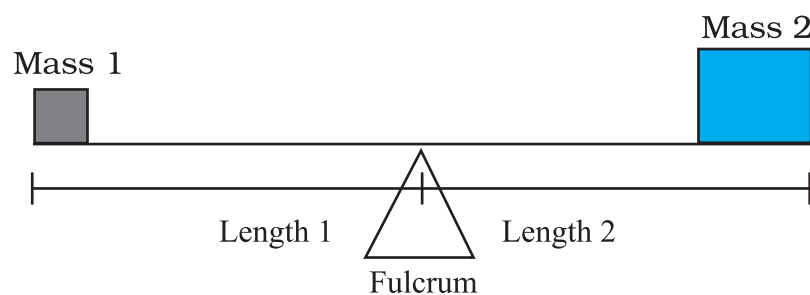
- 117. (a) Science Application:** The king cobra can reach a length of 558 cm. This is only about 60 per cent of the length of the largest reticulated python. Find the length of the largest reticulated python.



- (b) Physical Science Application:** Unequal masses will not balance on a fulcrum if they are at equal distance from it; one side will go up and the other side will go down.

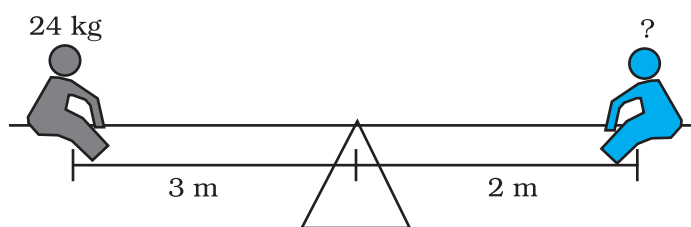
Unequal masses will balance when the following proportion is true:

$$\frac{\text{mass1}}{\text{length2}} = \frac{\text{mass2}}{\text{length1}}$$



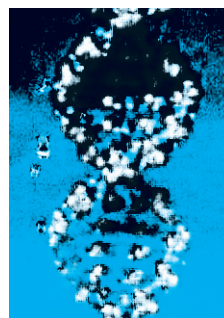
Two children can be balanced on a seesaw when

$\frac{\text{mass1}}{\text{length2}} = \frac{\text{mass2}}{\text{length1}}$. The child on the left and child on the right are balanced. What is the mass of the child on the right?



(c) Life Science Application

A DNA model was built using the scale 2 cm : 0.0000001 mm. If the model of the DNA chain is 17 cm long, what is the length of the actual chain?

**Check Understanding**

Often the time period is not given in years. Write each time period in terms of years.

3 months = _____ year.

4 months = _____ year.

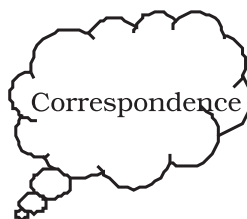
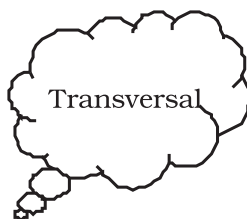
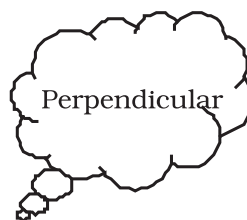
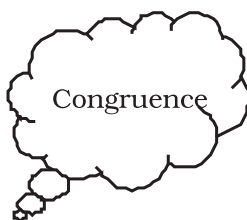
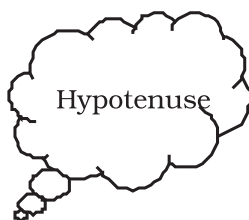
9 months = _____ year.

28 months = _____ years.

42 months = _____ years.

118. Language Application

Given below are few Mathematical terms.

**Find**

- The ratio of consonants to vowels in each of the terms.
- The percentage of consonants in each of the terms.

- 119. What's the Error?** An analysis showed that 0.06 per cent of the T-shirts made by one company were defective. A student says this is 6 out of every 100. What is the student's error?
- 120. What's the Error?** A student said that the ratios $\frac{3}{4}$ and $\frac{9}{16}$ were proportional. What error did the student make?
- 121. What's the Error?** A clothing store charges ₹ 1024 for 4 T-shirts. A student says that the unit price is ₹ 25.6 per T-shirt. What is the error? What is the correct unit price?
- 122.** A tea merchant blends two varieties of tea in the ratio of 5 : 4. The cost of first variety is ₹ 200 per kg and that of second variety is ₹ 300 per kg. If he sells the blended tea at the rate of ₹ 275 per kg, find out the percentage of her profit or loss.
- 123.** A piece of cloth 5 m long shrinks 10 per cent on washing. How long will the cloth be after washing?
- 124.** Nancy obtained 426 marks out of 600 and the marks obtained by Rohit are 560 out of 800. Whose performance is better?
- 125.** A memorial trust donates ₹ 5,00,000 to a school, the interest on which is to be used for awarding 3 scholarships to students obtaining first three positions in the school examination every year. If the donation earns an interest of 12 per cent per annum and the values of the second and third scholarships are ₹ 20,000 and ₹ 15,000 respectively, find out the value of the first scholarship.

Build Understanding

Many sales men work on commission. This means they earn an amount of money that is a per cent of their total sales. The per cent is the commission rate. The amount of money they receive is the commission. Often, their income is a combination of salary (base pay) plus commission.

You can use either a proportion or an equation to solve problems involving commission.

- 126.** Ambika got 99 per cent marks in Mathematics, 76 per cent marks in Hindi, 61 per cent in English, 84 per cent in Science, and 95% in Social Science. If each subject carries 100 marks, then find the percentage of marks obtained by Ambika in the aggregate of all the subjects.
- 127.** What sum of money lent out at 16 per cent per annum simple interest would produce ₹ 9600 as interest in 2 years?

Think and Discuss

1. **Explain** the meaning of each variable in the interest formula.
2. **Tell** what value should be used for t when referring to 6 months.
3. **Name** the different variables in the simple interest formula.
4. **Demonstrate** that doubling the time while halving the interest rate results in the same amount of simple interest.

- 128.** Harish bought a gas-chullah for ₹ 900 and later sold it to Archana at a profit of 5 per cent. Archana used it for a period of two years and later sold it to Babita at a loss of 20 per cent. For how much did Babita get it?
- 129.** Match each of the entries in Column I with the appropriate entries in Column II:

Column I	Column II
(i) 3:5	(A) ₹ 54
(ii) 2.5	(B) ₹ 47
(iii) 100%	(C) ₹ 53
(iv) $\frac{2}{3}$	(D) ₹ 160
(v) $6\frac{1}{4}\%$	(E) 60 %
(vi) 12.5 %	(F) 25 %
(vii) SP when CP = ₹ 50 and loss = 6 %	(G) $\frac{1}{16}$
(viii) SP when CP = ₹ 50 and profit = ₹ 4	(H) 250 %
(ix) Profit% when CP = ₹ 40 and SP = ₹ 50	(I) ₹ 159

- | | |
|---|-----------------------|
| (x) Profit% when CP = ₹ 50 and SP = ₹ 60 | (J) $66\frac{2}{3}\%$ |
| (xi) Interest when principal = ₹ 800,
Rate of interest = 10% per annum
and period = 2 years | (K) 20 % |
| (xii) Amount when principal = ₹ 150,
Rate of interest = 6% per annum
and period = 1 year | (L) 0. 125 |
| | (M) 3 : 2 |
| | (N) ₹ 164 |
| | (O) 3 : 3 |

- 130.** In a debate competition, the judges decide that 20 per cent of the total marks would be given for accent and presentation. 60 per cent of the rest are reserved for the subject matter and the rest are for rebuttal. If this means 8 marks for rebuttal, then find the total marks.

Build Understanding

The **per cent of increase** tells what per cent the amount of increase is of the original number.

To find the per cent of increase, express a ratio of the amount of increase to the original number as a per cent.

$$\text{Per cent of increase} = \frac{\text{amount of increase}}{\text{original number}} \times 100$$

- 131.** Divide ₹ 10000 in two parts so that the simple interest on the first part for 4 years at 12 per cent per annum may be equal to the simple interest on the second part for 4.5 years at 16 per cent per annum.
- 132.** ₹ 9000 becomes ₹ 18000 at simple interest in 8 years. Find the rate per cent per annum.
- 133.** In how many years will the simple interest on a certain sum be 4.05 times the principal at 13.5 per cent per annum?
- 134.** The simple interest on a certain sum for 8 years at 12 per cent per annum is ₹ 3120 more than the simple interest on the same sum for 5 years at 14 per cent per annum. Find the sum.

**Make a Plan**

- **Do you need an estimate or an exact answer?**

When you are solving a word problem, ask yourself whether you need an exact answer or whether an estimate is sufficient. For example, if the amounts given in the problem are approximate, only an approximate answer can be given. If an estimate is sufficient, you may wish to use estimation techniques to save time in your calculations.

- 135.** The simple interest on a certain sum for 2.5 years at 12 per cent per annum is ₹ 300 less than the simple interest on the same sum for 4.5 years at 8 per cent per annum. Find the sum.

136. Designing a Healthy Diet

When you design your healthy diet, you want to make sure that you meet the dietary requirements to help you grow into a healthy adult. As you plan your menu, follow the following guidelines

1. Calculate your ideal weight as per your height from the table given at the end of this question.
2. An active child should eat around 55.11 calories for each kilogram desired weight.
3. 55 per cent of calories should come from carbohydrates. There are 4 calories in each gram of carbohydrates.
4. 15 per cent of your calories should come from proteins. There are 4 calories in each gram of proteins.
5. 30 per cent of your calories may come from fats. There are 9 calories in each gram of fat.

Following is an example to design your own healthy diet.

Example

1. Ideal weight = 40 kg.
2. The number of calories needed = $40 \times 55.11 = 2204.4$
3. Calories that should come from carbohydrates
 $= 2204.4 \times 0.55 = 1212.42$ calories.

Therefore, required quantity of carbohydrates

$$= \frac{1212.42}{4} = 303.105 \text{ g} = 300 \text{ g. (approx).}$$

4. Calories that should come from proteins
 $= 2204.4 \times 0.15 = 330.66$ calories.

Therefore, required quantity of protein

$$= \frac{330.66}{4} \text{ g} = 82.66 \text{ g.}$$

5. Calories that may come from fat $= 2204.4 \times 0.3$
 $= 661.3$ calories.

Therefore, required quantity of fat

$$= \frac{661.3}{9} \text{ g} = 73.47 \text{ g.}$$

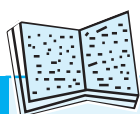
Answer the Given Questions

- Your ideal desired weight is _____ kg.
- The quantity of calories you need to eat is _____.
- The quantity of protein needed is _____ g.
- The quantity of fat required is _____ g.
- The quantity of carbohydrates required is _____ g.

Think and Discuss

- Tell** how finding commission is similar to finding sales tax.
- Explain** whether adding 6 per cent sales tax to a total gives the same result as finding 106 per cent of the total or not.
- Explain** how to find the price of an item if you know the total cost after 5 per cent sales tax.
- Explain** whether the sales tax on a ₹ 200 item would be double the sales tax on a ₹ 100 item. Justify your answer.

Ideal Height and Weight Proportion					
Men			Women		
Height		Weight	Height		Weight
Feet	cm	Kilograms	Feet	cm	Kilograms
5'	152	48	4'7"	140	34
5'1"	155	51	4'8"	142	36
5'2"	157	54	4'9"	145	39
5'3"	160	56	4'11"	150	43
5'4"	163	59	5'	152	45
5'5"	165	62	5'1"	155	48
5'6"	168	65	5'2"	157	50
5'7"	170	67	5'3"	160	52
5'8"	173	70	5'4"	163	55
5'9"	175	73	5'5"	165	57
5'10"	178	75	5'6"	168	59
5'11"	180	78	5'7"	170	61
6'	183	81	5'8"	173	64
6'1"	185	84	5'9"	175	66
6'2"	188	86	5'10"	178	68
6'3"	191	89	5'11"	180	70
6'4"	193	92			



Vocabulary

1. The word **principal** means “first”. What do you suppose **principal** means when referring to interest?
2. The word **commission** has the Latin prefix **com-**, which means “with,” and the Latin root **mis**, which means “send.” What do you think these Latin parts mean together when referring to money?
3. The word **per cent** contains the root word **cent**, which means “one hundred.” What do you think a **per cent** is?

137. 150 students are studying English, Maths or both. 62 per cent of students study English and 68 per cent are studying Maths. How many students are studying both?

138. Earth Science: The table lists the world's 10 largest deserts.

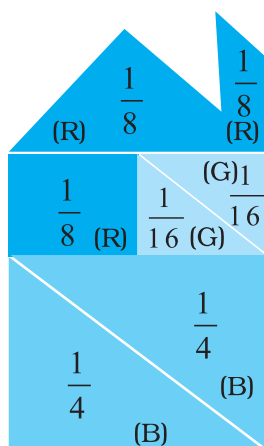
Largest Deserts in the World	
Desert	Area (km ²)
Sahara (Africa)	8,800,000
Gobi (Asia)	1,300,000
Australian Desert (Australia)	1,250,000
Arabian Desert (Asia)	850,000
Kalahari Desert (Africa)	580,000
Chihuahuan Desert (North America)	370,000
Takla Makan Desert (Asia)	320,000
Kara Kum (Asia)	310,000
Namib Desert (Africa)	310,000
Thar Desert (Asia)	260,000

- What are the mean, median and mode of the areas listed?
- How many times the size of the Gobi Desert is the Namib Desert?
- What percentage of the deserts listed are in Asia?
- What percentage of the total area of the deserts listed is in Asia?

139. Geography Application: Earth's total land area is about 148428950 km². The land area of Asia is about 30 per cent of this total. What is the approximate land area of Asia to the nearest square km?



140. The pieces of Tangrams have been rearranged to make the given shape.



By observing the given shape, answer the following questions:

- What percentage of total has been coloured?
 - (i) Red (R) = _____
 - (ii) Blue (B) = _____
 - (iii) Green (G) = _____
- Check that the sum of all the percentages calculated above should be 100.
- If we rearrange the same pieces to form some other shape, will the percentage of colours change?

(D) Applications

1. Healthy Diet

Keep a record of your diet for one day and compare with the nutritional guidelines given at the end of this unit and calculate the intake of each component. The table below gives recommendations of calories, proteins, calcium, carbohydrates and fat required for boys and girls between 11 and 14 years of age.

Component	Girls	Boys
Total Calories	48.6 per kg of body weight	55.11 per kg of body weight
Protein (g)	46	45
Calcium (mg)	1,200	1,200
Carbohydrates(g)	330	375
Fat (g)	less than 75	less than 80

Now complete the table given below to calculate the percentage of difference.

Component	Consumed	Recommended	% of Difference
Calories			
Protein			
Calcium			
Carbohydrates			
Fat			

Consumed > Recommended for any component implies excess intake of the component and

$$\text{per cent of difference} = \frac{\text{Consumed} - \text{Recommended}}{\text{Recommended}} \times 100$$

Recommended > Consumed for any component implies deficiency of component and

$$\text{per cent of difference} = \frac{\text{Recommended} - \text{Consumed}}{\text{Recommended}} \times 100$$

2. Nutrition Facts

Cut out a food label from any food item and analyse the nutritional value of the products. Use the information on the label to fill in the table given below and answer the questions that follow:

Sample of Food Label

1. The first place to start with at the Nutrition Facts Label is the serving size and the number of servings in the package. Pay attention to the serving size, especially how many servings there are in the food package. Then ask yourself, 'How many servings am I consuming'? (e.g. 1/2 serving, 1 serving or more). If you ate the whole package, you would eat (2 × 280) grams of this sample food. That doubles the calories and other nutrient numbers, including percentage daily values as shown in the sample label.

2. Calories and Calories from Fat

Calories provide a measure of how much energy you get from a serving of this food. The calorie section of the label can help you manage your weight (i.e. gain, lose or maintain).

3. The Nutrients : How much?

Limit these Nutrients

It shows some key nutrients that impact on your health and separates. Eating too much fat, saturated fat, cholesterol or sodium may increase your risk of certain chronic diseases, like heart disease, some cancers or high blood pressure.

4. Get Enough of these Nutrients

Eating enough of these nutrients can improve your health and help reduce the risk of some diseases and conditions.

5. Understanding the footnote

The * used after the heading "% Daily Value" refers to the Footnote in the lower part of the nutrition label, which tells that "% Daily Values are based on a 2,000 calorie diet".

Nutrition Facts

Serving Size 280 grams
Service Per Container 2

Amount Per Serving

Calories 320 **Calories from Fat 72**

% Daily Value

Total Fat 8g 12%

Saturated Fat-1.5g 8%

Cholesterol 5mg 2%

Sodium 780mg 32%

Total Carbohydrate 54g 18%

Dietary Fibre 6 gram 25%

Sugar 8g

Protein 11g 44%

Vitamin A 8% Vitamin C 40%

Calcium 0% Iron 2%

Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs:

	Calories:	2,000	2,500
Total Fat	Less than	65g	80g
Saturated Fat	Less than	20g	25g
Cholesterol	Less than	300mg	300mg
Sodium	Less than	2,400mg	2,400mg
Total Carbohydrate		300g	375g
Dietary Fibre		25g	30g
Protein		25g	

Calories per gram:

Fat 9 -Carbohydrate 4 Protein 4

How the Daily Values Relate to the % DVs

Look at the example below for another way to see how the Daily Values (DVs) relate to the %DVs and dietary guidance. For each nutrient listed there is a DV, a %DV, and dietary advice or a goal.

Examples of DVs versus %DVs

Based on a 2,000 Calorie Diet

Nutrient	DV		% DV	Goal
Total Fat	65 g	=	100% DV	Less than
Saturated Fat	20 g	=	100% DV	Less than
Cholesterol	300 mg	=	100% DV	Less than
Sodium	2400 mg	=	100% DV	Less than
Total Carbohydrate	300 g	=	100% DV	At least
Dietary Fibre	25 g	=	100% DV	At least

Note : 1. *The sodium should not be more than 3300 mg per day.*
 2. *The Dietary fibre should be between 25 – 35g per day.*

My food item is _____

Total Calories		% Daily value based on a 2000 calorie Diet
Proteins (g)		
Carbohydrates (g)		
Fat (g)		
Cholestrol (g)		
Sodium (mg)		
Fibre (g)		

What maximum percentage of total calories are from

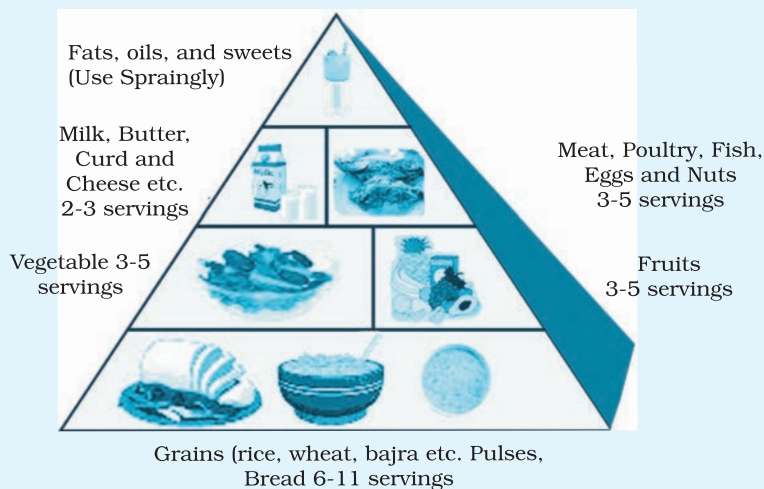
1. Fat _____.
2. Carbohydrates _____.
3. Proteins _____.
4. Others _____.

Check that the sum of all the percentages calculated above should be 100.

3. The Food Pyramid

The food guide pyramid should be kept in mind before planning an ideal menu. Everything on the pyramid is necessary for good nutrition.

Plan a menu for 1 day and classify the menu by categories in the food pyramid. Put a tick for each serving in the table given below. Calculate the total servings for each food group in the end and compare the total servings by the recommendations given in the food pyramid and answer the questions that follow:



Example: Suppose your lunch consists of rice, chapati, curd, dal, vegetable, an apple and one sweet. This means that you have consumed 2 servings of grains, 1 serving of pulses, 1 serving of dairy products, 1 serving of fruits and 1 serving of sweets.

Foods	Grains, (rice, wheat, bajara etc.) pulses, chapati	Fruits	Vegetables	Meat, poultry, fish, eggs, nuts etc.	Milk, curd, cheese, oils butter etc.	Fats and sweets
Food Group	I	II	III	IV	V	VI
Breakfast Juice, Milk, Banana, Chapati etc.						
Lunch Chapati, Vegetable, Curd, Rice etc.						
Snack Curd, Chips Soup, Popcorn etc.						
Dinner Salad, Vegetables, Rice, Dal etc.						
Total Servings						

1. What percentage of the total servings constitute fruits and vegetables in your diet?
2. Calculate the ratio of servings of food group-I items to food group item V of your diet.
3. Calculate the percentage of fats, oils and sweets you have consumed and compare with the given food pyramid.

4. Crossword Puzzle

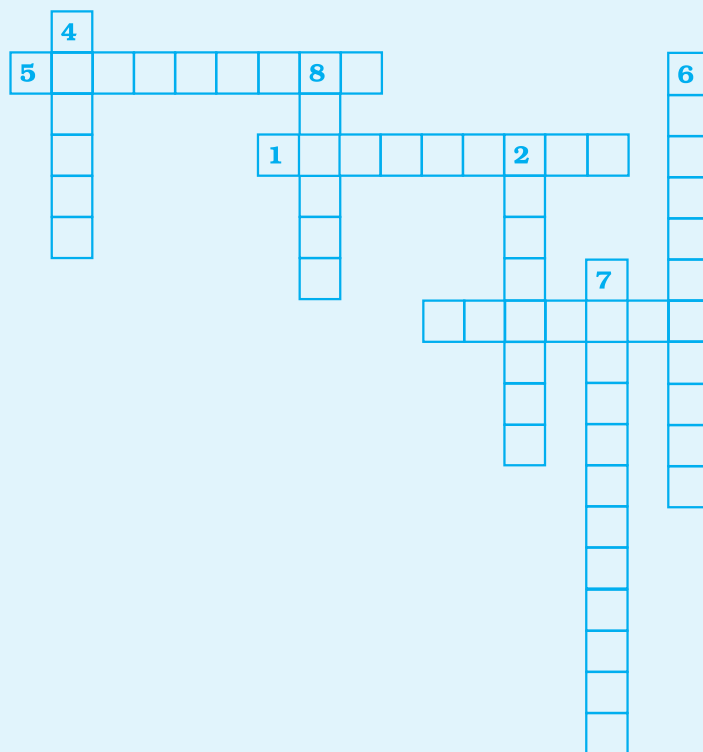
Solve the given crossword and then fill up the given boxes in puzzles 4, 5 and 6. Clues are given below in each puzzle for across as well as down fillings. Also for across and down clues, clue number is written at the corner of the boxes. Answers of clues have to fill up in their respective boxes.

Across

1. This includes cost as well as overhead costs
3. The term representing per hundred or out of hundred
5. The amount of money borrowed
7. The amount paid at when shopkeeper sells the things

Down

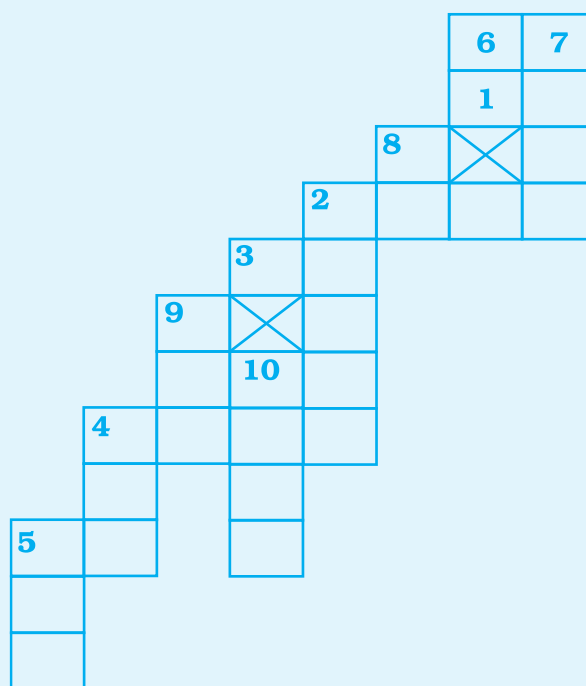
2. The extra money charged by borrower for using borrowed money for a given period of time.
4. Selling price – Cost price
6. Two equivalent ratios
8. Principal + Interest.



5. Cross Number Puzzle

Across

- Express 3 : 15 as per cent.
- A worker is paid ₹ 2850 for 15 days. What amount will he receive if he works for 8 days?
- 2% of 1 hour = _____ seconds.
- Find the sum for which interest paid after 3 years is ₹ 450 at 5 per cent rate of interest per annum.
- Price of a shirt decreased from ₹ 800 to ₹ 600 then the per cent decrease is _____.



Down

6. A number whose 25 per cent is 8.
7. Out of 15000 people, 60 per cent people voted. Find the number of people who did not vote.
8. Convert $\frac{12}{16}$ into per cent.
2. A TV is bought for ₹ 10,000 and sold at a profit of 20%. Find SP.
9. SI on a sum of ₹ 7000 at a rate of 3.5% per annum borrowed for 2 years.
10. Shalu spends 90 per cent of her salary. Find her salary if her saving is ₹. 900.
4. The new price of an article of Rs 350 if there is a 10 per cent increase in the price.
5. Cost of 12 bats if cost of 5 bats is ₹ 90.

6. Cross Number Puzzle

Across

1. $33\frac{1}{3}\%$ of 150
2. The interest on a sum of ₹ 1200 for 2 years at 10% pa
3. The cost price of a pen if SP = ₹ 7 and profit is 40%.
4. Total length of the journey if 25 per cent of that journey is 75 km.

Down

2. The profit I earned by selling a watch worth ₹ 800 for ₹ 992.
5. The selling price of an item if the CP = ₹ 130 and loss = 20%.
6. The principal if the I = ₹ 80, R = 10% pa and T = 2 years.
7. 25% of 50% of 72

1 _	—			5
		2 _	—	—
3	6 _	—		
	—		7	
4 _	—			

The Nutrition Counter

The following table shows a variety of foods and the calories, protein, fat, carbohydrates and calcium contained in per 100 gm of edible portion. These data are approximate.

Sl. No.	Name of the Food Stuff	Protein (N × 6.25) g.	Fat g.	Carbo- hydrates g.	Calcium mg.
CEREAL GRAINS AND PRODUCTS					
1.	BAJRA	11.6	5.0	67.5	42
2.	JOWAR	10.4	1.9	72.6	25
3.	MAIZE, tender	4.7	0.9	24.6	9
4.	RAGI	7.3	1.3	72.0	344
5.	RICE, parboiled, milled	6.4	0.4	79.0	9
6.	RICE, raw, milled	6.8	0.6	78.2	10
7.	RICE, flakes	6.6	1.2	77.3	20
8.	RICE, puffed	7.5	0.1	73.6	23
9.	SAMAI	7.7	4.7	67.0	17
10.	WHEAT, flour (whole)	12.1	1.7	69.4	48
11.	WHEAT, flour (refined)	11.0	0.9	73.9	23
12.	WHEAT, semolina	10.4	0.8	74.8	16
13.	WHEAT, vermicelli	8.7	0.4	78.3	22
14.	WHEAT, bread (brown)	8.8	1.4	49.0	18
15.	WHEAT, bread (white)	7.8	0.7	51.9	11
PULSES AND LEGUMES					
16.	BENGAL GRAM, dhal	20.8	5.6	59.8	56
17.	BLACK GRAM, dhal	24.0	1.4	59.6	154
18.	COW PEA	24.1	1.0	54.5	77
19.	GREEN GRAM, whole	24.	1.3	56.7	124
20.	KHESARI, dhal	28.2	0.6	56.6	90
21.	LENTIL	25.1	0.7	59.0	69
22.	MOTH BEANS	23.6	1.1	56.5	202
23.	PEAS green	7.2	0.1	15.9	20
24.	RAJMAH	22.9	1.3	60.6	260
25.	SOYABEAN	43.2	19.5	20.9	240
LEAFY VEGETABLES					
26.	BATHUA LEAVES	3.7	0.4	2.9	150
27.	BENGAL GRAM LEAVES	7.0	1.4	14.1	340
28.	CABBAGE	1.8	0.1	4.6	39
29.	CAULIFLOWER	5.9	1.3	7.6	626
30.	COLOCASIA LEAVES (green variety)	3.9	1.5	6.8	227

31.	CORIANDER LEAVES	3.3	0.6	6.3	184
32.	CURRY LEAVES	6.1	1.0	18.7	830
33.	DRUMSTICK LEAVES	6.7	1.7	12.5	440
34.	FENUGREEK LEAVES	4.4	0.9	6.0	395
35.	LETTUCE	2.1	0.3	2.5	50
36.	MINT	4.8	0.6	5.8	200
37.	MUSTARD LEAVES	4.0	0.6	3.2	155
38.	RADISH LEAVES	3.8	0.4	2.4	265
39.	SPINACH	2.0	0.7	2.9	73
40.	TURNIP GREENS	4.0	1.5	9.4	710
ROOTS AND TUBERS					
41.	BEET ROOT	1.7	0.1	8.8	18.3
42.	CARROT	0.9	0.2	10.6	80
43.	ONION big	1.2	0.1	11.1	46.9
44.	ONION small	1.8	0.1	12.6	40
45.	POTATO	1.6	0.1	22.6	10
46.	RADISH pink	0.6	0.3	6.8	50
47.	RADISH white	0.7	0.1	3.4	35
48.	SWEET POTATO	1.2	0.3	28.2	46
49.	TURNIP	0.5	0.2	6.2	30
OTHER VEGETABLES					
50.	BEANS, scarlet runner	7.4	1.0	29.8	50
51.	BITTER GOURD, small	2.1	1.0	10.6	23
52.	BITTER GOURD	0.2	0.1	2.5	20
53.	BRINJAL	1.4	0.3	4.0	18
54.	CAULIFLOWER	2.6	0.4	4.0	33
55.	COLOCASIA STEM	0.3	0.3	3.6	60
56.	CUCUMBER	0.4	0.1	2.5	10
57.	FIGS, red (<i>Ficus cunia</i>)	1.2	0.6	10.8	187
58.	FRENCH BEANS	1.7	0.1	4.5	50
59.	GIANT CHILLIES (capsicum)	1.3	0.3	4.3	10
60.	JACK FRUIT, tender	2.6	0.3	9.4	30
61.	JACK FRUIT, seeds	6.6	0.4	25.8	50
62.	LADIES FINGER	1.9	0.2	6.4	66
63.	MANGO, green	0.7	0.1	10.1	10
64.	ONION STALKS	0.9	0.2	8.9	50
65.	PUMPKIN fruit	2.2	0.8	5.8	120
66.	TOMATO, green	1.9	0.1	3.6	20
NUTS AND OIL SEEDS					
67.	ALMOND	20.8	58.9	10.5	230
68.	CASHEWNUT	21.2	46.9	22.3	50

69.	CHILGOZA	13.9	49.3	29.0	91
70.	COCONUT dry	6.8	62.3	18.4	400
71.	COCONUT fresh	4.5	41.6	13.0	10
72.	COCONUT water	1.4	0.1	4.4	24
73.	GROUNDNUT	25.3	40.1	26.1	90
74.	GROUNDNUT roasted	26.2	39.8	26.7	77
75.	PISTACHIO NUT	19.8	53.5	16.2	140
76.	WALNUT	15.6	64.5	11.0	100
77.	WATER MELON SEEDS (kernal)	34.1	52.6	4.5	100
CONDIMENTS AND SPICES					
78.	ASAFOETIDA	4.0	1.1	67.8	690
79.	CARDAMOM	10.2	2.2	42.1	130
80.	CHILLIES dry	15.9	6.2	31.6	160
81.	CHILLIES green	2.9	0.6	3.0	30
82.	CLOVES dry	5.2	8.9	46.0	740
83.	CLOVES green	2.3	5.9	24.1	310
84.	CORIANDER	14.1	16.1	21.6	630
85.	CUMIN SEEDS	18.7	15.0	36.6	1080
86.	FENUGREEK SEEDS	26.2	5.8	44.1	160
87.	GARLIC dry	6.3	0.1	29.8	30
88.	GINGER fresh	2.3	0.9	12.3	20
89.	MANGO POWDER	2.8	7.8	64.0	180
90.	PEPPER DRY (black)	11.5	6.8	49.2	460
91.	TAMARIND PULP	3.1	0.1	67.4	170
92.	TURMERIC	6.3	5.1	69.4	150
FRUITS					
93.	AMLA	0.5	0.1	13.7	50
94.	APPLE	0.2	0.5	13.4	10
95.	APRICOT fresh	1.0	0.3	11.6	20
96.	APRICOT dry	1.6	0.7	73.4	110
97.	BAEL FRUIT	1.8	0.3	31.8	85
98.	BANANA, ripe	1.2	0.3	27.2	17
99.	CHERRIES, red	1.1	0.5	13.8	24
100.	CURRENTS, black	2.7	0.5	75.2	130
101.	DATES dried	2.5	0.4	75.8	120
102.	DATES fresh	1.2	0.4	33.8	22
103.	GRAPES blue variety	0.6	0.4	13.1	20
104.	GRAPES pale green variety	0.6	0.4	13.1	20
105.	GUAVA, country	0.9	0.3	11.2	10
106.	JACK FRUIT	1.9	0.1	19.8	20
107.	LEMON	1.0	0.9	11.1	70
108.	LITCHI	1.1	0.2	13.6	10

109.	LIME	1.5	1.0	10.9	90
110.	MELON, musk	0.3	0.2	3.5	32
111.	MELON, water	0.2	0.2	3.3	11
112.	ORANGE	0.7	0.2	10.9	26
113.	ORANGE juice	0.2	0.1	1.9	5
114.	PAPAYA, ripe	0.6	0.1	7.2	17
115.	PEACHES	1.2	0.3	10.5	15
116.	PEARS	0.6	0.2	11.9	8
117.	PHALSA	1.3	0.9	14.7	129
118.	PINE APPLE	0.4	0.1	10.8	20
119.	PLUM	0.7	0.5	11.1	10
120.	POMEGRANATE	1.6	0.1	14.5	10
121.	RAISINS	1.8	0.3	74.6	87
122.	STRAWBERRY	0.7	0.2	9.8	30

FISHES AND OTHER SEA FOODS

123.	HILSA	21.8	19.4	2.9	180
124.	KATAL	19.5	2.4	2.9	530
125.	MACKEREL	18.9	1.7	0.5	429
126.	PRAWN	19.1	1.0	0.8	323
127.	ROHU	16.6	1.4	4.4	650
128.	SARDINE	21.0	1.9	–	90
129.	SHRIMP (small, dried)	68.1	8.5	–	4384

MEAT AND POULTRY

130.	BEEF muscle	22.6	2.6	–	10
131.	DUCK	21.6	4.8	0.1	4
132.	EGG, hen	13.3	13.3	–	60
133.	FOWL	25.9	0.6	–	25
134.	GOAT MEAT (lean)	21.4	3.6	–	12
135.	MUTTON, muscle	18.5	13.3	–	150
136.	PORK, muscle	18.7	4.4	–	30

MILK AND MILK PRODUCTS

137.	MILK buffalo's	4.3	6.5	5.0	210
138.	MILK cow's	3.2	4.1	4.4	120
139.	MILK goat's	3.3	4.5	4.6	170
140.	CURDS (cow's milk)	3.1	4.0	3.0	149
141.	BUTTER MILK	0.8	1.1	0.5	30
142.	SKIMMED MILK, liquid	2.5	0.1	4.6	120
143.	CHANNA, cow's milk	18.3	20.8	1.2	208
144.	CHANNA, buffalo's milk	13.4	23.0	7.9	480
145.	CHEESE	24.1	25.1	6.3	790
146.	KHOA (whole buffalo milk)	14.6	31.2	20.5	650

147.	SKIMMED MILK POWDER(cow's milk)	38.0	0.1	51.0	1370
148.	WHOLE MILK POWDER (cow's milk)	25.8	26.7	38.0	950

FATS AND EDIBLE OILS

149.	BUTTER	–	81.0	–	–
150.	GHEE (cow)	–	100.0	–	–
151.	HYDROGENATED OIL (fortified)	–	100.0	–	–
152.	COOKING OIL (Groundnut, Gingelly, Palmolein, Mustard, Coconut, etc)	–	100.0	–	–

SUGARS

153.	SUGARY CANE	0.1	0	99.4	12
154.	HONEY	0.3	0	79.5	5
155.	JAGGERY (cane)	0.4	0.1	95.0	80

BEVERAGES (Non-Alcoholic)

156.	SUGAR CANE JUICE	0.1	0.2	9.1	10
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UNIT 8

RATIONAL NUMBERS

(A) Main Concepts and Results

- A number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.
- All integers and fractions are rational numbers.
- If the numerator and denominator of a rational number are multiplied or divided by a non-zero integer, we get a rational number which is said to be equivalent to the given rational number.
- Rational numbers are classified as positive, zero or negative rational numbers. When the numerator and denominator both are positive integers or both are negative integers, it is a positive rational number. When either the numerator or the denominator is a negative integer, it is a negative rational number.
- The number 0 is neither a positive nor a negative rational number.
- There are unlimited number of rational numbers between two rational numbers.
- A rational number is said to be in the standard form, if its denominator is a positive integer and the numerator and denominator have no common factor other than 1.
- Two rational numbers with the same denominator can be added by adding their numerators, keeping with the same denominator.
- Two rational numbers with different denominators are added by first taking the LCM of the two denominators and then converting both the rational numbers to their equivalent forms having the LCM as the denominator and adding them as above.

- While subtracting two rational numbers, we add the additive inverse of the rational number to be subtracted to the other rational number.
- Product of rational numbers = $\frac{\text{Product of numerators}}{\text{Product of denominators}}$
- The reciprocal of a non-zero rational number $\frac{p}{q}$ is $\frac{q}{p}$.
- To divide one rational number by the other non-zero rational number, we multiply the first rational number by the reciprocal of the other.

(B) Solved Examples

In Examples 1 to 4, there are four options, out of which one is correct. Choose the correct one.

Example 1: Which of the following rational numbers is equivalent to $\frac{2}{3}$?

- (a) $\frac{3}{2}$ (b) $\frac{4}{9}$ (c) $\frac{4}{6}$ (d) $\frac{9}{4}$

Solution: Correct answer is (c).

Example 2: Which of the following rational numbers is in standard form?

- (a) $\frac{20}{30}$ (b) $\frac{10}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{-3}$

Solution: Correct answer is (c).

Example 3: The sum of $\frac{-3}{2}$ and $\frac{1}{2}$ is

- (a) -1 (b) -2 (c) 4 (d) 3

Solution: Correct answer is (a).

Example 4: The value of $-\frac{4}{3} - \frac{-1}{3}$ is

- (a) -2 (b) -3 (c) 2 (d) -1

Solution: Correct answer is (d).

In Examples 5 and 6, fill in the blanks to make the statements true.

Example 5: There are _____ number of rational numbers between two rational numbers.

Solution: Unlimited

Example 6: The rational number _____ is neither positive nor negative.

Solution: 0 (Zero).

In Examples 7 to 9, state whether the statements are True or False.

Example 7: In any rational number $\frac{p}{q}$, denominator is always a non-zero integer.

Solution: True.

Reading Strategy: Read a Lesson for Understanding

You need to be actively involved as you work through each lesson in your textbook. To begin with, find the lesson's objective given as main concepts and results.

Lesson Features

Learn to write rational numbers in equivalent forms



Identify the objectives of the lesson and look through the lesson to get a feel for how the objectives are met.

Example



Work through each example. The examples help to demonstrate the lesson objectives.

Think and Discuss



Check your understanding of the lesson by answering the exercise/questions.

Example 8: “To reduce the rational number to its standard form, we divide its numerator and denominator by their HCF”.

Solution: True.

Example 9: “All rational numbers are integers”.

Solution: False.

Solve

• Choose an Operation

To decide whether to add, subtract, multiply, or divide to solve a problem, you need to determine the action taking place in the problem.



- Divided by
- Quotient
- Divided into



- Subtracted from
- Minus
- Difference
- Less than
- Decreased by



- Added to
- Plus
- Sum
- More than



- Multiplied by
- Times
- Product
- Groups of

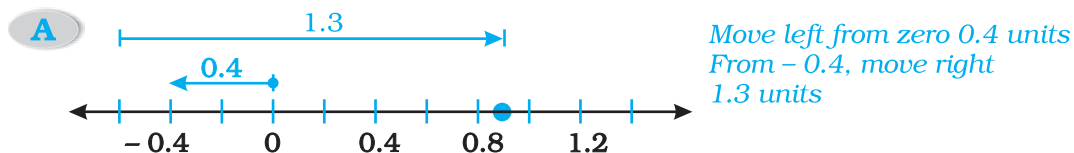
Example 10: List three rational numbers between $\frac{4}{5}$ and $\frac{5}{6}$.

Solution: We convert the rational numbers $\frac{4}{5}$ and $\frac{5}{6}$ into rational numbers with the same denominators.

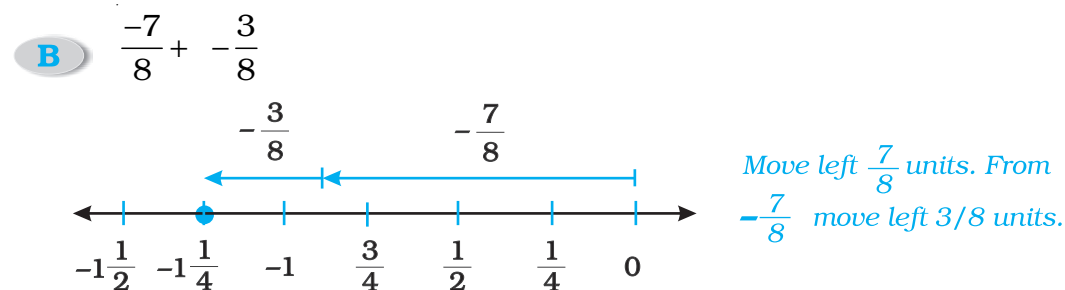
$$\frac{4}{5} = \frac{4}{5} \times \frac{6}{6} = \frac{24}{30}; \quad \frac{5}{6} = \frac{5}{6} \times \frac{5}{5} = \frac{25}{30}$$

Using a Number Line to Add Rational Numbers

Use a number line to find each sum.



You finish at 0.9, So $-0.4 + 1.3 = 0.9$



You finish at $-1\frac{1}{4}$. So $-\frac{7}{8} + \left(-\frac{3}{8}\right) = -1\frac{1}{4}$

So,

$$\frac{24}{30} = \frac{24}{30} \times \frac{4}{4} \quad \text{and} \quad \frac{25}{30} = \frac{25}{30} \times \frac{4}{4}$$

$$= \frac{96}{120} \qquad \qquad \qquad = \frac{100}{120}$$

or

Here,

$$\frac{96}{120} < \frac{97}{120} < \frac{98}{120} < \frac{99}{120} < \frac{100}{120}$$

So, the required numbers are $\frac{97}{120}$, $\frac{98}{120}$ and $\frac{99}{120}$

Alternate solution A rational number between

$\frac{4}{5}$ and $\frac{5}{6}$ is

$$= \frac{1}{2} \left(\frac{4}{5} + \frac{5}{6} \right) = \frac{49}{60}$$

another rational number

$$= \frac{1}{2} \left(\frac{4}{5} + \frac{49}{60} \right) = \frac{97}{120}$$

one more rational number

$$= \frac{1}{2} \left(\frac{49}{60} + \frac{5}{6} \right) = \frac{99}{120} = \frac{33}{40}$$

Therefore, three rational numbers between $\frac{4}{5}$ and $\frac{5}{6}$ are

$$\frac{49}{60}, \frac{97}{120} \text{ and } \frac{33}{40}$$

Note: There can be many set of answers.

ADDING AND SUBTRACTING WITH LIKE DENOMINATORS

Words	Numbers	Formula
To add or subtract rational numbers with the same denominator, add or subtract the numerators and keep the same denominator.	$\frac{1}{5} + \left(\frac{-4}{5} \right) = \frac{1+(-4)}{5}$ $= \frac{-3}{5}, \text{ or } \frac{-3}{5}$	$\frac{a}{d} + \frac{b}{d} = \frac{a+b}{d}$

Example 11: Which of the following pairs represent equivalent rational numbers ?

(i) $\frac{7}{12}$ and $\frac{28}{48}$ (ii) $\frac{-2}{-3}$ and $\frac{-16}{24}$

Solution:

(i) $\frac{7}{12}$ and $\frac{28}{48}$

Now, first rational number is $\frac{7}{12}$ and it is already in the standard form because there is no common factor in 7 and 12 other than 1.

So, $\frac{7}{12}$ is in its standard form _____(a)

Now, Consider $\frac{28}{48}$

$$28 = 2 \times 2 \times 7$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$\text{HCF} = 2 \times 2 = 4$$

Now, to reduce the rational numbers to its standard form, we divide the numerator and denominator by their HCF.

First we take HCF of 28 and 48:

$$\text{Now, } \frac{28}{48} = \frac{28 \div 4}{48 \div 4} = \frac{7}{12} \quad \text{_____ (b)}$$

From (a) and (b), we can say that the rational numbers

$\frac{7}{12}$ and $\frac{28}{48}$ are equivalent.

$$\text{(ii) } \frac{-2}{-3} \text{ and } \frac{-16}{24}$$

First we multiply the numerator and denominator of $\frac{-2}{-3}$ by (-1) , we get

$$\frac{-2}{-3} = \frac{(-2) \times (-1)}{(-3) \times (-1)} = \frac{2}{3} \quad \text{_____ (a)}$$

Now it is in its standard form.

$$\text{Now, Consider } \frac{16}{24}$$

HCF of 16 and 24 is $2 \times 2 \times 2 = 8$

$$16 = 2 \times 2 \times 2 \times 2$$

$$24 = 2 \times 2 \times 2 \times 3$$

$$\text{HCF} = 2 \times 2 \times 2 = 8$$

$$\text{So, } \frac{-16}{24} = \frac{-16 \div 8}{24 \div 8} = \frac{-2}{3} \quad \text{_____ (b)}$$

From (a) and (b), we can say that the rational numbers

$\frac{-2}{-3}$ and $\frac{-16}{24}$ are not equivalent.

Action	Operation
Combining numbers or putting numbers together	Addition
Taking away or finding out how far apart two numbers are	Subtraction
Combining groups	Multiplication
Splitting things into equal groups or finding how many equal groups you can make	Division

Example 12: Write four more rational numbers to complete the pattern:

$$\frac{-1}{3}, \frac{-2}{6}, \frac{-3}{9}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}.$$

Solution: By observing the above pattern, we find that denominator is multiple of 3. So we will increase this pattern in this way.

$$\frac{-2}{6} = \frac{-1 \times 2}{3 \times 2}, \frac{-3}{9} = \frac{-1 \times 3}{3 \times 3}, \frac{-4}{12} = \frac{-1 \times 4}{3 \times 4}$$

$$\frac{-1 \times 1}{3 \times 1} = \frac{-1}{3},$$

$$\frac{-1 \times 4}{3 \times 4} = \frac{-4}{12}$$

Thus, we observe a pattern in these numbers.

So, the other numbers would be

$$\frac{-1 \times 5}{3 \times 5} = \frac{-5}{15}, \frac{-1 \times 6}{3 \times 6} = \frac{-6}{18}, \frac{-1 \times 7}{3 \times 7} = \frac{-7}{21} \text{ and } \frac{-1 \times 8}{3 \times 8} = \frac{-8}{24}$$

DIVIDING RATIONAL NUMBERS IN FRACTION FORM

Words	Numbers	Algebra
To divide by a fraction, multiply by the reciprocal	$\frac{1}{7} \div \frac{4}{5} = \frac{1}{7} \times \frac{5}{4} = \frac{5}{28}$	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

Example 13: Find the sum of $-4\frac{5}{6}$ and $-7\frac{3}{4}$.

Solution:

$$-4\frac{5}{6} + \left(-7\frac{3}{4}\right)$$

$$= \frac{-29}{6} + \left(\frac{-31}{4}\right) = \frac{-29}{6} + \frac{-31}{4}$$

$$\begin{aligned}
 &= \frac{-29 \times 2}{12} + \frac{-31 \times 3}{12} \text{ [Since LCM of 6 and 4 is 12].} \\
 &= \frac{-29 \times 2 - 31 \times 3}{12} \\
 &= \frac{-58 - 93}{12} \\
 &= \frac{-151}{12}
 \end{aligned}$$

So, the required sum is $\frac{-151}{12}$.

Think and Discuss

1. **Give an example** of two denominators with no common factors.
2. **Tell** if $-2\frac{1}{5} - \left(-2\frac{3}{16}\right)$ is positive or negative. Explain.
3. **Explain** how to add $2\frac{2}{5} + 9\frac{1}{2}$, without first writing them as improper fractions.

Example 14: Find the product of $-2\frac{3}{4}$ and $5\frac{6}{7}$.

Solution: $-2\frac{3}{4} \times 5\frac{6}{7} = \frac{-11}{4} \times \frac{41}{7}$

Now, product of two rational numbers

$$= \frac{\text{Product of numerators}}{\text{Product of denominators}}$$

$$\text{So, } -2\frac{3}{4} \times 5\frac{6}{7} = \frac{-11}{4} \times \frac{41}{7} = \frac{-11 \times 41}{4 \times 7} = \frac{-451}{28}$$

Think and Discuss

1. **Explain** how you can be sure that a fraction is simplified.
2. **Give** the sign of a rational number in which the numerator is negative and the denominator is negative.

Example 15: Match column I to column II in the following:

Column I

(i) $\frac{3}{4} \div \frac{3}{4}$

(ii) $\frac{1}{2} \div \frac{4}{3}$

(iii) $\frac{2}{3} \div (-1)$

(iv) $\frac{3}{4} \div \frac{1}{2}$

(v) $\frac{5}{7} \div \left(-\frac{5}{7}\right)$

Column II

(a) -1

(b) $-\frac{2}{3}$

(c) $\frac{3}{2}$

(d) $\frac{3}{8}$

(e) 1

Solution: (i) \leftrightarrow (e), (ii) \leftrightarrow (d), (iii) \leftrightarrow (b), (iv) \leftrightarrow (c), (v) \leftrightarrow (a)

Application on Problem Solving Strategy



Example 16

Find the reciprocal of $\frac{2}{11} \div -\frac{5}{55}$.

Solution :



Understand and Explore the Problem

- What are you trying to find?
The reciprocal of the given number.



Plan a Strategy

- You know the division of rational numbers and the meaning of reciprocal. Apply this knowledge to find the reciprocal.



Solve

- **Given expression** $= \frac{2}{11} \div \frac{-5}{55} = \frac{2}{\cancel{11}} \times -\frac{\cancel{55}}{5} = -2$

- Now find out the reciprocal of -2

The reciprocal of -2 is $-\frac{1}{2}$.



Revise

- Multiply -2 and $-\frac{1}{2}$ and check whether you get 1 or not.

$$-2 \times -\frac{1}{2} = 1$$

Hence, our answer is correct, since we know that the product of a rational number with its reciprocal is always 1.

Think and Discuss

1. **Can you find** the reciprocal of $\frac{2}{11} \times \frac{5}{55}$?

2. **Can you compare** this reciprocal with the original number?

(C) Exercise

In each of the following questions 1 to 12, there are four options, out of which, only one is correct. Write the correct one.

1. A rational number is defined as a number that can be expressed in

the form $\frac{p}{q}$, where p and q are integers and

- (a) $q = 0$ (b) $q = 1$ (c) $q \neq 1$ (d) $q \neq 0$

2. Which of the following rational numbers is positive?

- (a) $\frac{-8}{7}$ (b) $\frac{19}{-13}$ (c) $\frac{-3}{-4}$ (d) $\frac{-21}{13}$

3. Which of the following rational numbers is negative?

- (a) $-\left(\frac{-3}{7}\right)$ (b) $\frac{-5}{-8}$ (c) $\frac{9}{8}$ (d) $\frac{3}{-7}$

4. In the standard form of a rational number, the common factor of numerator and denominator is always:

- (a) 0 (b) 1 (c) -2 (d) 2

5. Which of the following rational numbers is equal to its reciprocal?

- (a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0

6. The reciprocal of $\frac{1}{2}$ is

- (a) 3 (b) 2 (c) -1 (d) 0

7. The standard form of $\frac{-48}{60}$ is

- (a) $\frac{48}{60}$ (b) $\frac{-60}{48}$ (c) $\frac{-4}{5}$ (d) $\frac{-4}{-5}$

Number	Reciprocal	Product
$\frac{3}{4}$	$\frac{4}{3}$	$\frac{3}{4}\left(\frac{4}{3}\right)=1$
$-\frac{5}{12}$	$-\frac{12}{5}$	$-\frac{5}{12}\left(-\frac{12}{5}\right)=1$
6	$\frac{1}{6}$	$6\left(\frac{1}{6}\right)=1$

8. Which of the following is equivalent to $\frac{4}{5}$?
- (a) $\frac{5}{4}$ (b) $\frac{16}{25}$ (c) $\frac{16}{20}$ (d) $\frac{15}{25}$
9. How many rational numbers are there between two rational numbers?
- (a) 1 (b) 0 (c) unlimited (d) 100
10. In the standard form of a rational number, the denominator is always a
- (a) 0 (b) negative integer
(c) positive integer (d) 1
11. To reduce a rational number to its standard form, we divide its numerator and denominator by their
- (a) LCM (b) HCF (c) product (d) multiple
12. Which is greater number in the following:
- (a) $-\frac{1}{2}$ (b) 0 (c) $\frac{1}{2}$ (d) -2

RULES FOR MULTIPLYING TWO RATIONAL NUMBERS

If the signs of the factors are the same, the product is positive.

$$(+). (+) = (+) \text{ or } (-). (-) = (+)$$

If the signs of the factors are different, the product is negative

$$(+). (-) = (-) \text{ or } (-). (+) = (-)$$

In Questions 13 to 46, fill in the blanks to make the statements true.

13. $-\frac{3}{8}$ is a _____ rational number.
14. 1 is a _____ rational number.

15. The standard form of $\frac{-8}{-36}$ is _____.
16. The standard form of $\frac{18}{-24}$ is _____.
17. On a number line, $\frac{-1}{2}$ is to the _____ of zero (0).
18. On a number line, $\frac{4}{3}$ is to the _____ of zero (0).
19. $-\frac{1}{2}$ is _____ than $\frac{1}{5}$.
20. $-\frac{3}{5}$ is _____ than 0.
21. $\frac{-16}{24}$ and $\frac{20}{-16}$ represent _____ rational numbers.
22. $\frac{-27}{45}$ and $\frac{-3}{5}$ represent _____ rational numbers.
23. Additive inverse of $\frac{2}{3}$ is _____.
24. $\frac{-3}{5} + \frac{2}{5} =$ _____.
25. $\frac{-5}{6} + \frac{-1}{6} =$ _____.
26. $\frac{3}{4} \times \left(\frac{-2}{3}\right) =$ _____.
27. $\frac{-5}{3} \times \left(\frac{-3}{5}\right) =$ _____.
28. $\frac{-6}{7} = \frac{\quad}{42}$
29. $\frac{1}{2} = \frac{6}{\quad}$
30. $\frac{-2}{9} - \frac{7}{9} =$ _____.

In questions 31 to 35, fill in the boxes with the correct symbol $>$, $<$ or $=$.

31. $\frac{7}{-8} \square \frac{8}{9}$

32. $\frac{3}{7} \square \frac{-5}{6}$

33. $\frac{5}{6} \square \frac{8}{4}$

34. $\frac{-9}{7} \square \frac{4}{-7}$

35. $\frac{8}{8} \square \frac{2}{2}$

36. The reciprocal of _____ does not exist.

37. The reciprocal of 1 is _____.

38. $\frac{-3}{7} \div \left(\frac{-7}{3} \right) = \underline{\hspace{2cm}}$.

39. $0 \div \left(\frac{-5}{6} \right) = \underline{\hspace{2cm}}$.

40. $0 \times \left(\frac{-5}{6} \right) = \underline{\hspace{2cm}}$.

41. $\underline{\hspace{2cm}} \times \left(\frac{-2}{5} \right) = 1$.

42. The standard form of rational number -1 is _____.

43. If m is a common divisor of a and b , then $\frac{a}{b} = \frac{a \div m}{\underline{\hspace{2cm}}}$

44. If p and q are positive integers, then $\frac{p}{q}$ is a _____ rational number

and $\frac{p}{-q}$ is a _____ rational number.

45. Two rational numbers are said to be equivalent or equal, if they have the same _____ form.

46. If $\frac{p}{q}$ is a rational number, then q cannot be _____.

State whether the statements given in question 47 to 65 are True or False.

47. Every natural number is a rational number but every rational number need not be a natural number.
48. Zero is a rational number.
49. Every integer is a rational number but every rational number need not be an integer.
50. Every negative integer is not a negative rational number.
51. If $\frac{p}{q}$ is a rational number and m is a non-zero integer, then $\frac{p}{q} = \frac{p \times m}{q \times m}$
52. If $\frac{p}{q}$ is a rational number and m is a non-zero common divisor of p and q , then $\frac{p}{q} = \frac{p \div m}{q \div m}$.
53. In a rational number, denominator always has to be a non-zero integer.
54. If $\frac{p}{q}$ is a rational number and m is a non-zero integer, then $\frac{p \times m}{q \times m}$ is a rational number not equivalent to $\frac{p}{q}$.
55. Sum of two rational numbers is always a rational number.
56. All decimal numbers are also rational numbers.
57. The quotient of two rationals is always a rational number.
58. Every fraction is a rational number.
59. Two rationals with different numerators can never be equal.
60. 8 can be written as a rational number with any integer as denominator.
61. $\frac{4}{6}$ is equivalent to $\frac{2}{3}$.
62. The rational number $\frac{-3}{4}$ lies to the right of zero on the number line.
63. The rational numbers $\frac{-12}{-5}$ and $\frac{-7}{17}$ are on the opposite sides of zero on the number line.

64. Every rational number is a whole number.

65. Zero is the smallest rational number.

66. Match the following:

Column I

Column II

(i) $\frac{a}{b} \div \frac{a}{b}$

(a) $\frac{-a}{b}$

(ii) $\frac{a}{b} \div \frac{c}{d}$

(b) -1

(iii) $\frac{a}{b} \div (-1)$

(c) 1

(iv) $\frac{a}{b} \div \frac{-a}{b}$

(d) $\frac{bc}{ad}$

(v) $\frac{b}{a} \div \left(\frac{d}{c}\right)$

(e) $\frac{ad}{bc}$

67. Write each of the following rational numbers with positive denominators: $\frac{5}{-8}, \frac{15}{-28}, \frac{-17}{-13}$.

68. Express $\frac{3}{4}$ as a rational number with denominator:

(i) 36

(ii) -80

69. Reduce each of the following rational numbers in its lowest form:

(i) $\frac{-60}{72}$

(ii) $\frac{91}{-364}$

70. Express each of the following rational numbers in its standard form:

(i) $\frac{-12}{-30}$

(ii) $\frac{14}{-49}$

(iii) $\frac{-15}{35}$

(iv) $\frac{299}{-161}$

71. Are the rational numbers $\frac{-8}{28}$ and $\frac{32}{-112}$ equivalent? Give reason.

72. Arrange the rational numbers $\frac{-7}{10}, \frac{5}{-8}, \frac{2}{-3}, \frac{-1}{4}, \frac{-3}{5}$ in ascending order.

73. Represent the following rational numbers on a number line:

$\frac{3}{8}, \frac{-7}{3}, \frac{22}{-6}$.

- 74.** If $\frac{-5}{7} = \frac{x}{28}$, find the value of x .
- 75.** Give three rational numbers equivalent to:
- (i) $\frac{-3}{4}$ (ii) $\frac{7}{11}$
- 76.** Write the next three rational numbers to complete the pattern:
- (i) $\frac{4}{-5}, \frac{8}{-10}, \frac{12}{-15}, \frac{16}{-20}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}.$
- (ii) $\frac{-8}{7}, \frac{-16}{14}, \frac{-24}{21}, \frac{-32}{28}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}.$
- 77.** List four rational numbers between $\frac{5}{7}$ and $\frac{7}{8}$.
- 78.** Find the sum of
- (i) $\frac{8}{13}$ and $\frac{3}{11}$ (ii) $\frac{7}{3}$ and $\frac{-4}{3}$
- 79.** Solve:
- (i) $\frac{29}{4} - \frac{30}{7}$ (ii) $\frac{5}{13} - \frac{-8}{26}$
- 80.** Find the product of:
- (i) $\frac{-4}{5}$ and $\frac{-5}{12}$ (ii) $\frac{-22}{11}$ and $\frac{-21}{11}$
- 81.** Simplify:
- (i) $\frac{13}{11} \times \frac{-14}{5} + \frac{13}{11} \times \frac{-7}{5} + \frac{-13}{11} \times \frac{34}{5}$ (ii) $\frac{6}{5} \times \frac{3}{7} - \frac{1}{5} \times \frac{3}{7}$
- 82.** Simplify:
- (i) $\frac{3}{7} \div \left(\frac{21}{-55}\right)$ (ii) $1 \div \left(-\frac{1}{2}\right)$
- 83.** Which is greater in the following?
- (i) $\frac{3}{4}, \frac{7}{8}$ (ii) $-3\frac{5}{7}, 3\frac{1}{9}$
- 84.** Write a rational number in which the numerator is less than ' -7×11 ' and the denominator is greater than ' $12 + 4$ '.
- 85.** If $x = \frac{1}{10}$ and $y = \frac{-3}{8}$, then
evaluate $x + y$, $x - y$, $x \times y$ and $x \div y$.

86. Find the reciprocal of the following:

(i) $\left(\frac{1}{2} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times 6\right)$ (ii) $\frac{20}{51} \times \frac{4}{91}$

(iii) $\frac{3}{13} \div \frac{-4}{65}$ (iv) $\left(-5 \times \frac{12}{15}\right) - \left(-3 \times \frac{2}{9}\right)$

87. Complete the following table by finding the sums:

+	$-\frac{1}{9}$	$\frac{4}{11}$	$-\frac{5}{6}$
$\frac{2}{3}$			
$-\frac{5}{4}$		$\frac{-39}{44}$	
$-\frac{1}{3}$			

88. Write each of the following numbers in the form $\frac{p}{q}$, where p and q are integers:

- (a) six-eighths
- (b) three and half
- (c) opposite of 1
- (d) one-fourth
- (e) zero
- (f) opposite of three-fifths

89. If $p = m \times t$ and $q = n \times t$, then $\frac{p}{q} = \frac{\boxed{}}{\boxed{}}$

90. Given that $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers with different denominators and both of them are in standard form. To compare these rational numbers we say that:

(a) $\frac{\boxed{}}{\boxed{}} < \frac{\boxed{}}{\boxed{}}$, if $p \times s < r \times q$

(b) $\frac{p}{q} = \frac{r}{s}$, if _____ = _____

(c) $\frac{\boxed{}}{\boxed{}} > \frac{\boxed{}}{\boxed{}}$, if $p \times s > r \times q$

91. In each of the following cases, write the rational number whose numerator and denominator are respectively as under:

- (a) $5 - 39$ and $54 - 6$
 (b) $(-4) \times 6$ and $8 \div 2$
 (c) $35 \div (-7)$ and $35 - 18$
 (d) $25 + 15$ and $81 \div 40$

92. Write the following as rational numbers in their standard forms:

- (a) 35% (b) 1.2 (c) $-6\frac{3}{7}$
 (d) $240 \div (-840)$ (e) $115 \div 207$

93. Find a rational number exactly halfway between:

- (a) $-\frac{1}{3}$ and $\frac{1}{3}$ (b) $\frac{1}{6}$ and $\frac{1}{9}$ (c) $\frac{5}{-13}$ and $\frac{-7}{9}$ (d) $\frac{1}{15}$ and $\frac{1}{12}$

94. Taking $x = \frac{-4}{9}$, $y = \frac{5}{12}$ and $z = \frac{7}{18}$, find :

- (a) the rational number which when added to x gives y .
 (b) the rational number which subtracted from y gives z .
 (c) the rational number which when added to z gives us x .
 (d) the rational number which when multiplied by y to get x .
 (e) the reciprocal of $x + y$.
 (f) the sum of reciprocals of x and y .
 (g) $(x \div y) \times z$ (h) $(x - y) + z$
 (i) $x + (y + z)$ (j) $x \div (y \div z)$
 (k) $x - (y + z)$

95. What should be added to $\frac{-1}{2}$ to obtain the nearest natural number?

96. What should be subtracted from $\frac{-2}{3}$ to obtain the nearest integer?

97. What should be multiplied with $\frac{-5}{8}$ to obtain the nearest integer?

98. What should be divided by $\frac{1}{2}$ to obtain the greatest negative integer?

99. From a rope 68 m long, pieces of equal size are cut. If length of one piece is $4\frac{1}{4}$ m, find the number of such pieces.

100. If 12 shirts of equal size can be prepared from 27m cloth, what is length of cloth required for each shirt?

101. Insert 3 equivalent rational numbers between

(i) $\frac{-1}{2}$ and $\frac{1}{5}$ (ii) 0 and -10

102. Put the (✓), wherever applicable

Number	Natural Number	Whole Number	Integer	Fraction	Rational Number
(a) - 114					
(b) $\frac{19}{27}$					
(c) $\frac{623}{1}$					
(d) $-19\frac{3}{4}$					
(e) $\frac{73}{71}$					
(f) 0					

103. 'a' and 'b' are two different numbers taken from the numbers 1 – 50.

What is the largest value that $\frac{a-b}{a+b}$ can have? What is the largest value that $\frac{a+b}{a-b}$ can have?

104. 150 students are studying English, Maths or both. 62 per cent of the students are studying English and 68 per cent are studying Maths. How many students are studying both?

- 105.** A body floats $\frac{2}{9}$ of its volume above the surface. What is the ratio of the body submerged volume to its exposed volume? Re-write it as a rational number.

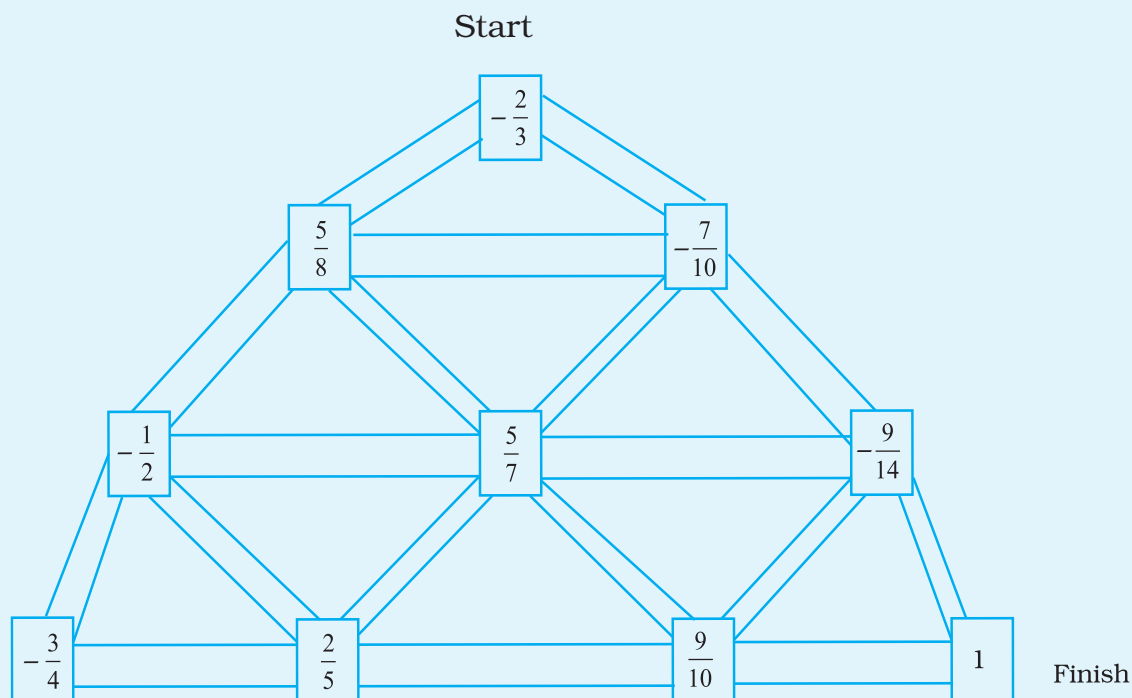
Find the odd one out of the following and give reason.

- 106.** (a) $\frac{4}{3} \times \frac{3}{4}$ (b) $\frac{-3}{2} \times \frac{-2}{3}$
 (c) $2 \times \frac{1}{2}$ (d) $\frac{-1}{3} \times \frac{3}{1}$
- 107.** (a) $\frac{4}{-9}$ (b) $\frac{-16}{36}$
 (c) $\frac{-20}{-45}$ (d) $\frac{28}{-63}$
- 108.** (a) $\frac{-4}{3}$ (b) $\frac{-7}{6}$
 (c) $\frac{-10}{3}$ (d) $\frac{-8}{7}$
- 109.** (a) $\frac{-3}{7}$ (b) $\frac{-9}{15}$
 (c) $\frac{+24}{20}$ (d) $\frac{+35}{25}$

- 110. What's the Error?** Chhaya simplified a rational number in this manner $\frac{-25}{-30} = \frac{-5}{6}$. What error did the student make?

(D) Applications

- Moving from start to finish by going from smaller to bigger rational numbers.

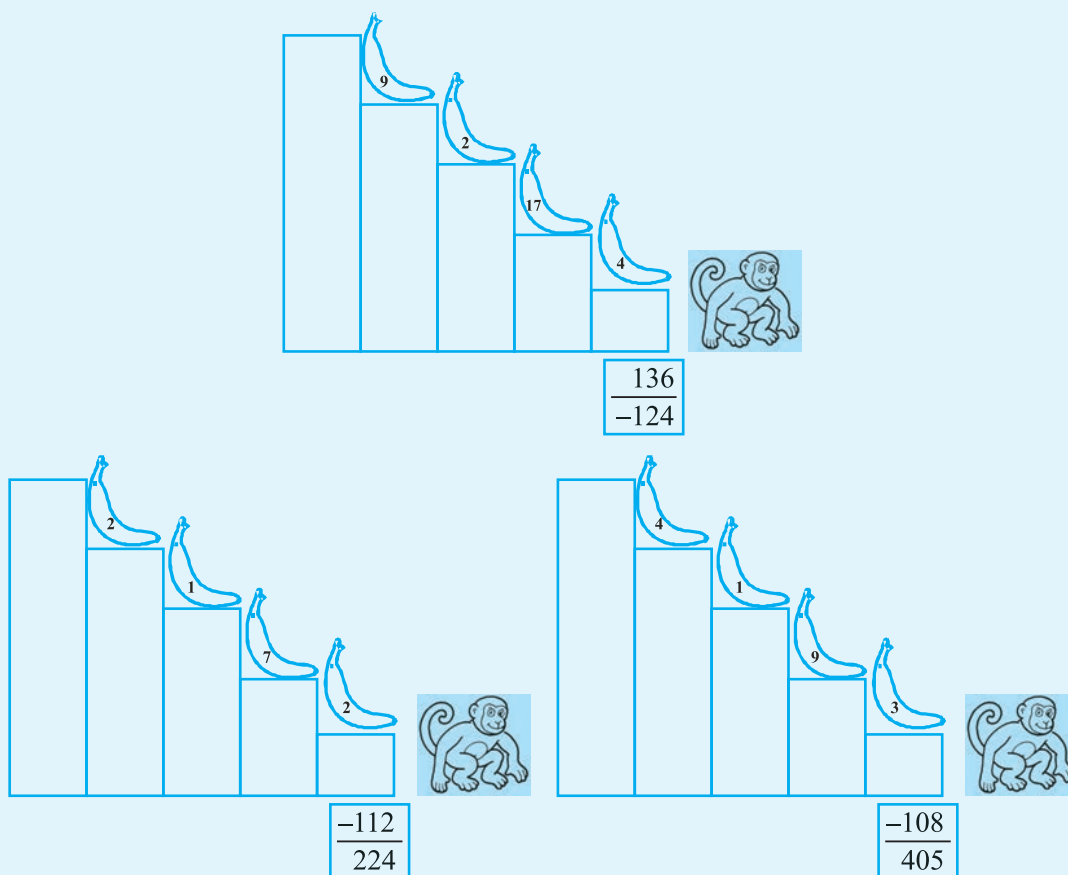


- Replace '*' by inserting an appropriate rational number between the given rational numbers.

$-\frac{1}{4}$		$-\frac{1}{6}$
*		*
$-\frac{1}{2}$		$-\frac{1}{5}$
*		*
$-\frac{1}{3}$		$-\frac{1}{7}$

0	*	-1
	-2	
	*	
	-3	
-4	*	-5

3. Three monkeys are climbing upstairs. They can only move ahead if they eat a banana with the common factor of their numerator and denominator on it. Which of the three monkeys will be able to reach till the end?



4. Crossword Puzzle

Solve the given crossword and then fill up the given boxes. Clues are given below for across as well as downward filling. Also, for across and down clues, clue number is written at the corner of boxes. Answers of clues have to be filled in their respective boxes.

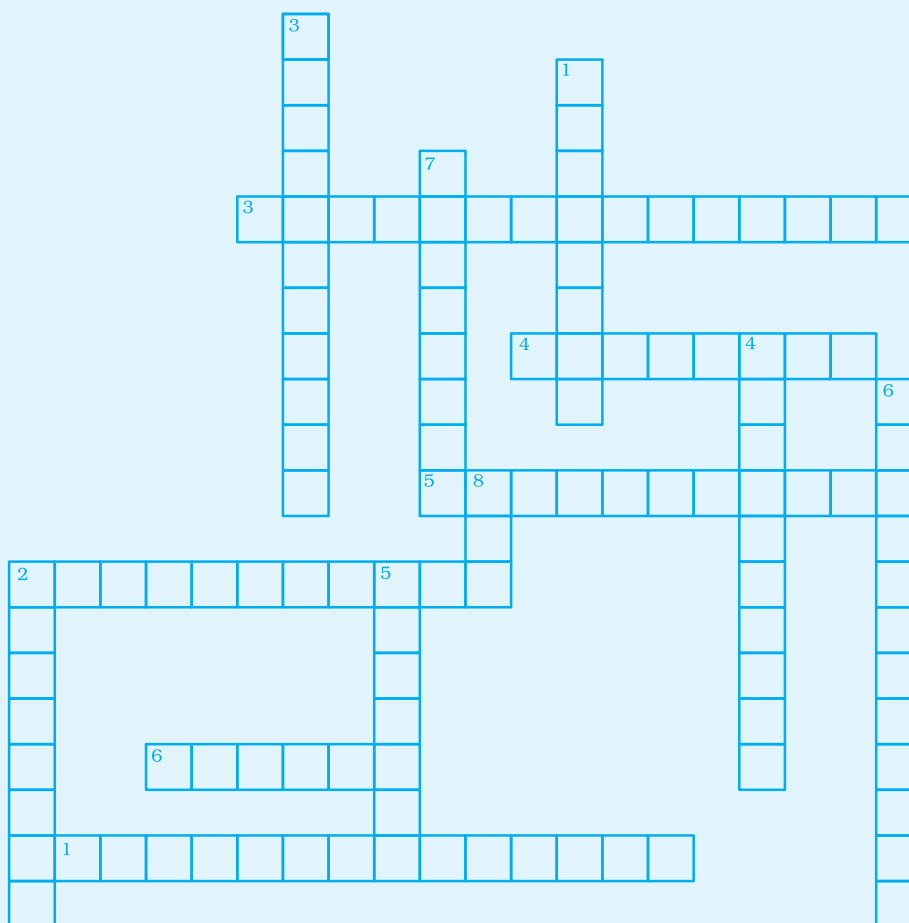
Down 1: $\frac{2}{3}$ and $\frac{-5}{4}$ are _____ numbers.

Down 2: The _____ inverse of $\frac{a}{f}$ is $-\frac{a}{f}$.

Down 3: The addition and multiplication of whole numbers, integers and rational numbers is _____

Down 4: Since, $\frac{1}{0}$ is not defined, hence 0 has no _____.

- Down 5:** Reciprocal is also known as multiplicative _____.
- Down 6 :** The number line extends _____ on both the sides.
- Down 7:** The _____ of two integers may not lead to the formation of another integer.
- Down 8:** The multiplication of a number by its reciprocal gives _____.
- Across 1:** There are _____ rational numbers between two integers.
- Across 2:** The multiplication of rational numbers is _____ commutative and _____.
- Across 3:** The addition and _____ of two whole numbers lead to the formation of another whole number.
- Across 4:** All the positive integers excluding 0 are known as _____ numbers.
- Across 5:** For any rational number a ; $a \div 0$ is _____.
- Across 6:** Rational numbers can be represented on a _____ line.



UNIT 9

PERIMETER AND AREA

(A) Main Concepts and Results

- Perimeter of a closed figure is the distance around it while area is the measure of the part of plane or region enclosed by it.
- Perimeter of a regular polygon = Number of sides \times Length of one side.
- Perimeter of a square = $4 \times$ side



Fig. 9.1

- Perimeter of a rectangle = $2(l + b)$

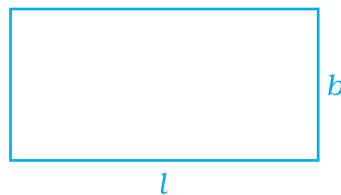


Fig. 9.2

- Area of square = side \times side
- Area of rectangle = $l \times b$
- Area of parallelogram = $b \times h$
- Area of triangle = $\frac{1}{2} \times b \times h$

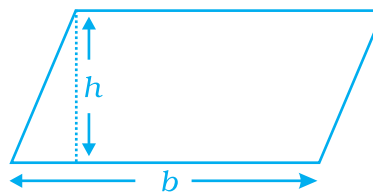


Fig. 9.3

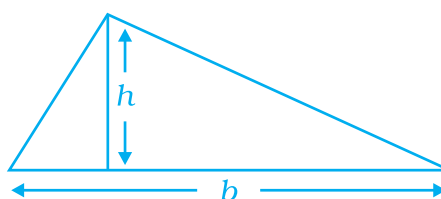


Fig. 9.4

- The distance around a circle is known as its circumference.
- The ratio of circumference and diameter of a circle is a constant and is denoted by π (pi).
- Approximate value of π is taken as $\frac{22}{7}$ or 3.14
- Circumference of a circle of radius r is $2\pi r$,
- Area of a circle of radius r is πr^2 .

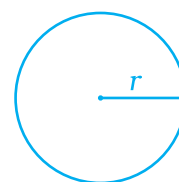
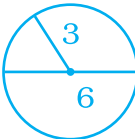


Fig. 9.5

CIRCUMFERENCE OF A CIRCLE		
Words	Numbers	Formula
The circumference C of a circle is π times the diameter d , or 2π times the radius r .	 $ \begin{aligned} C &= \pi \times 6 \\ &= 2\pi \times 3 \\ &= 18.8 \text{ units} \end{aligned} $	$ \begin{aligned} C &= \pi d \\ \text{or} \\ C &= 2\pi r \end{aligned} $

(B) Solved Examples

In Examples 1 and 2, there are four options, out of which one is correct. Choose the correct one.

Example 1: Following rectangle is composed of 8 congruent parts.

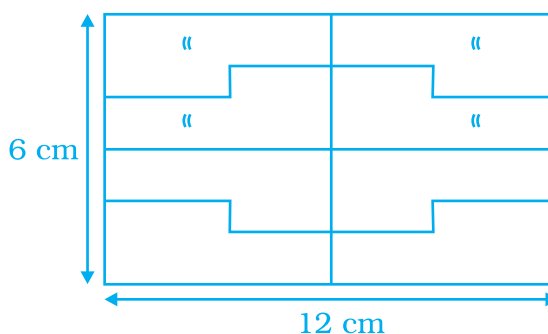


Fig. 9.6

Area of each part is

- (a) 72 cm^2 (b) 36 cm^2 (c) 18 cm^2 (d) 9 cm^2

Solution: Correct answer is (d).

Example 2: Area of a right triangle is 54 cm^2 . If one of its legs is 12 cm long, its perimeter is

- (a) 18 cm (b) 27 cm (c) 36 cm (d) 54 cm

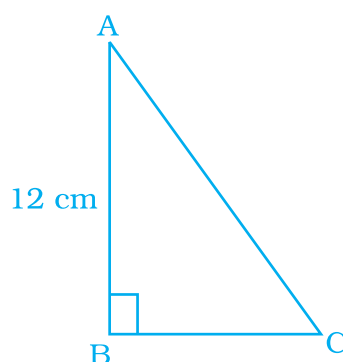
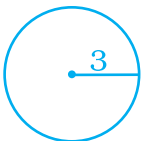


Fig. 9.7

Solution: Correct answer is (c).

AREA OF A CIRCLE

Words	Numbers	Formula
The area A of a circle is π times the square of the radius r .	 $ \begin{aligned} A &= \pi \times 3^2 \\ &= 9\pi \\ &= 28.3 \text{ units} \end{aligned} $	$A = \pi r^2$

In Examples 3 to 6, fill in the blanks to make it a statement true.

Example 3: Area of parallelogram QPON is _____ cm^2 .

Solution: 48 cm^2

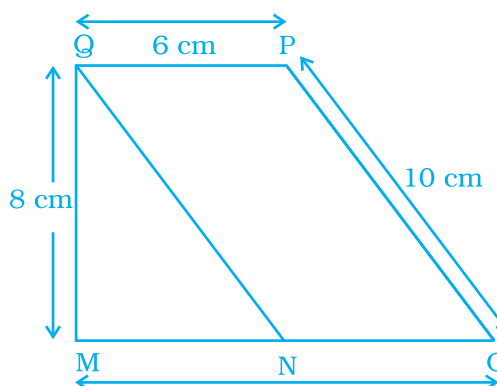


Fig. 9.8

Example 4: 1 hectare = _____ cm^2

Solution: 10,00,00,000

Example 5: _____ squares of each side 1 m makes a square of side 5 km.

Solution: 2,50,00,000

Example 6: All the congruent triangles have _____ area.

Solution: equal

In Examples 7 to 10, state whether the statements are True or False.

Example 7: All the triangles equal in area are congruent.

Solution: False

Example 8: The area of any parallelogram ABCD, is $AB \times BC$.

Solution: False.

Example 9: Ratio of the circumference and the diameter of a circle is more than 3.

Solution: True

Example 10: A nursery school play ground is 160 m long and 80 m wide. In it $80 \text{ m} \times 80 \text{ m}$ is kept for swings and in the remaining portion, there is 1.5 m wide path parallel to its width and parallel to its remaining length as shown in Fig. 9.9. The remaining area is covered by grass. Find the area covered by grass.

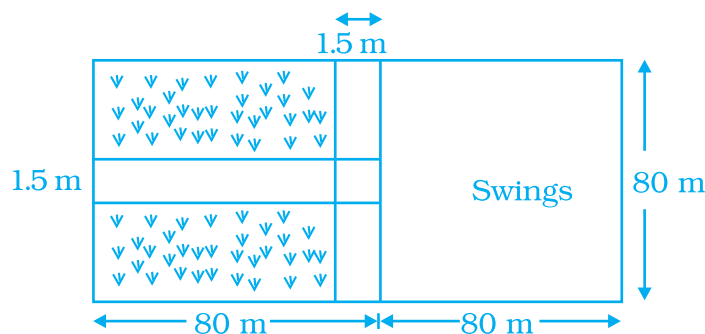


Fig. 9.9

Solution : Area of school playground is $160 \text{ m} \times 80 \text{ m} = 12800 \text{ m}^2$

Area kept for swings = $80 \text{ m} \times 80 \text{ m} = 6400 \text{ m}^2$

Area of path parallel to the width of playground

$$= 80 \text{ m} \times 1.5 \text{ m} = 120 \text{ m}^2$$

Area of path parallel to the remaining length of playground

$$= 80 \text{ m} \times 1.5 \text{ m} = 120 \text{ m}^2.$$

Area common to both paths = $1.5 \text{ m} \times 1.5 \text{ m} = 2.25 \text{ m}^2$.

[since it is taken twice for measurement it is to be

subtracted from the area of paths]

Total area covered by both the paths

$$= (120 + 120 - 2.25) \text{ m}^2$$

$$= 237.75 \text{ m}^2.$$

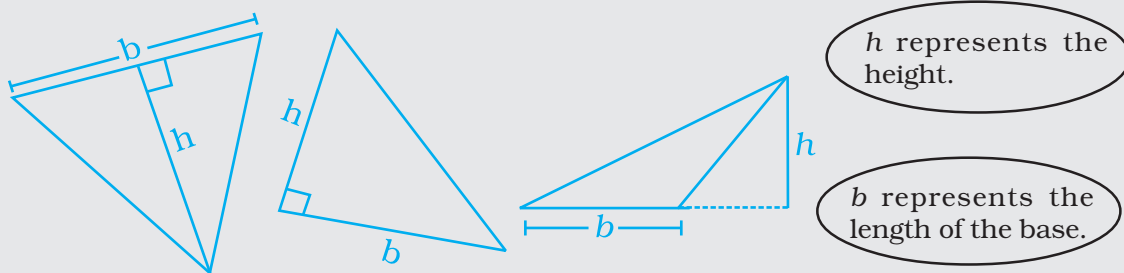
Area covered by grass = Area of school playground – (Area kept for swings + Area covered by paths)

$$= 12800 \text{ m}^2 - [6400 + 237.75] \text{ m}^2$$

$$= (12800 - 6637.75) \text{ m}^2$$

$$= 6162.25 \text{ m}^2.$$

Any side of a triangle can be the **base**. The diagrams below show the length of the **base** (b) and the **height** (h) of several triangles.



Example 11:

In Fig. 9.10, ABCD is a parallelogram, in which $AB = 8$ cm, $AD = 6$ cm and altitude $AE = 4$ cm. Find the altitude corresponding to side AD .

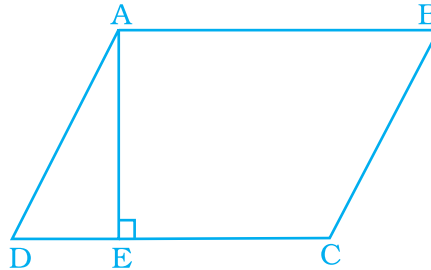


Fig. 9.10

Solution:

Area of parallelogram ABCD = $AB \times AE = 8 \times 4 \text{ cm}^2$
 $= 32 \text{ cm}^2$

Let altitude corresponding to AD be h . Then,

$$h \times AD = 32$$

or $h \times 6 = 32$

or $h = \frac{32}{6} = \frac{16}{3}$

Thus, altitude corresponding to AD is $\frac{16}{3}$ cm.

Example 12:

A rectangular shaped swimming pool with dimensions $30 \text{ m} \times 20 \text{ m}$ has 5 m wide cemented path along its length and 8 m wide path along its width (as shown in Fig. 9.11). Find the cost of cementing the path at the rate of Rs 200 per m^2 .

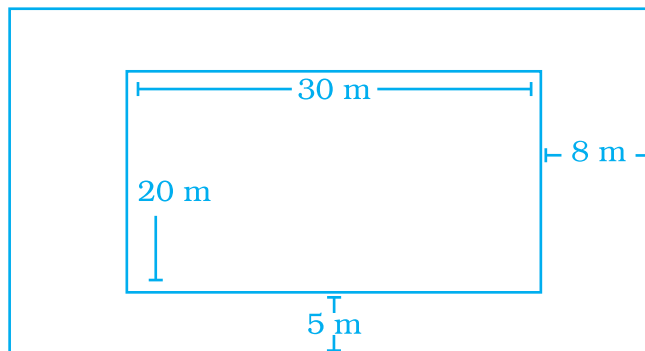


Fig. 9.11

Solution: Area covered by swimming pool = $30 \text{ m} \times 20 \text{ m} = 600 \text{ m}^2$.

Length of outer rectangle = $(30 + 8 + 8) \text{ m} = 46 \text{ m}$

and its breadth = $(20 + 5 + 5) \text{ m} = 30 \text{ m}$

So, the area of outer rectangle

$$= 46 \text{ m} \times 30 \text{ m} = 1380 \text{ m}^2.$$

Area of cemented path =

Area of outer rectangle – Area of swimming pool

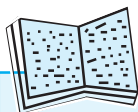
$$= (1380 - 600) \text{ m}^2 = 780 \text{ m}^2.$$

Cost of cementing 1 m^2 path = ₹ 200

So, total cost of cementing the path

$$= ₹ 780 \times 200$$

$$= ₹ 156000$$



Vocabulary

To become familiar with some of the vocabulary terms consider the following.

1. The word **circumference** contains the prefix circum-, which means “around”. What do you think about the circumference of a circle?
2. The Greek prefix peri- means “around,” and the root meter means “means of measuring.” “What do you suppose **perimeter** means?”
3. The Greek prefix dia- means “across.” What do you think about the **diameter** of a circle?

Example 13: Circumference of a circle is 33 cm. Find its area.

Solution: Let the radius of the circle be r .

Then, $2\pi r = 33$

$$\text{i.e., } r = \frac{33}{2\pi} = \frac{33}{2} \times \frac{7}{22} = \frac{21}{4}$$

Thus, radius is $\frac{21}{4}$ cm

$$\text{So, area of the circle} = \pi r^2 = \frac{22}{7} \cdot \frac{21}{4} \cdot \frac{21}{4} = \frac{693}{8}$$

Thus, area of the circle is $\frac{693}{8}$ cm².

Example 14: Rectangle ABCD is formed in a circle as shown in Fig. 9.12. If AE = 8 cm and AD = 5 cm, find the perimeter of the rectangle.

Solution:

$$DE = EA + AD = (8 + 5)\text{cm} = 13 \text{ cm}$$

DE is the radius of the circle.

Also, DB is the radius of the circle.

Next, AC = DB [Since diagonals of a rectangle are equal in length]

Therefore, AC = 13 cm.

$$\text{From } \triangle ADC, DC^2 = AC^2 - AD^2 = 13^2 - 5^2 = 169 - 25 = 144 = 12^2$$

$$\text{So, } DC = 12$$

Thus, length of DC is 12 cm.

$$\begin{aligned} \text{Hence, perimeter of the rectangle ABCD} \\ = 2(12 + 5)\text{cm} = 34 \text{ cm.} \end{aligned}$$

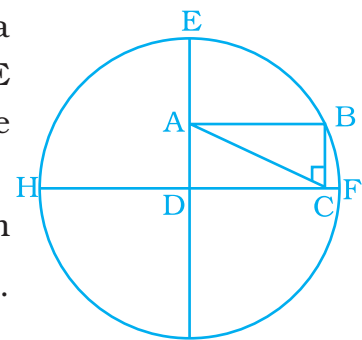
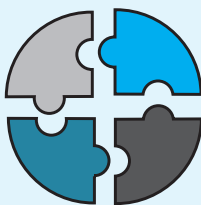


Fig. 9.12

Application on Problem Solving Strategy



Example 15

Find the area of a parallelogram shaped shaded region of Fig. 9.13. Also, find the area of each triangle. What is the ratio of area of shaded portion to the remaining area of rectangle?

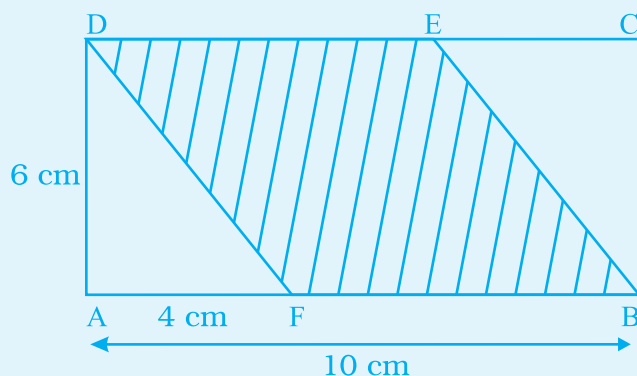


Fig. 9.13

Solution:**Understand and Explore the Problem**

- What information is given in the question?
 - (i) It is given that ABCD is a rectangle whose $l = 10$ cm and $b = 6$ cm.
 - (ii) In the figure $AF = 4$ cm
 - (iii) To find the area of shaded region.

**Plan a Strategy**

- First recall the areas of a triangle and a rectangle
 Area of a rectangle = length \times breadth
 Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{altitude}$
- In the Fig. 9.13, DAF is a right triangle in which $\angle A = 90^\circ$.
 ABCD is a rectangle and DEBF is a parallelogram,
 Since $\triangle DAF \cong \triangle BCE$, therefore their areas will be equal.

**Solve**

- Area of $\triangle DAF = \frac{1}{2} \times 4 \times 6 \text{ cm}^2$

- Area of rectangle = $l \times b$

$$= 10 \text{ cm} \times 6 \text{ cm} = 60 \text{ cm}^2$$
- Area of shaded region = Area of rectangle – Area of $\triangle DAF$

$$- \text{Area of } \triangle BCE = (60 - 12 - 12) \text{ cm}^2$$

$$= (60 - 24) \text{ cm}^2 = 36 \text{ cm}^2$$
- Area of remaining part = Area of Rectangle – Area of shaded portion

$$= (60 - 36) \text{ cm}^2 = 24 \text{ cm}^2$$

$$\begin{aligned}\text{Ratio} &= \text{Area of shaded portion} : \text{Area of remaining rectangle} \\ &= 36 : 24 = 3 : 2\end{aligned}$$


Revise

- Area of shaded portion + Area of remaining portion = Area of rectangle
- That is, $(36 + 24) \text{ cm}^2 = 60 \text{ cm}^2$

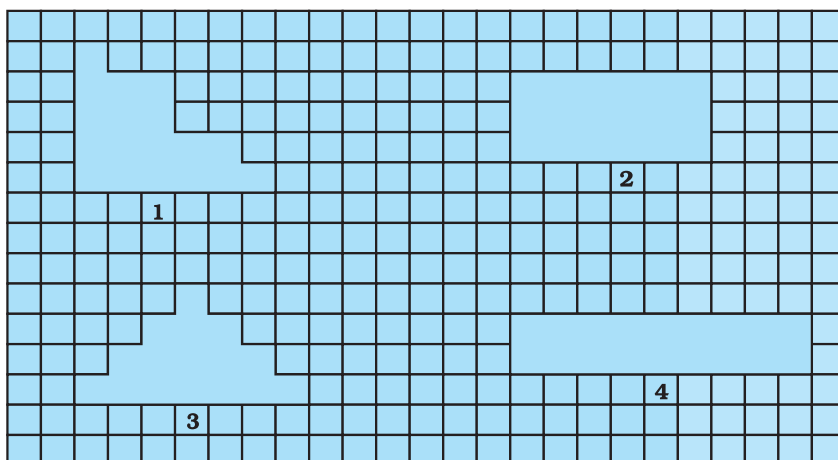
Think and Discuss

1. We can also calculate area of shaded portion by using area of parallelogram. Think what would be its base and altitude.
2. Can you frame, questions in which areas of all the plane figures rectangle, square, triangle and a parallelogram are to be calculated?

(C) Exercise

In the Questions 1 to 37, there are four options, out of which one is correct. Choose the correct one.

- 1.** Observe the shapes 1, 2, 3 and 4 in the figures. Which of the following statements is not correct?



- (a) Shapes 1, 3 and 4 have different areas and different perimeters.
 (b) Shapes 1 and 4 have the same area as well as the same perimeter.
 (c) Shapes 1, 2 and 4 have the same area.
 (d) Shapes 1, 3 and 4 have the same perimeter.

Think and Discuss

- Compare** the area of a rectangle with base b and height h with the area of a rectangle with base $2b$ and height $2h$.
 - Express** the formulas for the area and perimeter of a square using s for the length of a side.
2. A rectangular piece of dimensions $3\text{ cm} \times 2\text{ cm}$ was cut from a rectangular sheet of paper of dimensions $6\text{ cm} \times 5\text{ cm}$ (Fig. 9.14).

Area of remaining sheet of paper is

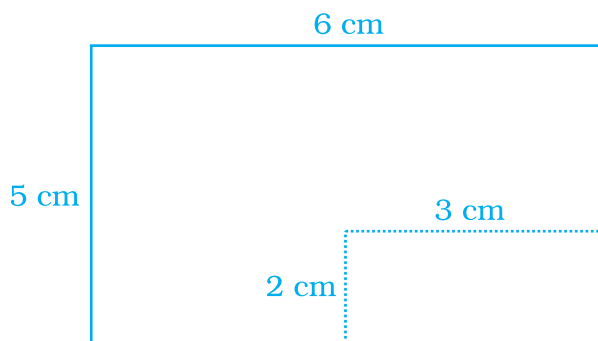


Fig. 9.14

- (a) 30 cm^2 (b) 36 cm^2 (c) 24 cm^2 (d) 22 cm^2

3. 36 unit squares are joined to form a rectangle with the least perimeter. Perimeter of the rectangle is
 (a) 12 units (b) 26 units
 (c) 24 units (d) 36 units
4. A wire is bent to form a square of side 22 cm. If the wire is rebent to form a circle, its radius is
 (a) 22 cm (b) 14 cm (c) 11 cm (d) 7 cm

Think and Discuss

1. Give the formula for the area of a circle in terms of the diameter d .

5. Area of the circle obtained in Question 4 is
 (a) 196 cm^2 (b) 212 cm^2 (c) 616 cm^2 (d) 644 cm^2
6. Area of a rectangle and the area of a circle are equal. If the dimensions of the rectangle are $14 \text{ cm} \times 11 \text{ cm}$, then radius of the circle is
 (a) 21 cm (b) 10.5 cm (c) 14 cm (d) 7 cm.
7. Area of shaded portion in Fig. 9.15 is
 (a) 25 cm^2 (b) 15 cm^2 (c) 14 cm^2 (d) 10 cm^2

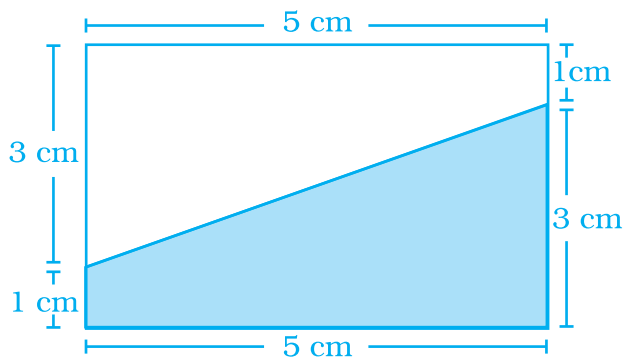


Fig. 9.15

8. Area of parallelogram ABCD (Fig. 9.16) is not equal to
 (a) $DE \times DC$ (b) $BE \times AD$ (c) $BF \times DC$ (d) $BE \times BC$

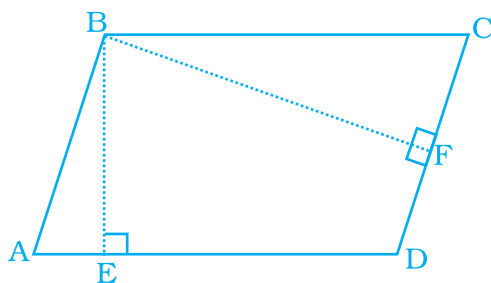


Fig. 9.16

Think and Discuss

- Describe** what happens to the area of a triangle when the base is doubled and the height remains the same.
- Describe** what happens to the area of a parallelogram when the length of its base is doubled but the height remains the same.

9. Area of triangle MNO of Fig. 9.17 is

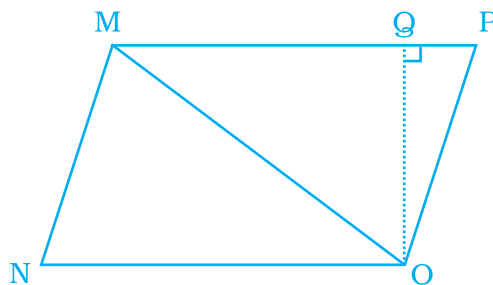


Fig. 9.17

- (a) $\frac{1}{2} MN \times NO$ (b) $\frac{1}{2} NO \times MO$ (c) $\frac{1}{2} MN \times OQ$ (d) $\frac{1}{2} NO \times OQ$
10. Ratio of area of $\triangle MNO$ to the area of parallelogram MNOP in the same figure 9.17 is
- (a) 2 : 3 (b) 1 : 1 (c) 1 : 2
(d) 2 : 1
11. Ratio of areas of $\triangle MNO$, $\triangle MOP$ and $\triangle MPQ$ in Fig. 9.18 is
- (a) 2 : 1 : 3 (b) 1 : 3 : 2 (c) 2 : 3 : 1 (d) 1 : 2 : 3

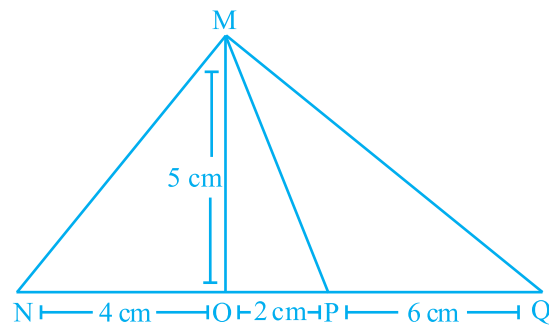


Fig. 9.18

12. In Fig. 9.19, EFGH is a parallelogram, altitudes FK and FI are 8 cm and 4 cm respectively. If $EF = 10$ cm, then area of EFGH is

- (a) 20 cm^2 (b) 32 cm^2
(c) 40 cm^2 (d) 80 cm^2

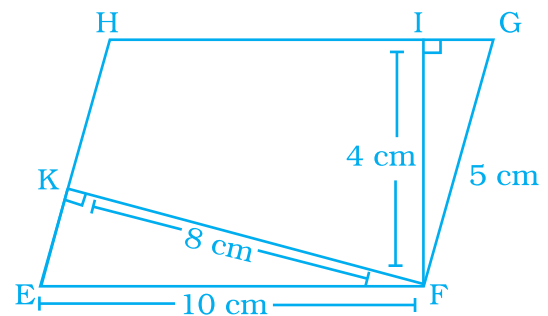


Fig. 9.19

The Taj Mahal, a world famous structure, is the most visited attraction in India. It was created in the 17th century by Emperor Shah Jahan to honour the memory of his beloved wife Mumtaz Mahal. The design of the Taj Mahal is based on the number four and its multiples.



Think about it

- The garden at the Taj Mahal was laid out in four squares of the same size. Each square was divided into four flower beds, with 400 flowers in each bed. How many flowers were in the garden?
- The central chamber of the Taj Mahal was built in the shape of an octagon. How is an octagon related to the number 4?

13. In reference to a circle the value of π is equal to

- (a) $\frac{\text{area}}{\text{circumference}}$ (b) $\frac{\text{area}}{\text{diameter}}$
 (c) $\frac{\text{circumference}}{\text{diameter}}$ (d) $\frac{\text{circumference}}{\text{radius}}$

14. Circumference of a circle is always

- (a) more than three times of its diameter
 (b) three times of its diameter
 (c) less than three times of its diameter
 (d) three times of its radius

15. Area of triangle PQR is 100 cm^2 (Fig. 9.20). If altitude QT is 10 cm, then its base PR is

- (a) 20 cm (b) 15 cm (c) 10 cm (d) 5 cm

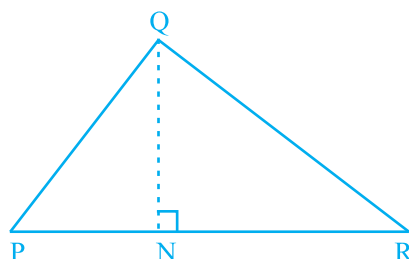


Fig. 9.20

16. In Fig. 9.21, if $PR = 12 \text{ cm}$, $QR = 6 \text{ cm}$ and $PL = 8 \text{ cm}$, then QM is

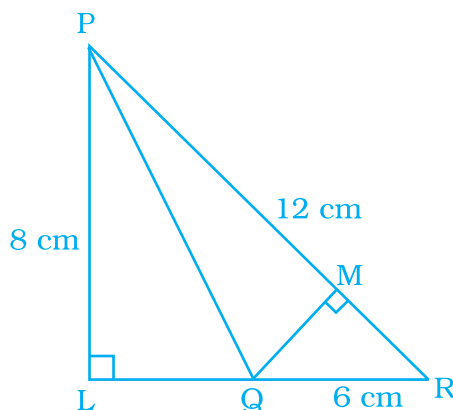


Fig. 9.21

- (a) 6 cm (b) 9 cm (c) 4 cm (d) 2 cm

17. In Fig. 9.22 $\triangle MNO$ is a right-angled triangle. Its legs are 6 cm and 8 cm long. Length of perpendicular NP on the side MO is

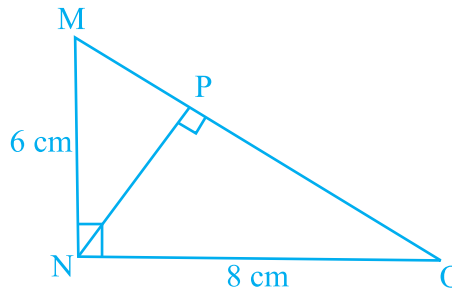
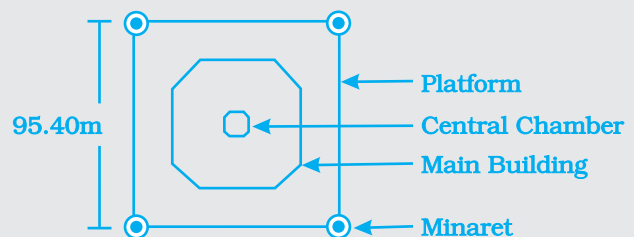


Fig. 9.22

- (a) 4.8 cm (b) 3.6 cm (c) 2.4 cm (d) 1.2 cm
18. Area of a right-angled triangle is 30 cm^2 . If its smallest side is 5 cm, then its hypotenuse is
(a) 14 cm (b) 13 cm (c) 12 cm (d) 11 cm
19. Circumference of a circle of diameter 5 cm is
(a) 3.14 cm (b) 31.4 cm (c) 15.7 cm (d) 1.57 cm
20. Circumference of a circle disc is 88 cm. Its radius is
(a) 8 cm (b) 11 cm (c) 14 cm (d) 44 cm



- a. The Taj Mahal stands on a square platform that is 95.40 m on each side. What is the area of this square in square metres?
- b. The floor area of the main building is 3214 m^2 . What is the area of the part of the platform that is not covered by the main building?

- 21.** Length of tape required to cover the edges of a semicircular disc of radius 10 cm is
 (a) 62.8 cm (b) 51.4 cm (c) 31.4 cm (d) 15.7 cm
- 22.** Area of circular garden with diameter 8 m is
 (a) 12.56 m² (b) 25.12 m² (c) 50.24 m² (d) 200.96 m²
- 23.** Area of a circle with diameter ' m ' radius ' n ' and circumference ' p ' is
 (a) $2\pi n$ (b) πm^2 (c) πp^2 (d) πn^2
- 24.** A table top is semicircular in shape with diameter 2.8 m. Area of this table top is
 (a) 3.08 m² (b) 6.16 m² (c) 12.32 m² (d) 24.64 m²
- 25.** If $1\text{ m}^2 = x\text{ mm}^2$, then the value of x is
 (a) 1000 (b) 10000 (c) 100000
 (d) 1000000
- 26.** If p squares of each side 1mm makes a square of side 1cm, then p is equal to
 (a) 10 (b) 100 (c) 1000 (d) 10000
- 27.** 12 m² is the area of
 (a) a square with side 12 m
 (b) 12 squares with side 1m each
 (c) 3 squares with side 4 m each
 (d) 4 squares with side 3 m each
- 28.** If each side of a rhombus is doubled, how much will its area increase?
 (a) 1.5 times (b) 2 times (c) 3 times (d) 4 times
- 29.** If the sides of a parallelogram are increased to twice its original lengths, how much will the perimeter of the new parallelogram?
 (a) 1.5 times (b) 2 times (c) 3 times (d) 4 times
- 30.** If radius of a circle is increased to twice its original length, how much will the area of the circle increase?
 (a) 1.4 times (b) 2 times (c) 3 times (d) 4 times
- 31.** What will be the area of the largest square that can be cut out of a circle of radius 10 cm?
 (a) 100 cm² (b) 200 cm² (c) 300 cm² (d) 400 cm²

Thirty-seven specialists including artists, stone cutters, engineers, architects, calligraphers, and inlayers designed the Taj Mahal and supervised the 20,000 workers who built it.

This section of flooring from a terrace at the Taj Mahal is inlaid with white marble and red sandstone tiles.

What geometric shapes do you see in the pattern in the floor?

The design and construction of the terrace must have involved measuring lengths and finding areas.



- 32.** What is the radius of the largest circle that can be cut out of the rectangle measuring 10 cm in length and 8 cm in breadth?
 (a) 4 cm (b) 5 cm (c) 8 cm (d) 10 cm
- 33.** The perimeter of the figure ABCDEFGHIJ is
 (a) 60 cm (b) 30 cm (c) 40 cm (d) 50 cm

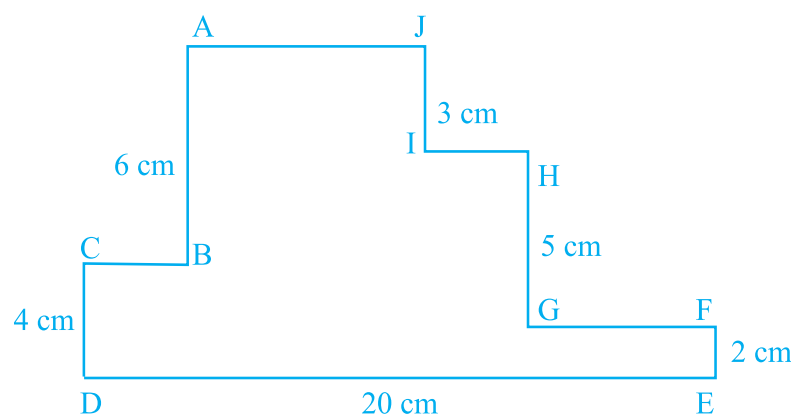


Fig. 9.23

- 34.** The circumference of a circle whose area is $81\pi r^2$, is
 (a) $9\pi r$ (b) $18\pi r$ (c) $3\pi r$ (d) $81\pi r$
- 35.** The area of a square is 100 cm^2 . The circumference (in cm) of the largest circle cut of it is
 (a) 5π (b) 10π (c) 15π (d) 20π

36. If the radius of a circle is tripled, the area becomes
 (a) 9 times (b) 3 times (c) 6 times (d) 30 times
37. The area of a semicircle of radius $4r$ is
 (a) $8\pi r^2$ (b) $4\pi r^2$ (c) $12\pi r^2$ (d) $2\pi r^2$

In Questions 38 to 56, fill in the blanks to make the statements true.

38. Perimeter of a regular polygon = length of one side \times _____.

39. If a wire in the shape of a square is rebent into a rectangle, then the _____ of both shapes remain same, but _____ may vary.

40. Area of the square MNOP of Fig. 9.24 is 144 cm^2 . Area of each triangle is _____.

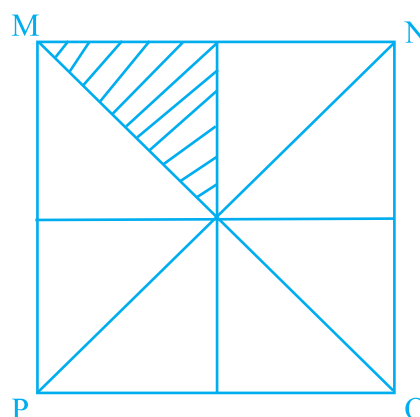


Fig. 9.24

41. In Fig. 9.25, area of parallelogram BCEF is _____ cm^2 where ACDF is a rectangle.

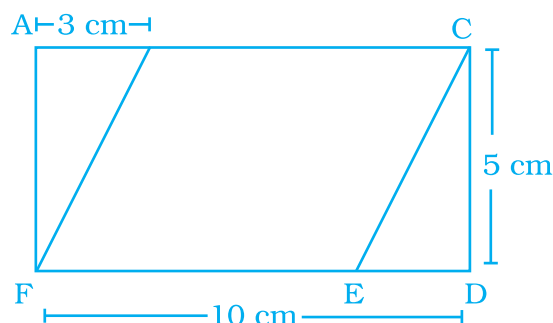


Fig. 9.25

42. To find area, any side of a parallelogram can be chosen as _____ of the parallelogram.
43. Perpendicular dropped on the base of a parallelogram from the opposite vertex is known as the corresponding _____ of the base.
44. The distance around a circle is its _____.

45. Ratio of the circumference of a circle to its diameter is denoted by symbol _____.
46. If area of a triangular piece of cardboard is 90 cm^2 , then the length of altitude corresponding to 20 cm long base is _____ cm.
47. Value of π is _____ approximately.
48. Circumference 'C' of a circle can be found by multiplying diameter 'd' with _____.
49. Circumference 'C' of a circle is equal to $2\pi \times$ _____.
50. $1 \text{ m}^2 =$ _____ cm^2 .
51. $1 \text{ cm}^2 =$ _____ mm^2 .
52. 1 hectare = _____ m^2 .
53. Area of a triangle = $\frac{1}{2}$ base \times _____.
54. $1 \text{ km}^2 =$ _____ m^2 .
55. Area of a square of side 6 m is equal to the area of _____ squares of each side 1 cm.
56. $10 \text{ cm}^2 =$ _____ m^2 .

In Questions 57 to 72, state whether the statements are True or False.

57. In Fig. 9.26, perimeter of (ii) is greater than that of (i), but its area is smaller than that of (i).

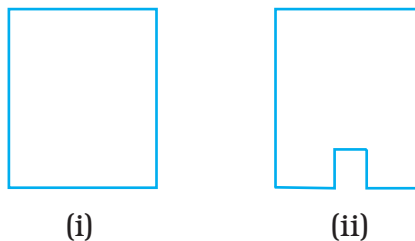
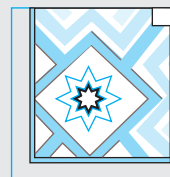


Fig. 9.26

Some of the designs created on the walls of the Taj Mahal can be made using rectangles and triangles. You can use what you know about the area of parallelograms to find the area of triangles.



58. In Fig. 9.27,

(a) area of (i) is the same as the area of (ii).

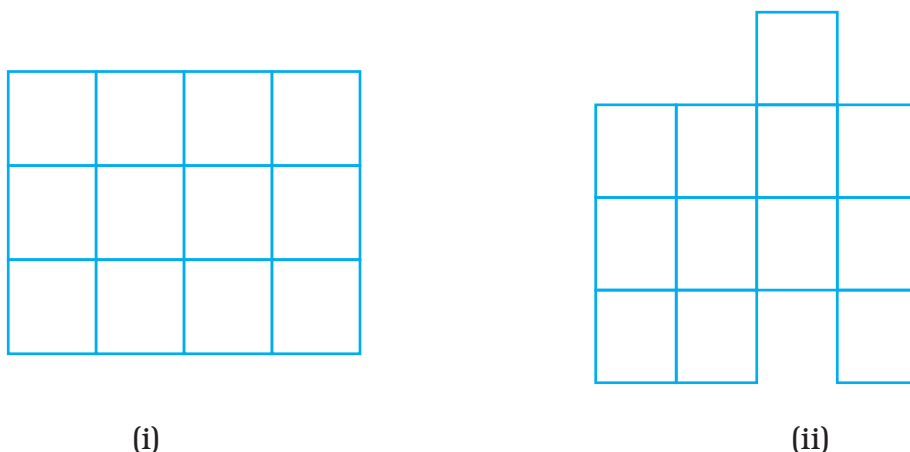


Fig. 9.27

(b) Perimeter of (ii) is the same as (i).

(c) If (ii) is divided into squares of unit length, then its area is 13 unit squares.

(d) Perimeter of (ii) is 18 units.

59. If perimeter of two parallelograms are equal, then their areas are also equal.

60. All congruent triangles are equal in area.

61. All parallelograms having equal areas have same perimeters.

Observe all the four triangles FAB, EAB, DAB and CAB as shown in Fig. 9.28:

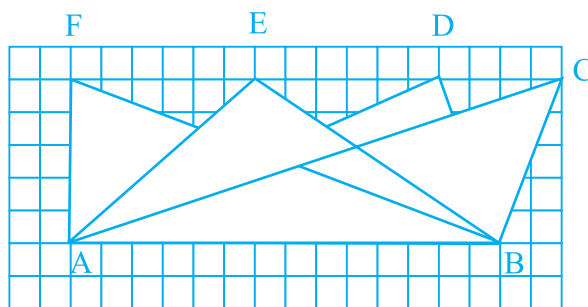


Fig. 9.28

Now answer Questions 62 to 65:

- 62.** All triangles have the same base and the same altitude.
- 63.** All triangles are congruent.
- 64.** All triangles are equal in area.
- 65.** All triangles may not have the same perimeter.
- 66.** In Fig. 9.29 ratio of the area of triangle ABC to the area of triangle ACD is the same as the ratio of base BC of triangle ABC to the base CD of triangle ACD.

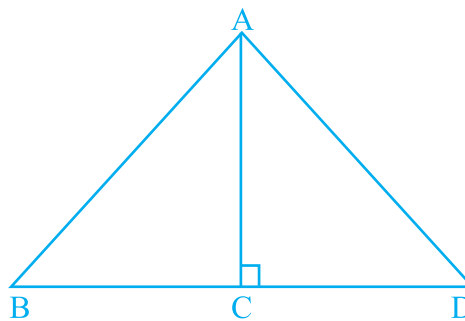


Fig. 9.29

- 67.** Triangles having the same base have equal area.
- 68.** Ratio of circumference of a circle to its radius is always $2\pi : 1$.
- 69.** 5 hectare = 500 m²
- 70.** An increase in perimeter of a figure always increases the area of the figure.
- 71.** Two figures can have the same area but different perimeters.
- 72.** Out of two figures if one has larger area, then its perimeter need not to be larger than the other figure.
- 73.** A hedge boundary needs to be planted around a rectangular lawn of size 72 m × 18 m. If 3 shrubs can be planted in a metre of hedge, how many shrubs will be planted in all?

- 74.** People of Khejadli village take good care of plants, trees and animals. They say that plants and animals can survive without us, but we can not survive without them. Inspired by her elders Amrita marked some land for her pets (camel and ox) and plants. Find the ratio of the areas kept for animals and plants to the living area.

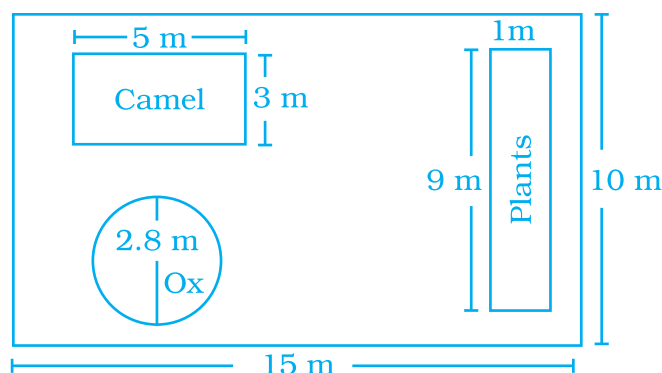
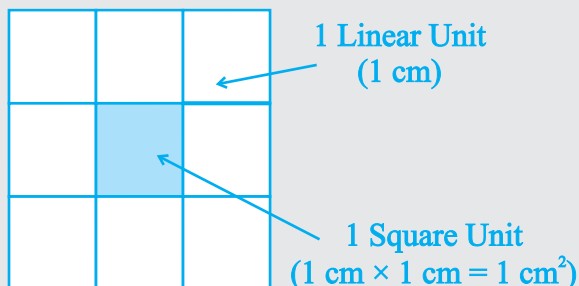


Fig. 9.30

- 75.** The perimeter of a rectangle is 40 m. Its length is four metres less than five times its breadth. Find the area of the rectangle.
- 76.** A wall of a room is of dimensions 5 m \times 4 m. It has a window of dimensions 1.5 m \times 1 m and a door of dimensions 2.25 m \times 1 m. Find the area of the wall which is to be painted.
- 77.** Rectangle MNOP is made up of four congruent rectangles (Fig. 9.31). If the area of one of the rectangles is 8 m² and breadth is 2 m, then find the perimeter of MNOP.

Square units are also used to measure area in the metric system. Since each small square is 1 cm by 1 cm, it has an area of **1 square centimetre (1 cm²)**.



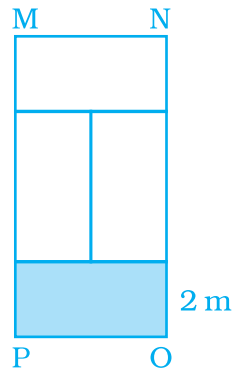


Fig. 9.31

- 78.** In Fig. 9.32, area of $\triangle AFB$ is equal to the area of parallelogram ABCD. If altitude EF is 16 cm long, find the altitude of the parallelogram to the base AB of length 10 cm. What is the area of $\triangle DAO$, where O is the mid point of DC?

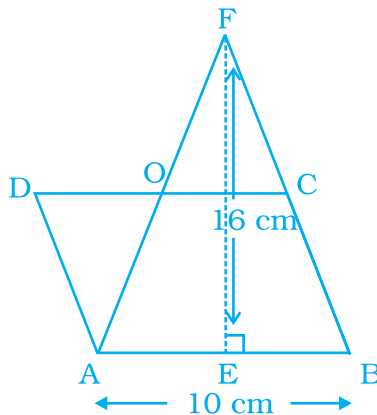


Fig. 9.32

Did You Know

Area is expressed in square units, such as square metre or square centimetres. You can abbreviate square units by writing the abbreviation for the unit followed by a power raised 2. For example, an abbreviation for square metre is m^2 .

Volume is expressed in cubic units. You can abbreviate cubic units by writing the abbreviation for the unit followed by a power raised 3. For example, an abbreviation for cubic centimetres is cm^3 .

- 79.** Ratio of the area of $\triangle WXY$ to the area of $\triangle WZY$ is 3 : 4 (Fig. 9.33). If the area of $\triangle WXZ$ is 56 cm^2 and $WY = 8 \text{ cm}$, find the lengths of XY and YZ .

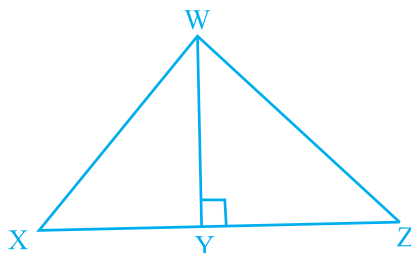


Fig. 9.33

- 80.** Rani bought a new field that is next to one she already owns (Fig. 9.34). This field is in the shape of a square of side 70 m. She makes a semi circular lawn of maximum area in this field.
- Find the perimeter of the lawn.
 - Find the area of the square field excluding the lawn.

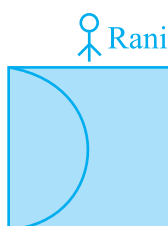


Fig. 9.34

- 81.** In Fig. 9.35, find the area of parallelogram ABCD if the area of shaded triangle is 9 cm^2 .

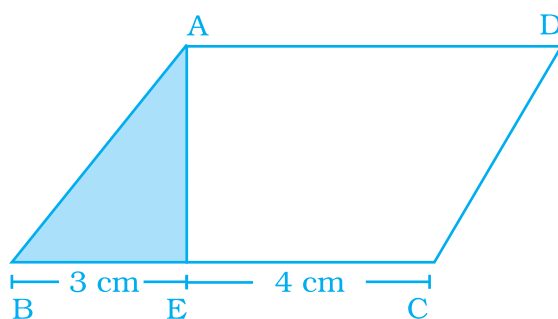


Fig. 9.35

- 82.** Pizza factory has come out with two kinds of pizzas. A square pizza of side 45 cm costs ₹ 150 and a circular pizza of diameter 50 cm costs ₹160 (Fig. 9.36). Which pizza is a better deal?

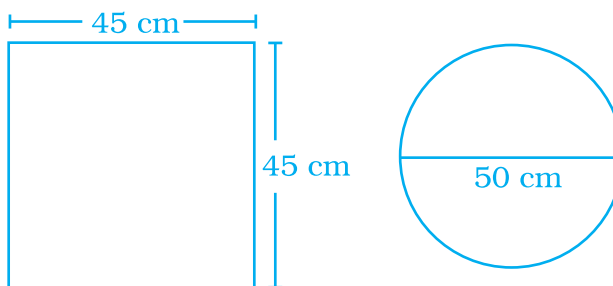


Fig. 9.36

- 83.** Three squares are attached to each other as shown in Fig. 9.37. Each square is attached at the mid point of the side of the square to its right. Find the perimeter of the complete figure.

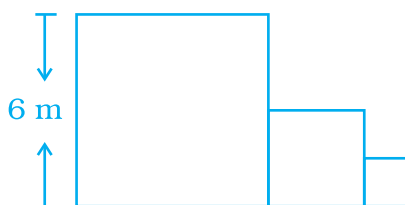
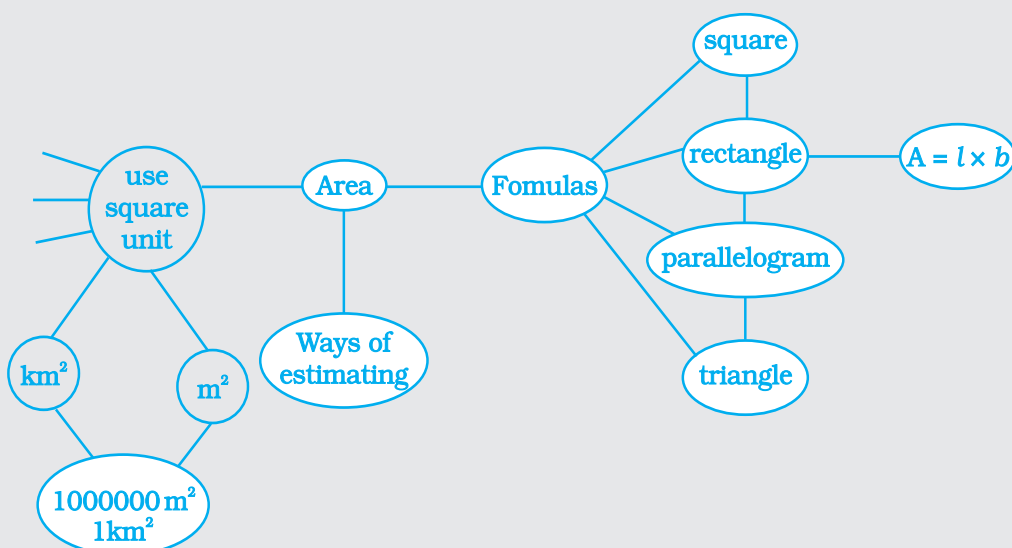


Fig. 9.37

Visual displays can help you relate ideas and organise information.

Copy and extend the concept map to connect ideas you have learned about area. Add on units of measure, formulas, and notes about relationships.



84. In Fig. 9.38, ABCD is a square with $AB = 15$ cm. Find the area of the square BDFE.

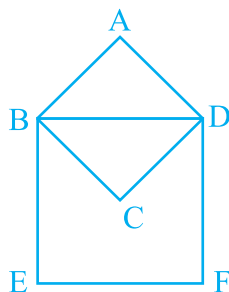


Fig. 9.38

85. In the given triangles of Fig. 9.39, perimeter of $\triangle ABC =$ perimeter of $\triangle PQR$. Find the area of $\triangle ABC$.

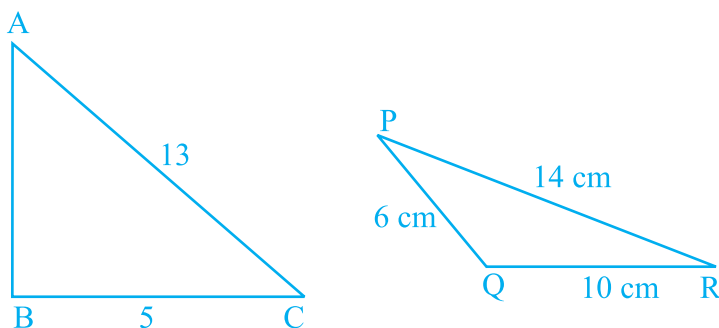


Fig. 9.39

86. Altitudes MN and MO of parallelogram $MGHK$ are 8 cm and 4 cm long respectively (Fig. 9.40). One side GH is 6 cm long. Find the perimeter of $MGHK$.

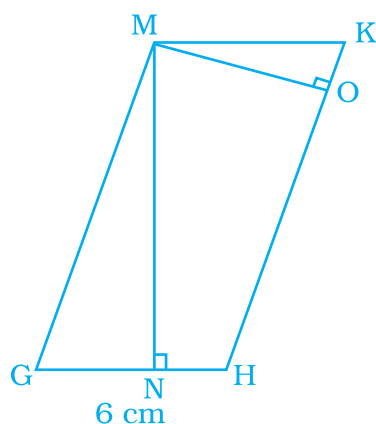


Fig. 9.40

87. In Fig. 9.41, area of ΔPQR is 20 cm^2 and area of ΔPQS is 44 cm^2 . Find the length RS , if PQ is perpendicular to QS and QR is 5 cm .

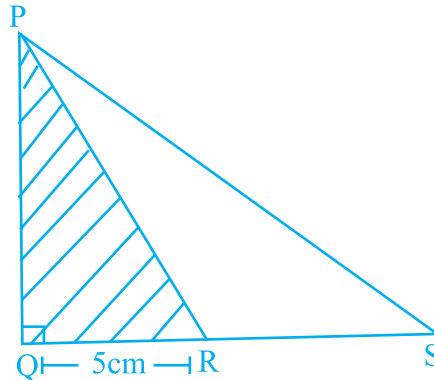


Fig. 9.41

88. Area of an isosceles triangle is 48 cm^2 . If the altitudes corresponding to the base of the triangle is 8 cm , find the perimeter of the triangle.
89. Perimeter of a parallelogram shaped land is 96 m and its area is 270 square metres. If one of the sides of this parallelogram is 18 m , find the length of the other side. Also, find the lengths of altitudes l and m (Fig. 9.42).

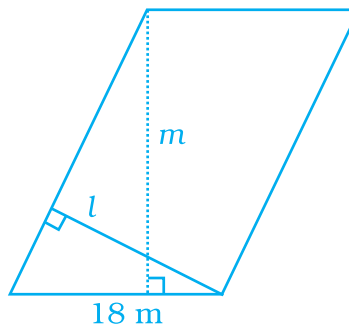
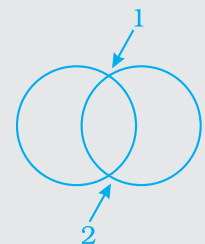


Fig. 9.42

Circles

What is the maximum number of times that six circles of the same size can intersect? To find the answer, start by drawing two circles that are of the same size. What is the greatest number of times they can intersect? Add another circle, and another, and so on.



- 90.** Area of a triangle PQR right-angled at Q is 60 cm^2 (Fig. 9.43). If the smallest side is 8 cm long, find the length of the other two sides.

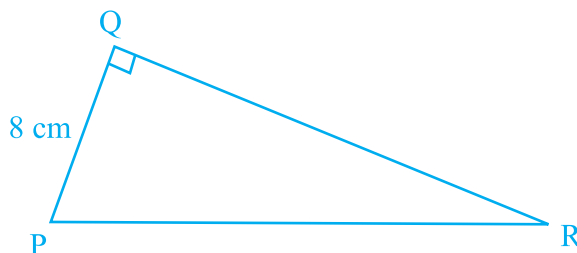


Fig. 9.43

- 91.** In Fig. 9.44 a rectangle with perimeter 264 cm is divided into five congruent rectangles. Find the perimeter of one of the rectangles.

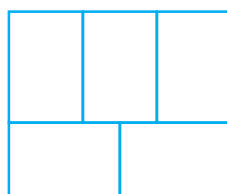


Fig. 9.44

- 92.** Find the area of a square inscribed in a circle whose radius is 7 cm (Fig. 9.45).

[Hint: Four right-angled triangles joined at right angles to form a square]

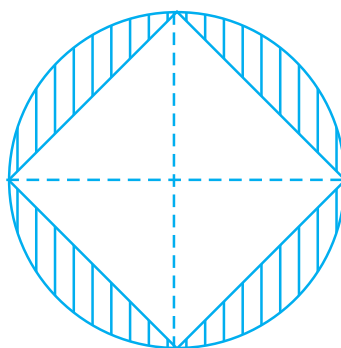


Fig. 9.45

- 93.** Find the area of the shaded portion in question 92.

In Questions 94 to 97 find the area enclosed by each of the following figures :

94.

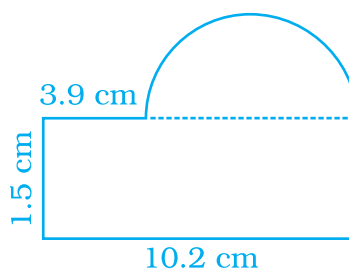


Fig. 9.46

95.

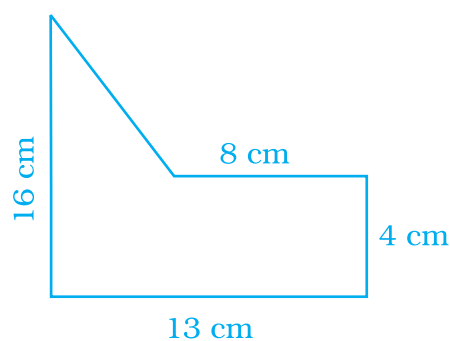


Fig. 9.47

96.

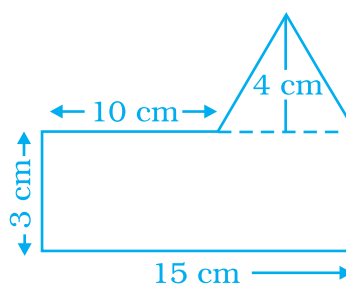


Fig. 9.48

97.

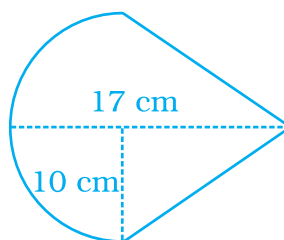


Fig. 9.49

In Questions 98 and 99 find the areas of the shaded region:

98.

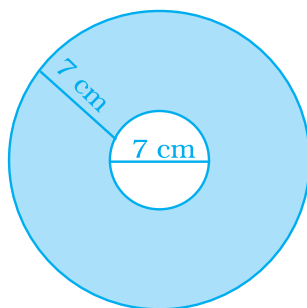


Fig. 9.50

99.

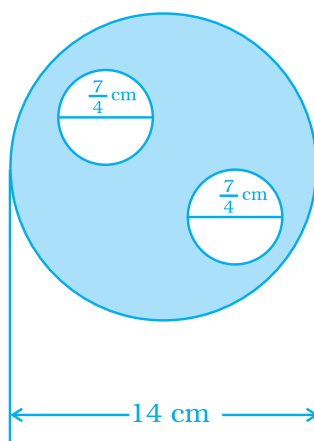


Fig. 9.51

100. A circle with radius 16 cm is cut into four equal parts and rearranged to form another shape as shown in Fig. 9.52:

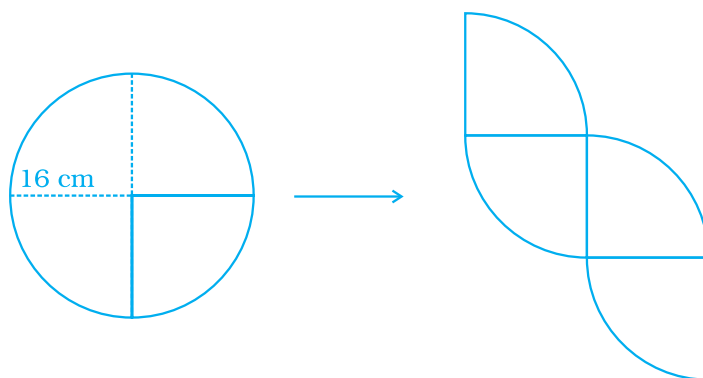


Fig. 9.52

Does the perimeter change? If it does change, by how much does it increase or decrease?

- 101.** A large square is made by arranging a small square surrounded by four congruent rectangles as shown in Fig. 9.53. If the perimeter of each of the rectangle is 16 cm, find the area of the large square.

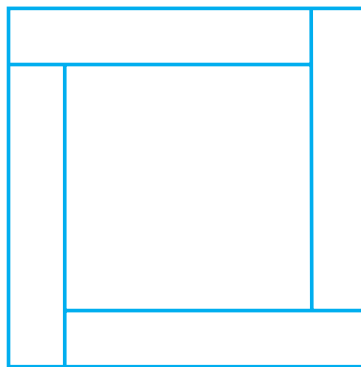
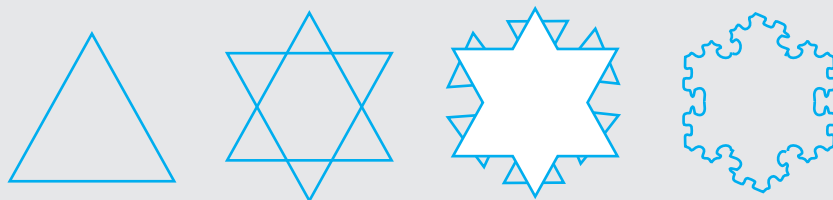


Fig. 9.53

The figures show how a fractal called the Koch snowflake is formed. It is constructed by first drawing an equilateral triangle. Then triangles with sides one-third the length of the original sides are added to the middle of each side. The second step is then repeated over and over again.



The area and perimeter of each figure is larger than that of the one before it. However, the area of any figure is never greater than the area of the shaded box, while the perimeters increase without bound.

- 102.** ABCD is a parallelogram in which AE is perpendicular to CD (Fig. 9.54). Also $AC = 5$ cm, $DE = 4$ cm, and the area of $\triangle AED = 6$ cm². Find the perimeter and area of ABCD.

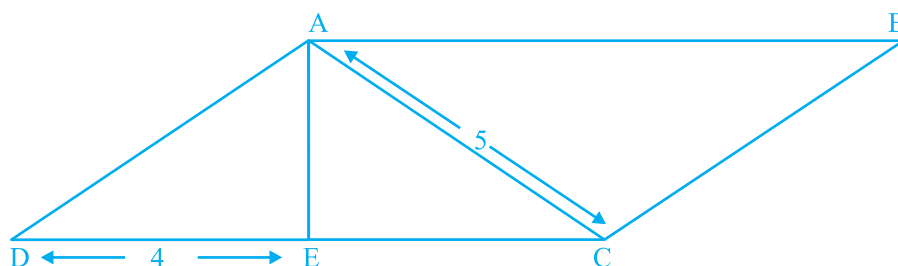


Fig. 9.54

- 103.** Ishika has designed a small oval race track for her remote control car. Her design is shown in the figure 9.55. What is the total distance around the track? Round your answer to the nearest whole cm.

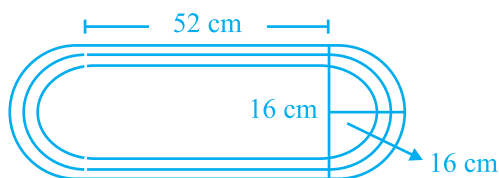
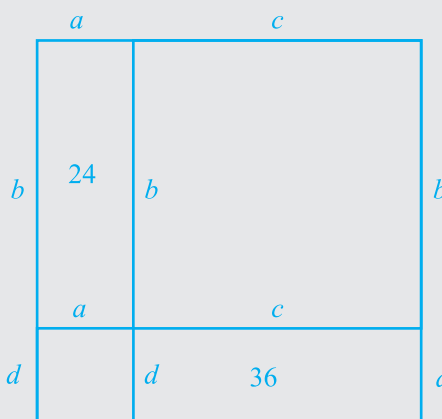
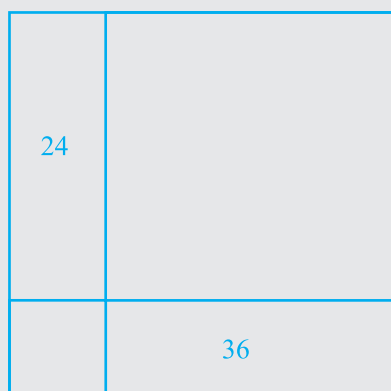


Fig. 9.55

Shape up

Rectangles

The square below has been divided into four rectangles. The areas of two of the rectangles are given. If the length of each of the segments in the diagram is an integer, what is the area of the original square?



(**Hint:** Remember $a + c = b + d$)

Use different lengths and a different answer to create your own version of this puzzle.

- 104.** A table cover of dimensions 3 m 25 cm \times 2 m 30 cm is spread on a table. If 30 cm of the table cover is hanging all around the table, find the area of the table cover which is hanging outside the top of the table. Also find the cost of polishing the table top at ₹ 16 per square metre.
- 105.** The dimensions of a plot are 200 m \times 150 m. A builder builds 3 roads which are 3 m wide along the length on either side and one in the middle. On either side of the middle road he builds houses to sell. How much area did he get for building the houses?
- 106.** A room is 4.5 m long and 4 m wide. The floor of the room is to be covered with tiles of size 15 cm by 10 cm. Find the cost of covering the floor with tiles at the rate of ₹ 4.50 per tile.
- 107.** Find the total cost of wooden fencing around a circular garden of diameter 28 m, if 1m of fencing costs ₹ 300.
- 108.** Priyanka took a wire and bent it to form a circle of radius 14 cm. Then she bent it into a rectangle with one side 24 cm long. What is the length of the wire? Which figure encloses more area, the circle or the rectangle?
- 109.** How much distance, in metres, a wheel of 25 cm radius will cover if it rotates 350 times?



Revise

- Does your solution answer the question?

When you think you have solved a problem, think again. Your answer may not really be the solution to the problem. For example, you may solve an equation to find the value or a variable, but to find the answer the problem is asking for, the value of the variable may need to be substituted into an expression.

- 110.** A circular pond is surrounded by a 2 m wide circular path. If outer circumference of circular path is 44 m, find the inner circumference of the circular path. Also find area of the path.
- 111.** A carpet of size 5 m \times 2 m has 25 cm wide red border. The inner part of the carpet is blue in colour (Fig. 9.56). Find the area of blue portion. What is the ratio of areas of red portion to blue portion?

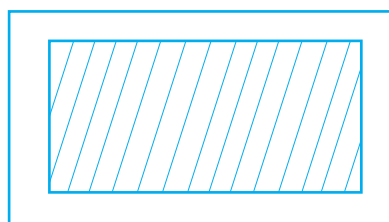


Fig. 9.56

- 112.** Use the Fig. 9.57 showing the layout of a farm house:

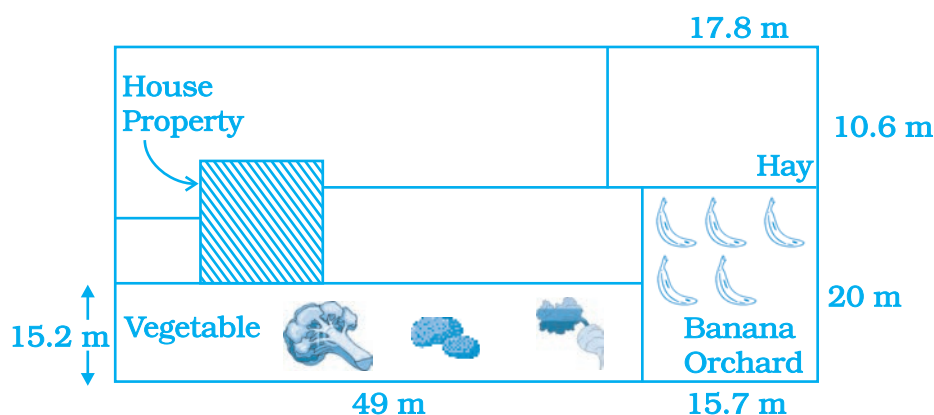


Fig. 9.57

- What is the area of land used to grow hay?
- It costs ₹ 91 per m^2 to fertilise the vegetable garden. What is the total cost?
- A fence is to be enclosed around the house. The dimensions of the house are 18.7 m \times 12.6 m. At least how many metres of fencing are needed?
- Each banana tree required 1.25 m^2 of ground space. How many banana trees can there be in the orchard?

113. Study the layout given below in Fig. 9.58 and answer the questions:

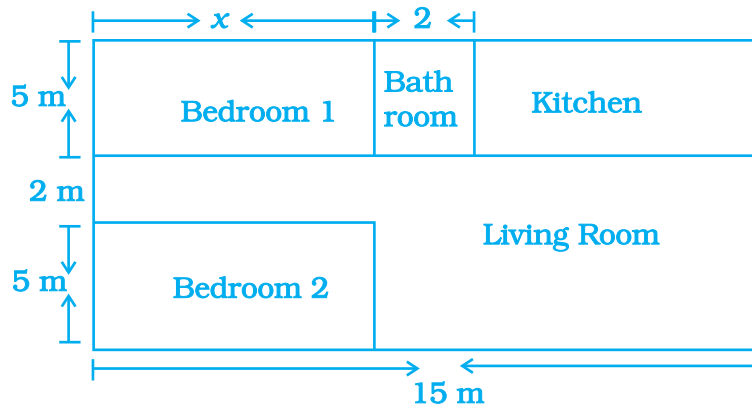


Fig. 9.58

- Write an expression for the total area covered by both the bedrooms and the kitchen.
 - Write an expression to calculate the perimeter of the living room.
 - If the cost of carpeting is ₹ 50/m², write an expression for calculating the total cost of carpeting both the bedrooms and the living room.
 - If the cost of tiling is ₹ 30/m², write an expression for calculating the total cost of floor tiles used for the bathroom and kitchen floors.
 - If the floor area of each bedroom is 35 m², then find x .
- 114.** A 10 m long and 4 m wide rectangular lawn is in front of a house. Along its three sides a 50 cm wide flower bed is there as shown in Fig. 9.59. Find the area of the remaining portion.

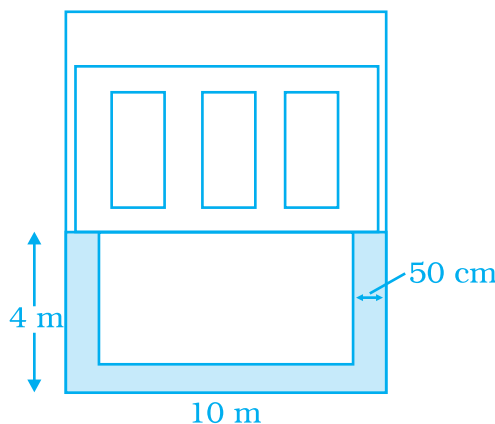


Fig. 9.59

- 115.** A school playground is divided by a 2 m wide path which is parallel to the width of the playground, and a 3 m wide path which is parallel to the length of the ground (Fig. 9.60). If the length and width of the playground are 120 m and 80 m respectively, find the area of the remaining playground.

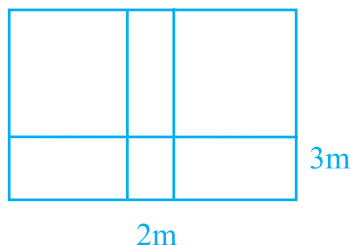


Fig. 9.60

- 116.** In a park of dimensions 20 m \times 15 m, there is a L shaped 1m wide flower bed as shown in Fig. 9.61. Find the total cost of manuring for the flower bed at the rate of Rs 45 per m².

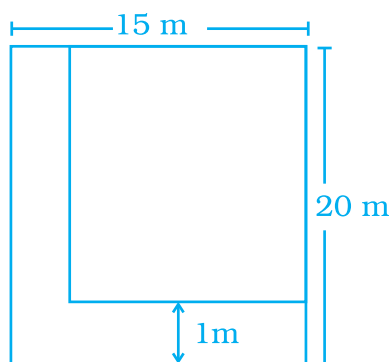


Fig. 9.61

- 117.** Dimensions of a painting are 60 cm \times 38 cm. Find the area of the wooden frame of width 6 cm around the painting as shown in Fig. 9.62.

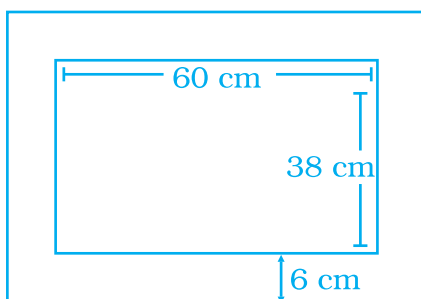


Fig. 9.62

- 118.** A design is made up of four congruent right triangles as shown in Fig. 9.63. Find the area of the shaded portion.

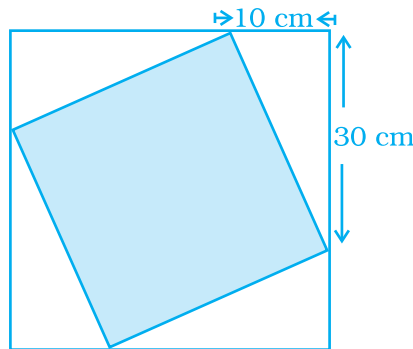


Fig. 9.63

- 119.** A square tile of length 20 cm has four quarter circles at each corner as shown in Fig. 9.64(i). Find the area of shaded portion. Another tile with same dimensions has a circle in the centre of the tile [Fig. 9.64 (ii)]. If the circle touches all the four sides of the square tile, find the area of the shaded portion. In which tile, area of shaded portion will be more? (Take $\pi = 3.14$)

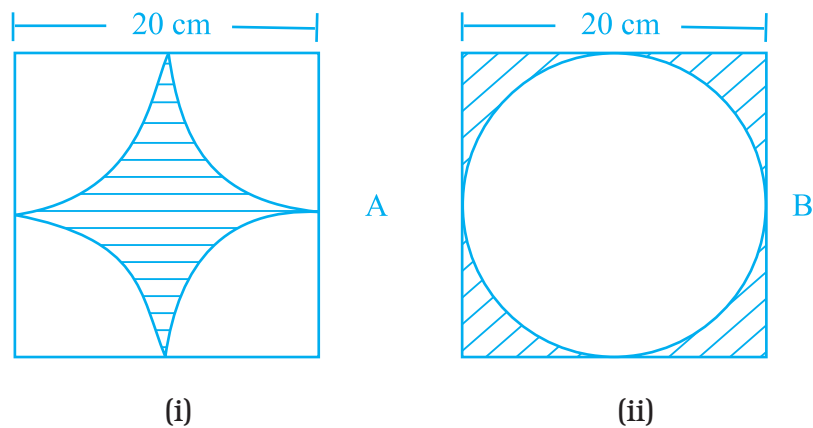


Fig. 9.64

- 120.** A rectangular field is 48 m long and 12 m wide. How many right triangular flower beds can be laid in this field, if sides including the right angle measure 2 m and 4 m, respectively?

- 121.** Ramesh grew wheat in a rectangular field that measured 32 metres long and 26 metres wide. This year he increased the area for wheat by increasing the length but not the width. He increased the area of the wheat field by 650 square metres. What is the length of the expanded wheat field?
- 122.** In Fig. 9.65, triangle AEC is right-angled at E, B is a point on EC, BD is the altitude of triangle ABC, $AC = 25$ cm, $BC = 7$ cm and $AE = 15$ cm. Find the area of triangle ABC and the length of DB.

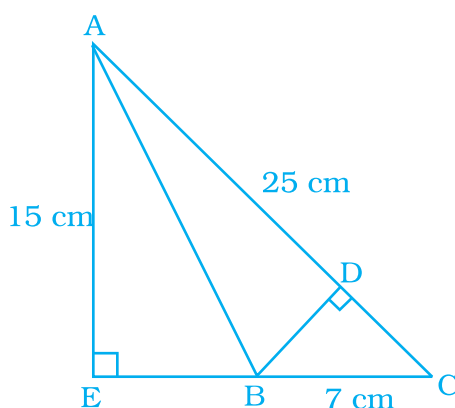


Fig. 9.65

123.



Can you help me to figure out how many pieces of $1.5 \text{ cm} \times 2 \text{ cm}$ chocolate can I cut from a $18 \text{ cm} \times 18 \text{ cm}$ sheet of chocolate?



I Raju facing a problem. Can you help me?



What is the problem Raju?



Hmm...., Let us take the help of our friends to find the number of pieces!!

- 124.** Calculate the area of shaded region in Fig. 9.66, where all of the short line segments are at right angles to each other and 1 cm long.

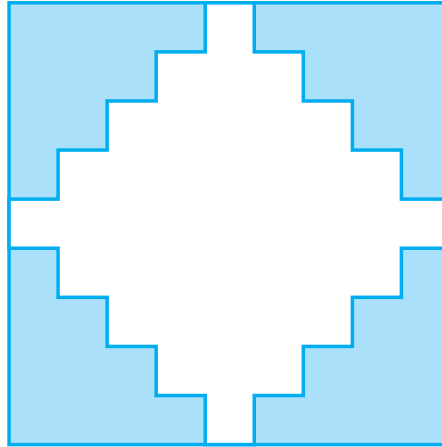


Fig. 9.66

- 125.** The plan and measurement for a house are given in Fig. 9.67. The house is surrounded by a path 1m wide.

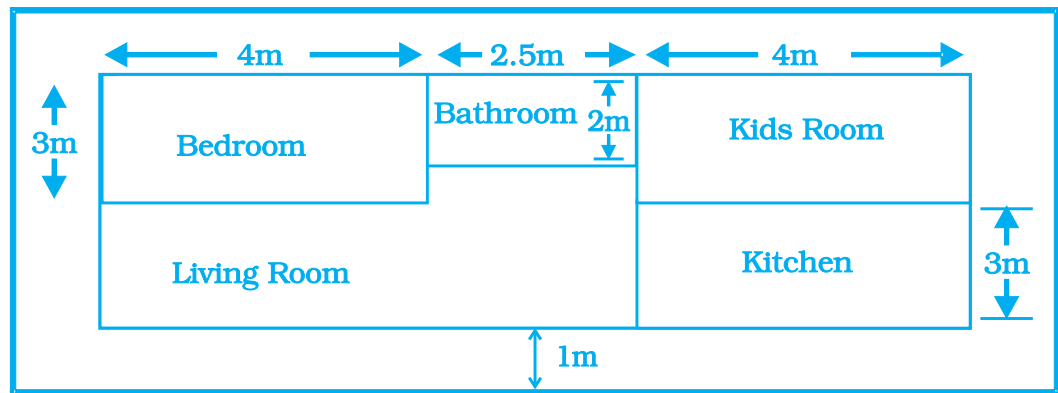


Fig. 9.67

Find the following:

- Cost of paving the path with bricks at rate of ₹ 120 per m^2 .
- Cost of wooden flooring inside the house except the bathroom at the cost of ₹ 1200 per m^2 .
- Area of Living Room.

- 126.** Architects design many types of buildings. They draw plans for houses, such as the plan shown in Fig. 9.68:

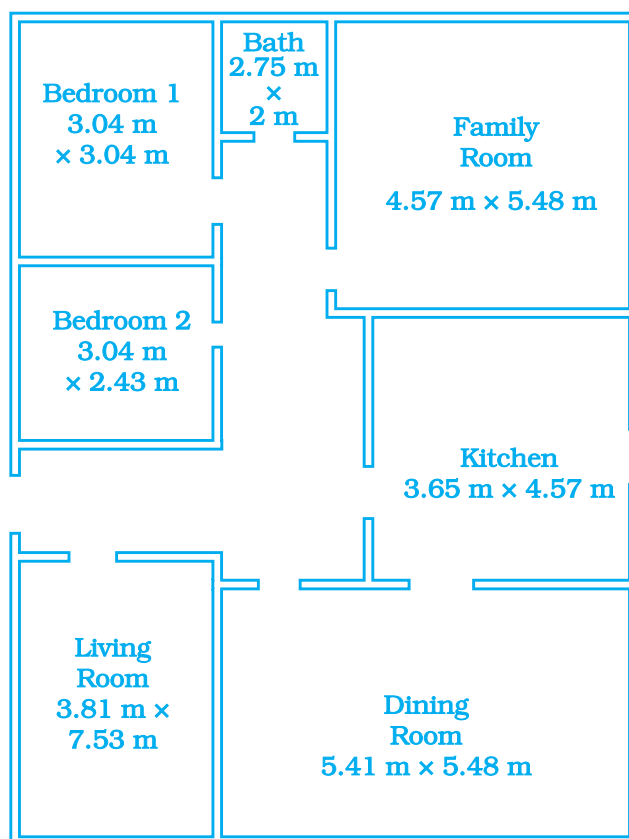


Fig. 9.68

An architect wants to install a decorative moulding around the ceilings in all the rooms. The decorative moulding costs ₹ 500/metre.

- (a) Find how much moulding will be needed for each room.
- (i) family room (ii) living room (iii) dining room
 - (iv) bedroom 1 (v) bedroom 2

(b) The carpet costs ₹ 200/m². Find the cost of carpeting each room.

(c) What is the total cost of moulding for all the five rooms.

- 127.** ABCD is a given rectangle with length as 80 cm and breadth as 60 cm. P, Q, R, S are the mid points of sides AB, BC, CD, DA respectively. A circular *rangoli* of radius 10 cm is drawn at the centre as shown in Fig. 9.69. Find the area of shaded portion.

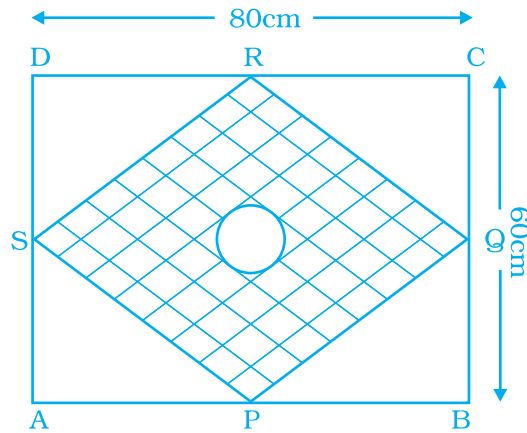


Fig. 9.69

- 128.** 4 squares each of side 10 cm have been cut from each corner of a rectangular sheet of paper of size 100 cm \times 80 cm. From the remaining piece of paper, an isosceles right triangle is removed whose equal sides are each of 10 cm length. Find the area of the remaining part of the paper.
- 129.** A dinner plate is in the form of a circle. A circular region encloses a beautiful design as shown in Fig. 9.70. The inner circumference is 352 mm and outer is 396 mm. Find the width of circular design.

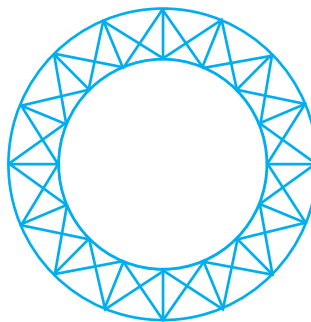


Fig. 9.70

- 130.** The moon is about 384000 km from earth and its path around the earth is nearly circular. Find the length of path described by moon in one complete revolution. (Take $\pi = 3.14$)

- 131.** A photograph of Billiard/Snooker table has dimensions as $\frac{1}{10}$ th of its actual size as shown in Fig. 9.71:

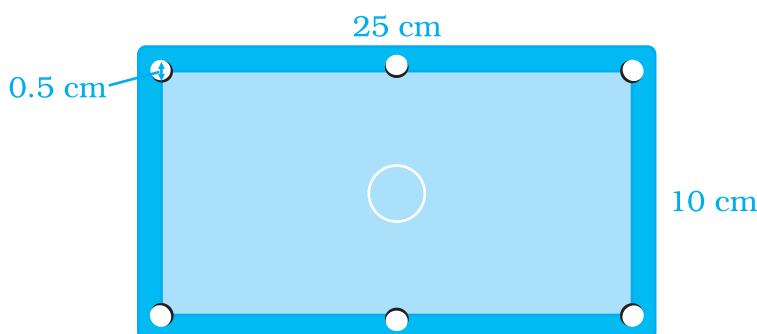


Fig. 9.71

The portion excluding six holes each of diameter 0.5 cm needs to be polished at rate of ₹ 200 per m^2 . Find the cost of polishing.

(D) Applications

For (1) –(4): For the dimensions of the field / court refer the diagram given at the end of the unit.

1. Find the dimensions of a Basket Ball court.
 - (i) Calculate the perimeter of the court.
 - (ii) Calculate the total area of the court.
 - (iii) Find the total area of the bigger central circle of the court.
 - (iv) Find the area of the smaller central circle.
 - (v) Find the difference of areas found in part (iii) and (iv).
2. Find the dimensions of a Badminton court.
 - (i) Calculate the perimeter of the court.
 - (ii) Calculate the total area of the court.
 - (iii) Find the total area of any one side boundaries of the court.
 - (iv) Find the area of a left service court.

3. In a foot ball field, calculate the
 - (i) total area of the 2 goal posts.
 - (ii) total area covered by the field.
 - (iii) the perimeter of the field.
4. In a hockey field, calculate the
 - (i) area included inside the shooting circles.
 - (ii) the perimeter of Hockey ground.
5. Complete the following data by using the formula for circumference of a circle.
 Circumference of a circle = $2\pi r$
 r = radius of the circle

	Radius	Diameter	Circumference
Foot ball			71 cm
Basket ball		24.8 cm	
Cricket ball			23 cm
Volley ball	10.3 cm		
Hockey ball			22.4 cm
Lawn Tennis ball		6.35 cm	
Shot put	65 mm		

(Circumference of a ball is used in the sense of circumference of the circle with the same radius).

6. Observe the two rectangles given in Fig. 9.72:
 Rectangle A has greater area but its perimeter is less than rectangle B.

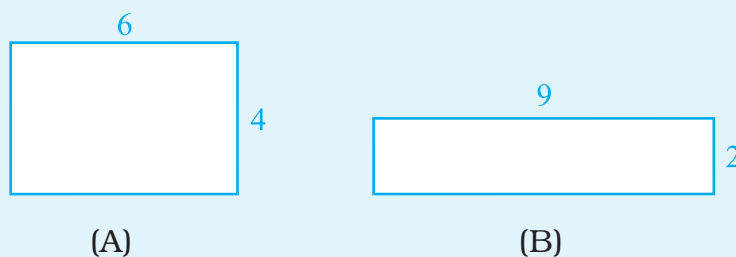


Fig. 9.72

Now draw the following pair of rectangles:

- (i) having same area but different perimeter.
- (ii) having same perimeter but different areas.
- (iii) One has larger area but smaller perimeter than other.
- (iv) Area of one rectangle is three times the area of other rectangle but both have the same perimeters.

7. Puzzle

In this puzzle, called a “Squared square,” squares of different sizes are contained within one big rectangle. The goal is to find out the sizes of the squares with the questions marks. By comparing known length of lines make some deductions to find out the sizes that are missing. Each number stands for the length of the side in that square.

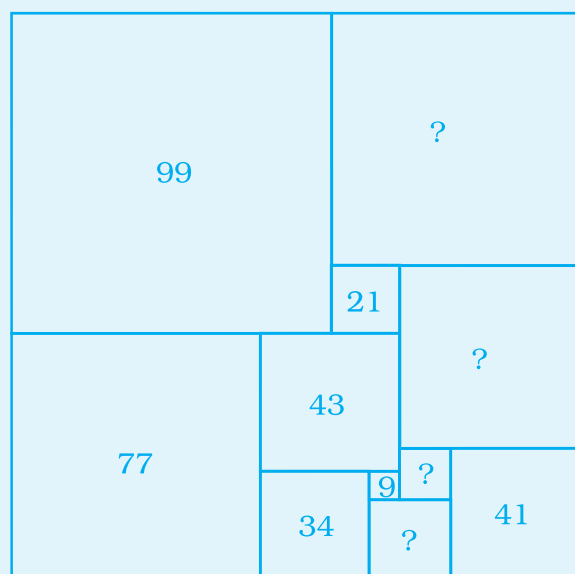
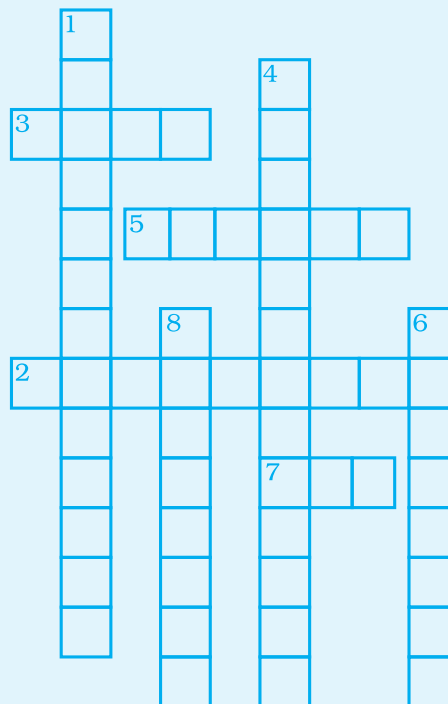


Fig. 9.73

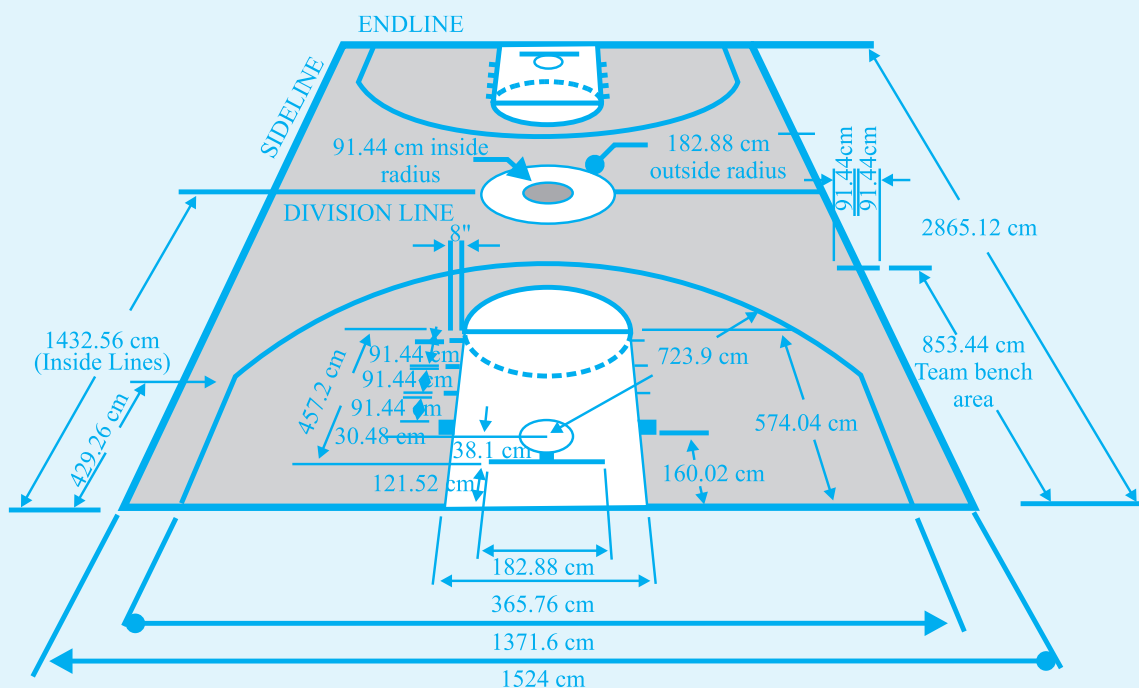
8. Cross-word Puzzle

Solve the given crossword and then fill up the given boxes. Clues are given below for across as well as downward filling. Also for across and down clues, clue number is written at the corner of boxes. Answers of clues have to fill in their respective boxes.

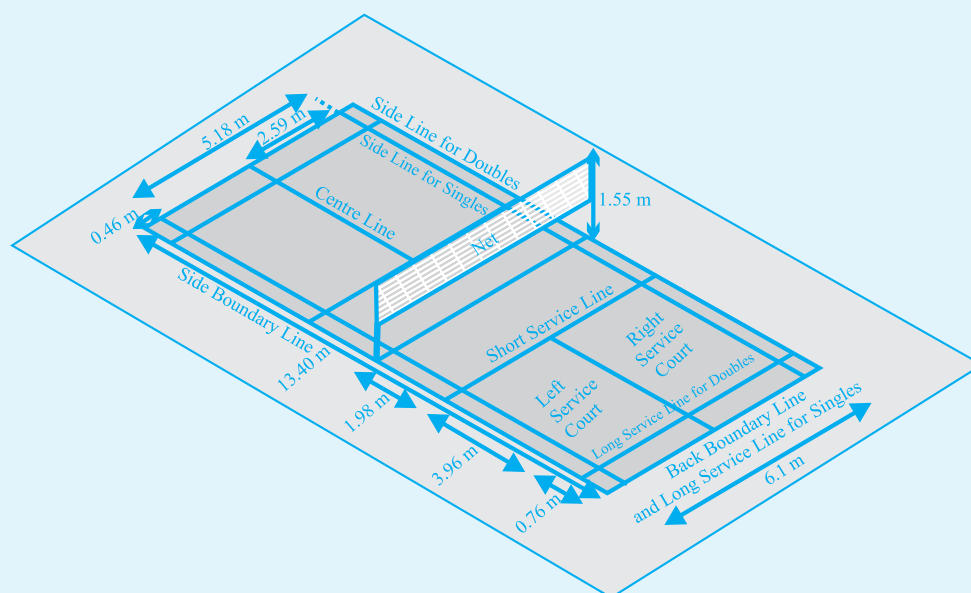
1. $2\pi r = \underline{\hspace{2cm}}$ of a circle of radius r.s.
2. $2(l + b) = \underline{\hspace{2cm}}$ of a rectangle.
3. $\pi r^2 = \underline{\hspace{2cm}}$ of a circle of radius r.
4. base \times height = Area of a $\underline{\hspace{2cm}}$.
5. side \times side = Area of a $\underline{\hspace{2cm}}$.
6. Area of $\underline{\hspace{2cm}}$ = $\frac{1}{2} \times$ base \times altitude.
7. $10000\text{m}^2 = \underline{\hspace{2cm}}$ hectare.
8. $\underline{\hspace{2cm}} = 2 \times$ radius.



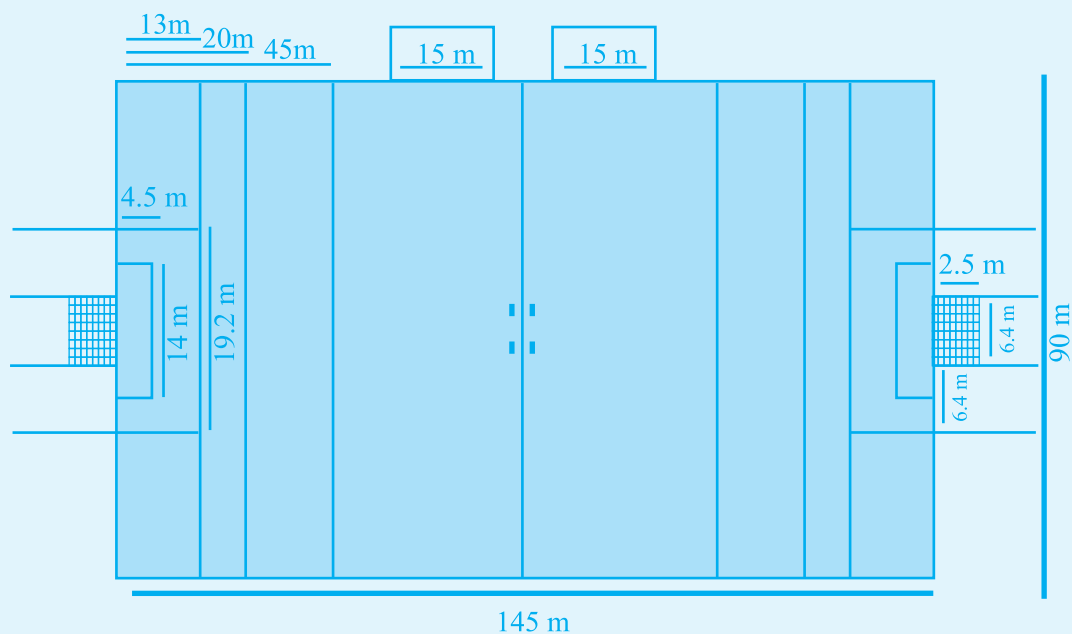
For Activity Q.1. Basket Ball Court



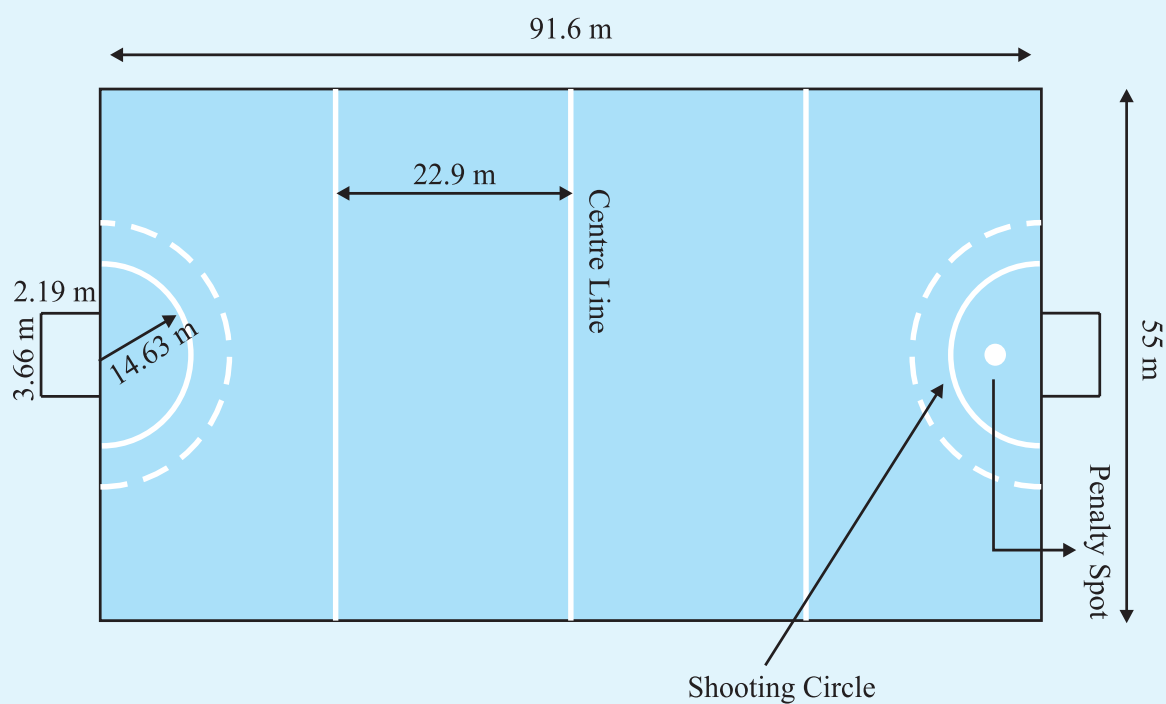
For Activity Q.2. Badminton Court



For Activity Q.3. Foot ball Field



For Activity Q.4. Hockey Ground



UNIT 10

ALGEBRAIC EXPRESSIONS

(A) Main Concepts and Results

- Algebraic expression is formed from variables and constants using different operations.
- Expressions are made up of terms.
- A term is the product of factors. Factors may be numerical as well as algebraic (literal).
- Coefficient is the numerical factor in a term. Sometimes, any factor in a term is called the coefficient of the remaining part of the term.
- The terms having the same algebraic factors are called like terms.
- The terms having different algebraic factors are called unlike terms.
- Expression with one term is called a 'Monomial'.
- Expression with two unlike terms is called a 'Binomial'.
- Expression with three unlike terms is called a 'Trinomial'.
- In general, an expression with one or more than one term (with non-negative integral exponents of the variables) is called a 'Polynomial'.
- The sum (or difference) of two like terms is a like term with coefficient equal to the sum (or difference) of coefficients of the two like terms.

- When we add (or subtract) two algebraic expressions, the like terms are added (or subtracted) and the unlike terms are written as they are.
- To find the value of an expression, we substitute the values of the variables in the expression and then simplify.
- Rules and formulas in mathematics are written in a concise and general form using algebraic expressions.

(B) Solved Examples

In Examples 1 to 3, there are four options, out of which one is correct. Write the correct answer.

Example 1: The like terms in $3x(3 - 2y)$ and $2(xy + x^2)$ are

- | | |
|---------------------|-----------------------|
| (a) $9x$ and $2x^2$ | (b) $-6xy$ and $2xy$ |
| (c) $9x$ and $2xy$ | (d) $-6xy$ and $2x^2$ |

Solution : The correct answer is (b).

Expressions are used to write word problems in math terms.

Expressions are like **instructions** that tell you what you have to do to a number or variable.

In words	Expression
A number x is increased by 7	$x + 7$
A number y is decreased by 7	$y - 7$
A number a is multiplied by 7	$a \times 7$
A number k is divided by 7	$k \div 7$

Sometimes you might have to describe a **real-life situation** using a mathematical expression.

You need to imagine what would happen to a quantity, and write that down using variables, and $+$, $-$, \times and \div .

When you change a **variable expression** to a **word expression** you, can say the **same thing** in several **different ways**.

+ : Instead of “2 **added to** x ”, you could say “ x **increased by** 2,” or “2 **more than** x ,” or “**the sum of** x and 2.”

– : “2 **subtracted from** x ” means the same as “2 **less than** x ,” or “ x decreased by 2.”

× : “ x **multiplied by** 2” means the same as “**the product of** x and 2,” “ x **times** 2,” or “**twice** x .”

÷ : you could say: either “ x **divided by** 3” or “**one third of** x .”

Example 2: The coefficient of xy in $3x^2zy + 7xyz - 2z^2x$ is
(a) $3z$ (b) -2 (c) $7yz$ (d) $7z$

Solution: Correct answer is (d).

Example 3: The factors of the term $-xy^2$ are
(a) $x \times y \times y$ (b) $-1 \times y \times y$
(c) $-1 \times x \times y$ (d) $-1 \times x \times y \times y$

Solution: Correct answer is (d).

In Examples 4 to 7, fill in the blanks to make the statements true.

Example 4: An algebraic expression having one or more terms with non-negative integral exponents of the variables is called _____.

Solution: Polynomials

Example 5: Numerical factor in any term of a polynomial is called _____ of the term.

Solution: Numerical coefficient or coefficient.

Example 6: The terms with different algebraic factors are called _____.

Solution: Unlike terms

Example 7: The terms with same algebraic factors are called _____.

Solution: Like terms

In Examples 8 to 10, state whether the statements are True or False.

Example 8: An expression with two terms is called a binomial.

Solution: True

Example 9: Every polynomial is a monomial.

Solution: False

Example 10: The value of a variable is fixed.

Solution: False

Example 11: Twice the sum of length x and breadth y of a rectangle is the perimeter of a rectangle. Write the expression for perimeter.

Solution: Perimeter of rectangle = $2 (\text{Length} + \text{Breadth})$
 $= 2 (x + y) = 2x + 2y$

Example 12: Identify the term containing u^2 in the expression $u^3 + 3u^2v + 3uv^2 + v^3$ and write its coefficient.

Solution: Term containing $u^2 = 3u^2v$
 Coefficient of $u^2 = 3v$

A Variable Represents an Unknown Number

In algebra you'll often have to work with numbers whose values you don't know. When you solve math problems, you can use a **letter** or a **symbol** to stand in for the number. The letter or symbol is called a **variable**.

Coefficient \curvearrowright $2k + 4$ \curvearrowleft Constant
 Variable \curvearrowright

The number that the variable is being multiplied by is called the **coefficient** – like the 2 above.

Any number not joined to a variable is called a **constant** – like the 4 above. It's called that because its value doesn't **change**, even if the value of the variable changes.

A term is a group of numbers and variables. One or more terms added together make an expression. For example, in the expression above, $2k$ is one term and 4 is another term. In the expression $3 + 4x - 5wyz$, the terms are 3, $4x$ and $-5wyz$.

An Expression is a Mathematical Phrase

Expressions are **mathematical phrases** that may contain **numbers**, **operations** and **variables**. The operations act like a set of instructions that tell you what to do with the numbers and variables. For example, $2k + 4$ tells you to double k , then add four to it.

There are two types of expressions – **numeric** and **variable**.

- **Numeric expressions** have numbers in them, and often operations – but they don't include any variables:

$$\rightarrow 5 + 13$$

$$\rightarrow 2 \times 5 - 6$$

$$\rightarrow 8 + 7 \div 6$$

- **Variable expressions** have variables in them, and may also include numbers and operations :

$$\rightarrow 5h$$

$$\rightarrow 5x$$

$$\rightarrow 5k + 4$$

Example 13: Simplify the expression by combining the like terms:

$$7x^3 - 3x^2y + xy^2 + x^2y - y^3$$

Solution: Rearranging the terms in the given expression, we get

$$7x^3 - 3x^2y + x^2y + xy^2 - y^3$$

$$= 7x^3 + (-3x^2y) + x^2y + xy^2 - y^3$$

$$= 7x^3 + (-3 + 1)x^2y + xy^2 - y^3 \text{ [Using distributive property]}$$

$$= 7x^3 + (-2)x^2y + xy^2 - y^3$$

$$= 7x^3 - 2x^2y + xy^2 - y^3$$

Example 14: Subtract the sum of $-3x^3y^2 + 2x^2y^3$ and $-3x^2y^3 - 5y^4$ from $x^4 + x^3y^2 + x^2y^3 + y^4$.

Solution:

$$\begin{array}{r}
 - 3x^3y^2 + 2x^2y^3 \\
 + \quad - 3x^2y^3 - 5y^4 \\
 \hline
 - 3x^3y^2 - x^2y^3 - 5y^4 \\
 \hline
 \end{array}$$

$$\text{Sum} = - 3x^3y^2 - x^2y^3 - 5y^4$$

$$\text{Now, } x^4 + x^3y^2 + x^2y^3 + y^4$$

$$- 3x^3y^2 - x^2y^3 - 5y^4$$

$$\begin{array}{ccc}
 (+) & (+) & (+)
 \end{array}$$

$$\text{difference} = \frac{x^4 + 4x^3y^2 + 2x^2y^3 + 6y^4}{}$$

Build Understanding

The parts of a variable expression that are separated by addition or subtraction signs are called **terms**. The variable expression $x + 3y + 2x - 4y^2$ contains four terms : x , $3y$, $2x$ and $- 4y^2$. The terms x and $2x$ are like terms because they have the same variable raised to the same power. The terms $3y$ and $4y^2$ are unlike terms because they have different variable parts.

Variables are all well and good, but they're only useful when you use them to solve math problems. You can use variables and numbers to describe a problem in math terms — it's called an expression.

Example 15: Find the value of the following expressions at $a = 1$ and $b = -2$:

(i) $a^2 + b^2 + 3ab$

(ii) $a^3 + a^2b + ab^2 + b^3$

Solution:

(i) Value of $a^2 + b^2 + 3ab$ at $a = 1$ and $b = -2$

$$= (1)^2 + (-2)^2 + 3 (1)(-2)$$

$$= 1 + 4 - 6$$

$$= 5 - 6$$

$$= - 1$$

(ii) Value of $a^3 + a^2b + ab^2 + b^3$ at $a = 1$ and $b = -2$

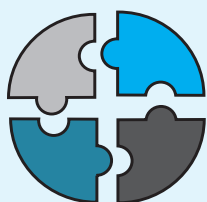
$$= (1)^3 + (1)^2(-2) + (1)(-2)^2 + (-2)^3$$

$$= 1 - 2 + 4 - 8$$

$$= 5 - 10$$

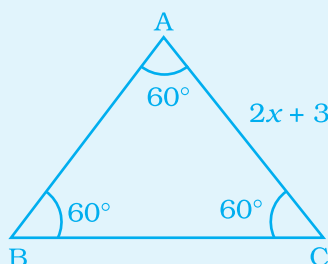
$$= -5$$

Application on Problem Solving Strategy



Example 16

Find each side of an equilateral triangle given below, if its perimeter is 240 cm.



Solution:



Understand and Explore the Problem

- What information is given in the question?
 $\triangle ABC$ is an equilateral triangle. Hence $AB = BC = CA$.
- What are we trying to find?
 The value of one of the sides of the equilateral triangle.
- Is there any information that is not needed?
 The measure of each angle of $\triangle ABC$.



Plan a Strategy

- In an equilateral triangle, all sides are equal. Therefore, three times each side is same as perimeter.



Solve

- $3 \times \text{length of one side} = \text{perimeter}$

Therefore, $3 \times (2x + 3) = 240$

Thus, $2x + 3 = \frac{240}{3} = 80$

$\Rightarrow 2x + 3 = 80$

$\Rightarrow 2x = 80 - 3$

$\Rightarrow 2x = 77$

$\Rightarrow x = \frac{77}{2}$

Therefore, $x = 38.5$

Hence, Side = $2x + 3 = (2 \times 38.5) + 3 = 80$ cm.



Revise

- The above answer is verified by multiplying side with 3 and comparing the result with given perimeter.

$3 \times 80 = 240 = \text{Perimeter given in question.}$

Think and Discuss

Draw this triangle on your copy and measure the angles of the triangle. What do you observe?

(C) Exercise

In each of the questions 1 to 16, out of the four options, only one is correct. Write the correct answer.

1. An algebraic expression containing three terms is called a
(a) monomial (b) binomial (c) trinomial (d) All of these
2. Number of terms in the expression $3x^2y - 2y^2z - z^2x + 5$ is
(a) 2 (b) 3 (c) 4 (d) 5

Check Understanding

Identify the following terms as like or unlike.

(a) $3x, 3y$

(b) $7k, -3k$

(c) $2p, 2pt$

- 3.** The terms of expression $4x^2 - 3xy$ are:
 (a) $4x^2$ and $-3xy$ (b) $4x^2$ and $3xy$
 (c) $4x^2$ and $-xy$ (d) x^2 and xy
- 4.** Factors of $-5x^2 y^2 z$ are
 (a) $-5 \times x \times y \times z$ (b) $-5 \times x^2 \times y \times z$
 (c) $-5 \times x \times x \times y \times y \times z$ (d) $-5 \times x \times y \times z^2$
- 5.** Coefficient of x in $-9xy^2z$ is
 (a) $9yz$ (b) $-9yz$ (c) $9y^2z$ (d) $-9y^2z$
- 6.** Which of the following is a pair of like terms?
 (a) $-7xy^2z, -7x^2yz$ (b) $-10xyz^2, 3xyz^2$
 (c) $3xyz, 3x^2y^2z^2$ (d) $4xyz^2, 4x^2yz$
- 7.** Identify the binomial out of the following:
 (a) $3xy^2 + 5y - x^2y$ (b) $x^2y - 5y - x^2y$
 (c) $xy + yz + zx$ (d) $3xy^2 + 5y - xy^2$
- 8.** The sum of $x^4 - xy + 2y^2$ and $-x^4 + xy + 2y^2$ is
 (a) Monomial and polynomial in y (b) Binomial and Polynomial
 (c) Trinomial and polynomial (d) Monomial and polynomial in x
- 9.** The subtraction of 5 times of y from x is
 (a) $5x - y$ (b) $y - 5x$ (c) $x - 5y$ (d) $5y - x$
- 10.** $-b - 0$ is equal to
 (a) $-1 \times b$ (b) $1 - b - 0$ (c) $0 - (-1) \times b$ (d) $-b - 0 - 1$

11. The side length of the top of square table is x . The expression for perimeter is:
 (a) $4 + x$ (b) $2x$ (c) $4x$ (d) $8x$
12. The number of scarfs of length half metre that can be made from y metres of cloth is :
 (a) $2y$ (b) $\frac{y}{2}$ (c) $y + 2$ (d) $y + \frac{1}{2}$
13. $123x^2y - 138x^2y$ is a like term of :
 (a) $10xy$ (b) $-15xy$ (c) $-15xy^2$ (d) $10x^2y$
14. The value of $3x^2 - 5x + 3$ when $x = 1$ is
 (a) 1 (b) 0 (c) -1 (d) 11
15. The expression for the number of diagonals that we can make from one vertex of a n sided polygon is:
 (a) $2n + 1$ (b) $n - 2$ (c) $5n + 2$ (d) $n - 3$

Evaluating Variable Expressions

A variable is a letter that is used to represent one or more numbers. The numbers are the values of the variable. A variable expression is a collection of numbers, variables and operations. Here are some examples.

VARIABLE EXPRESSION	MEANING	OPERATION
$8y = 8 \times y = 8(y)$	8 times y	Multiplication
$\frac{16}{b} = 16 \div b$	16 divided by b	Division
$4 + s$	4 plus s	Addition
$9 - x$	9 minus x	Subtraction

The expression $8y$ is usually not written as $8 \times y$ because of possible confusion of symbol ' \times ' with the variable x . Replacing each variable in an expression by a number is called **evaluating the expression**. The resulting number is the value of the expression.

Write the variable expression



Substitute values for variables



Simplify the numerical expression

16. The length of a side of square is given as $2x + 3$. Which expression represents the perimeter of the square?

- (a) $2x + 16$ (b) $6x + 9$ (c) $8x + 3$ (d) $8x + 12$

In questions 17 to 32, fill in the blanks to make the statements true.

17. Sum or difference of two like terms is _____.

18. In the formula, area of circle = πr^2 , the numerical constant of the expression πr^2 is _____.

19. $3a^2b$ and $-7ba^2$ are _____ terms.

20. $-5a^2b$ and $-5b^2a$ are _____ terms.

21. In the expression $2\pi r$, the algebraic variable is _____.

22. Number of terms in a monomial is _____.

23. Like terms in the expression $n(n + 1) + 6(n - 1)$ are _____ and _____.

24. The expression $13 + 90$ is a _____.

25. The speed of car is 55 km/hrs. The distance covered in y hours is _____.

26. $x + y + z$ is an expression which is neither monomial nor _____.

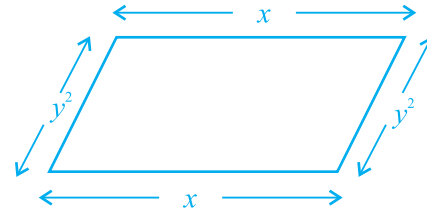
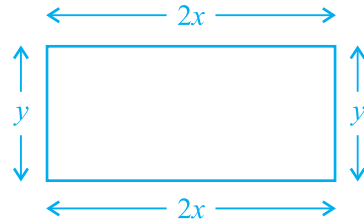
Expressions can be Described in Words

To show you understand an expression, you need to be able to explain **what it means** in words. You can write a **word expression** to represent the numeric or variable expression.

27. If $(x^2y + y^2 + 3)$ is subtracted from $(3x^2y + 2y^2 + 5)$, then coefficient of y in the result is _____.

28. $-a - b - c$ is same as $-a - (\text{_____})$.

29. The unlike terms in perimeters of following figures are _____ and _____.



30. On adding a monomial _____ to $-2x + 4y^2 + z$, the resulting expression becomes a binomial.
31. $3x + 23x^2 + 6y^2 + 2x + y^2 + \underline{\hspace{2cm}} = 5x + 7y^2$.
32. If Rohit has $5xy$ toffees and Shantanu has $20yx$ toffees, then Shantanu has _____ more toffees.

In questions 33 to 52, state whether the statements given are True or False.

33. $1 + \frac{x}{2} + x^3$ is a polynomial.
34. $(3a - b + 3) - (a + b)$ is a binomial.
35. A trinomial can be a polynomial.
36. A polynomial with more than two terms is a trinomial.
37. Sum of x and y is $x + y$.
38. Sum of 2 and p is $2p$.
39. A binomial has more than two terms.
40. A trinomial has exactly three terms.
41. In like terms, variables and their powers are the same.
42. The expression $x + y + 5x$ is a trinomial.
43. $4p$ is the numerical coefficient of q^2 in $-4pq^2$.
44. $5a$ and $5b$ are unlike terms.

45. Sum of $x^2 + x$ and $y + y^2$ is $2x^2 + 2y^2$.
46. Subtracting a term from a given expression is the same as adding its additive inverse to the given expression.
47. The total number of planets of Sun can be denoted by the variable n .
48. In like terms, the numerical coefficients should also be the same.
49. If we add a monomial and binomial, then answer can never be a monomial.
50. If we subtract a monomial from a binomial, then answer is atleast a binomial.
51. When we subtract a monomial from a trinomial, then answer can be a polynomial.
52. When we add a monomial and a trinomial, then answer can be a monomial.

Using Tables to Identify and Extend Patterns

Make a table that shows the number of triangles in each figure. Then tell how many triangles are in the fifth figure of the pattern. Use drawings to justify your answer.



Figure 1

Figure 2

Figure 3

The table shows the number of triangles in each figure.

Figure	1	2	3	4	5
Number of Triangles	2	4	6	8	10

+2 +2 +2 +2

The pattern is to add 2 triangles each time.

Figure 4 has $6 + 2 = 8$ triangles. Figure 5 has $8 + 2 = 10$ triangles.

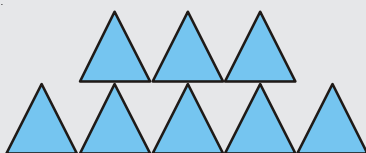


Figure 4

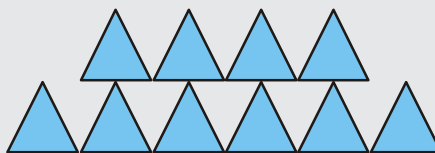
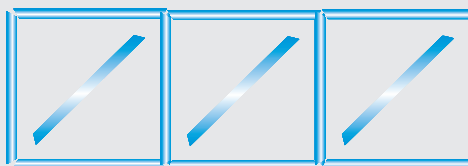
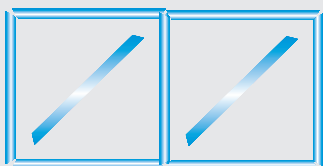
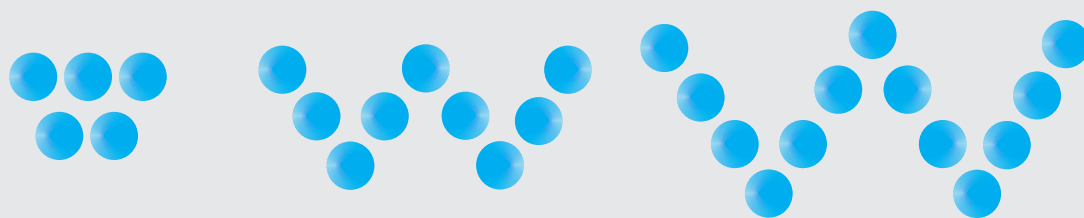
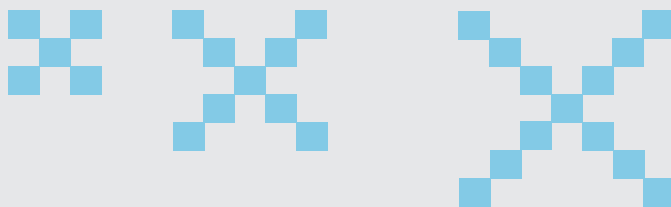


Figure 5

- 53.** Write the following statements in the form of algebraic expressions and write whether it is monomial, binomial or trinomial.
- x is multiplied by itself and then added to the product of x and y .
 - Three times of p and two times of q are multiplied and then subtracted from r .
 - Product of p , twice of q and thrice of r .
 - Sum of the products of a and b , b and c and c and a .
 - Perimeter of an equilateral triangle of side x .
 - Perimeter of a rectangle with length p and breadth q .
 - Area of a triangle with base m and height n .
 - Area of a square with side x .
 - Cube of s subtracted from cube of t .
 - Quotient of x and 15 multiplied by x .
 - The sum of square of x and cube of z .
 - Two times q subtracted from cube of q .
- 54.** Write the coefficient of x^2 in the following:
- $x^2 - x + 4$
 - $x^3 - 2x^2 + 3x + 1$
 - $1 + 2x + 3x^2 + 4x^3$
 - $y + y^2x + y^3x^2 + y^4x^3$
- 55.** Find the numerical coefficient of each of the terms :
- x^3y^2z , xy^2z^3 , $-3xy^2z^3$, $5x^3y^2z$, $-7x^2y^2z^2$
 - $10xyz$, $-7xy^2z$, $-9xyz$, $2xy^2z$, $2x^2y^2z^2$
- 56.** Simplify the following by combining the like terms and then write whether the expression is a monomial, a binomial or a trinomial.
- $3x^2yz^2 - 3xy^2z + x^2yz^2 + 7xy^2z$
 - $x^4 + 3x^3y + 3x^2y^2 - 3x^3y - 3xy^3 + y^4 - 3x^2y^2$
 - $p^3q^2r + pq^2r^3 + 3p^2qr^2 - 9p^2qr^2$
 - $2a + 2b + 2c - 2a - 2b - 2c - 2b + 2c + 2a$
 - $50x^3 - 21x + 107 + 41x^3 - x + 1 - 93 + 71x - 31x^3$

Number Strips and Expressions

Four sequences of patterns start as shown below:



The four patterns are different.

1. What do the four patterns have in common?

You may continue the sequence of each pattern as far as you want.

2. How many squares, dots, stars or bars will the 10th figure of each sequence have?

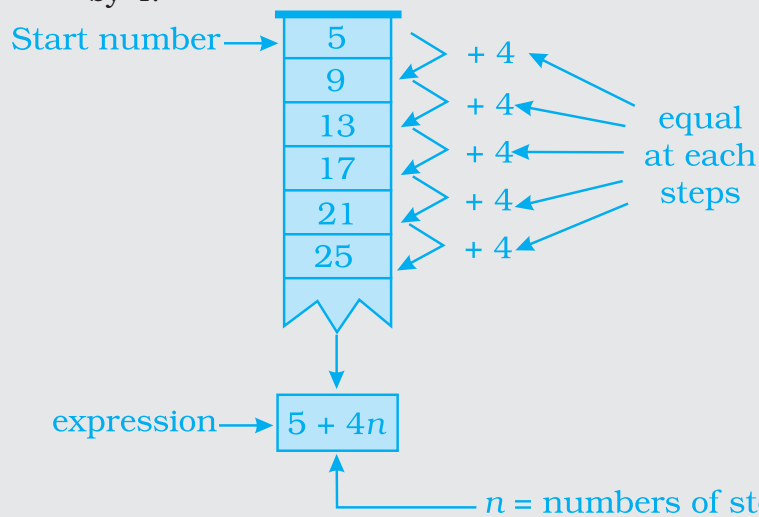
57. Add the following expressions:

- (a) $p^2 - 7pq - q^2$ and $-3p^2 - 2pq + 7q^2$
- (b) $x^3 - x^2y - xy^2 - y^3$ and $x^3 - 2x^2y + 3xy^2 + 4y^3$
- (c) $ab + bc + ca$ and $-bc - ca - ab$
- (d) $p^2 - q + r$, $q^2 - r + p$ and $r^2 - p + q$
- (e) $x^3y^2 + x^2y^3 + 3y^4$ and $x^4 + 3x^2y^3 + 4y^4$
- (f) $p^2qr + pq^2r + pqr^2$ and $-3pq^2r - 2pqr^2$
- (g) $uv - vw$, $vw - wu$ and $wu - uv$
- (h) $a^2 + 3ab - bc$, $b^2 + 3bc - ca$ and $c^2 + 3ca - ab$
- (i) $\frac{5}{8}p^4 + 2p^2 + \frac{5}{8}$; $\frac{1}{8} - 17p + \frac{9}{8}p^2$ and $p^5 - p^3 + 7$
- (j) $t - t^2 - t^3 - 14$; $15t^3 + 13 + 9t - 8t^2$; $12t^2 - 19 - 24t$
and $4t - 9t^2 + 19t^3$

Arithmetic Sequence

The common properties of the four sequences of patterns on previous page are:

- the first figure has 5 elements (squares, dots, stars or bars);
- with each step in the row of figures, the number of elements grows by 4.



So, the four sequences of patterns correspond to the same number sequence.

Remark: To reach the 50th number in the strip, you need 49 steps.

So take $n = 49$ and you find the 50th number:

$$5 + 4 \times 49 = 201.$$

58. Subtract

- (a) $-7p^2qr$ from $-3p^2qr$.
- (b) $-a^2 - ab$ from $b^2 + ab$.
- (c) $-4x^2y - y^3$ from $x^3 + 3xy^2 - x^2y$.
- (d) $x^4 + 3x^3y^3 + 5y^4$ from $2x^4 - x^3y^3 + 7y^4$.
- (e) $ab - bc - ca$ from $-ab + bc + ca$.
- (f) $-2a^2 - 2b^2$ from $-a^2 - b^2 + 2ab$.
- (g) $x^3y^2 + 3x^2y^2 - 7xy^3$ from $x^4 + y^4 + 3x^2y^2 - xy^3$.
- (h) $2(ab + bc + ca)$ from $-ab - bc - ca$.
- (i) $4.5x^5 - 3.4x^2 + 5.7$ from $5x^4 - 3.2x^2 - 7.3x$.
- (j) $11 - 15y^2$ from $y^3 - 15y^2 - y - 11$.

59. (a) What should be added to $x^3 + 3x^2y + 3xy^2 + y^3$ to get $x^3 + y^3$?

- (b) What should be added to $3pq + 5p^2q^2 + p^3$ to get $p^3 + 2p^2q^2 + 4pq$?

60. (a) What should be subtracted from $2x^3 - 3x^2y + 2xy^2 + 3y^3$ to get $x^3 - 2x^2y + 3xy^2 + 4y^3$?

- (b) What should be subtracted from $-7mn + 2m^2 + 3n^2$ to get $m^2 + 2mn + n^2$?

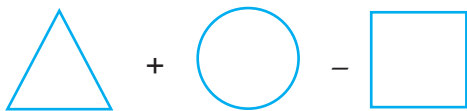
61. How much is $21a^3 - 17a^2$ less than $89a^3 - 64a^2 + 6a + 16$?**62.** How much is $y^4 - 12y^2 + y + 14$ greater than $17y^3 + 34y^2 - 51y + 68$?**63.** How much does $93p^2 - 55p + 4$ exceed $13p^3 - 5p^2 + 17p - 90$?**64.** To what expression must $99x^3 - 33x^2 - 13x - 41$ be added to make the sum zero?**Think and Discuss**

1. **Describe** two different number patterns that begin with 3, 6, ...
2. **Tell** when it would be useful to make a table to help you identify and extend a pattern.

- 65.** Subtract $9a^2 - 15a + 3$ from unity.
- 66.** Find the values of the following polynomials at $a = -2$ and $b = 3$:
- (a) $a^2 + 2ab + b^2$ (b) $a^2 - 2ab + b^2$
- (c) $a^3 + 3a^2b + 3ab^2 + b^3$ (d) $a^3 - 3a^2b + 3ab^2 - b^3$
- (e) $\frac{a^2 + b^2}{3}$ (f) $\frac{a^2 - b^2}{3}$
- (g) $\frac{a}{b} + \frac{b}{a}$ (h) $a^2 + b^2 - ab - b^2 - a^2$
- 67.** Find the values of following polynomials at $m = 1$, $n = -1$ and $p = 2$:
- (a) $m + n + p$ (b) $m^2 + n^2 + p^2$
- (c) $m^3 + n^3 + p^3$ (d) $mn + np + pm$
- (e) $m^3 + n^3 + p^3 - 3mnp$ (f) $m^2n^2 + n^2p^2 + p^2m^2$
- 68.** If $A = 3x^2 - 4x + 1$, $B = 5x^2 + 3x - 8$ and $C = 4x^2 - 7x + 3$, then find:
- (i) $(A + B) - C$
- (ii) $B + C - A$
- (iii) $A + B + C$
- 69.** If $P = -(x - 2)$, $Q = -2(y + 1)$ and $R = -x + 2y$, find a , when $P + Q + R = ax$.
- 70.** From the sum of $x^2 - y^2 - 1$, $y^2 - x^2 - 1$ and $1 - x^2 - y^2$ subtract $-(1 + y^2)$.
- 71.** Subtract the sum of $12ab - 10b^2 - 18a^2$ and $9ab + 12b^2 + 14a^2$ from the sum of $ab + 2b^2$ and $3b^2 - a^2$.
- 72.** Each symbol given below represents an algebraic expression:

$$\triangle = 2x^2 + 3y, \quad \bigcirc = 5x^2 + 3x, \quad \square = 8y^2 - 3x^2 + 2x + 3y$$

The symbols are then represented in the expression:



Find the expression which is represented by the above symbols.

- 73.** Observe the following nutritional chart carefully:

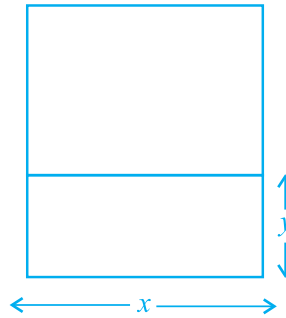
Food Item (Per Unit = 100g)	Carbohydrates
Rajma	60g
Cabbage	5g
Potato	22g
Carrot	11g
Tomato	4g
Apples	14g

Write an algebraic expression for the amount of carbohydrates in 'g' for

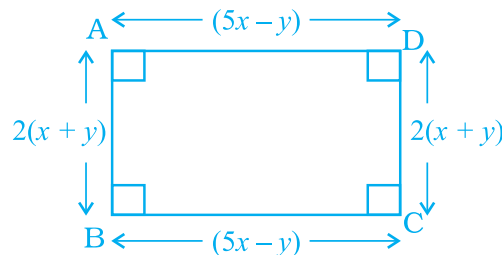
- (a) y units of potatoes and 2 units of rajma (b) $2x$ units tomatoes and y units apples.

- 74.** Arjun bought a rectangular plot with length x and breadth y and then sold a triangular part of it whose base is y and height is z . Find the area of the remaining part of the plot.
- 75.** Amisha has a square plot of side m and another triangular plot with base and height each equal to m . What is the total area of both plots?
- 76.** A taxi service charges ₹ 8 per km and levies a fixed charge of ₹ 50. Write an algebraic expression for the above situation, if the taxi is hired for x km.

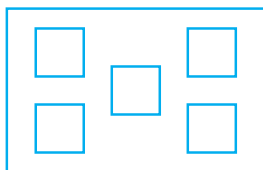
- 77.** Shiv works in a mall and gets paid ₹ 50 per hour. Last week he worked for 7 hours and this week he will work for x hours. Write an algebraic expression for the money paid to him for both the weeks.
- 78.** Sonu and Raj have to collect different kinds of leaves for science project. They go to a park where Sonu collects 12 leaves and Raj collects x leaves. After some time Sonu loses 3 leaves and Raj collects $2x$ leaves. Write an algebraic expression to find the total number of leaves collected by both of them.
- 79.** A school has a rectangular play ground with length x and breadth y and a square lawn with side x as shown in the figure given below. What is the total perimeter of both of them combined together?



- 80.** The rate of planting the grass is ₹ x per square metre. Find the cost of planting the grass on a triangular lawn whose base is y metres and height is z metres.
- 81.** Find the perimeter of the figure given below:




- 82.** In a rectangular plot, 5 square flower beds of side $(x + 2)$ metres each have been laid (see figure given below). Find the total cost of fencing the flower beds at the cost of ₹ 50 per 100 metres:

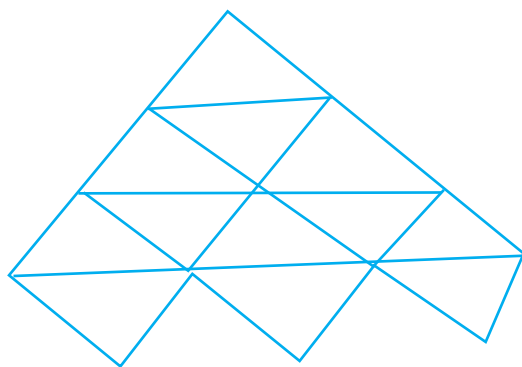


- 83.** A wire is $(7x - 3)$ metres long. A length of $(3x - 4)$ metres is cut for use. Now, answer the following questions:

- (a) How much wire is left?
- (b) If this left out wire is used for making an equilateral triangle. What is the length of each side of the triangle so formed?

- 84.** Rohan's mother gave him ₹ $3xy^2$ and his father gave him ₹ $5(xy^2 + 2)$. Out of this total money he spent ₹ $(10 - 3xy^2)$ on his birthday party. How much money is left with him?

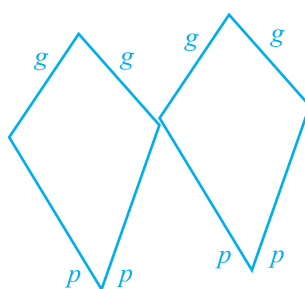
- 85.** (i) A triangle is made up of 2 red sticks and 1 blue sticks . The length of a red stick is given by r and that of a blue stick is given by b . Using this information, write an expression for the total length of sticks in the pattern given below:



- (ii) In the given figure, the length of a green side is given by g and that of the red side is given by p .



Write an expression for the following pattern. Also write an expression if 100 such shapes are joined together.



- 86.** The sum of first n natural numbers is given by $\frac{1}{2}n^2 + \frac{1}{2}n$. Find
- The sum of first 5 natural numbers.
 - The sum of first 11 natural numbers.
 - The sum of natural numbers from 11 to 30.

87. The sum of squares of first n natural numbers is given by $\frac{1}{6}n(n+1)(2n+1)$ or $\frac{1}{6}(2n^3 + 3n^2 + n)$. Find the sum of squares of the first 10 natural numbers.

88. The sum of the multiplication table of natural number ' n ' is given by $55 \times n$. Find the sum of

- (a) Table of 7
- (b) Table of 10
- (c) Table of 19

89. If $\triangle_x = 2x + 3$, $\square_x = \frac{3}{2}x + 7$ and $\bigcirc_x = x - 3$,

then find the value of :

(i) $2 \triangle_6 + \square_3 - \bigcirc_1$

(ii) $\frac{1}{2} \square_2 + \bigcirc_8 - 3 \triangle_0$

90. If $\triangle_x = \frac{3}{4}x - 2$ and $\diamond_x = x + 6$, then find the value of:

(i) $\triangle_{10} - \diamond_4$

(ii) $2 \diamond_{12} - \frac{3}{2} \triangle_1$

Translate each of the following algebraic expressions Question 91 to 94 into words.

91. $4b - 3$

92. $8(m + 5)$

93. $\frac{7}{8-x}$

94. $17\left(\frac{16}{w}\right)$

95. (i) **Critical Thinking** Write two different algebraic expressions for the word phrase “ $\left(\frac{1}{4}\right)$ of the sum of x and 7.”
- (ii) **What’s the Error?** A student wrote an algebraic expression for “5 less than a number n divided by 3” as $\frac{n}{3} - 5$. What error did the student make?
- (iii) **Write About it** Shashi used addition to solve a word problem about the weekly cost of commuting by toll tax for ₹ 15 each day. Ravi solved the same problem by multiplying. They both got the correct answer. How is this possible?
96. **Challenge** Write an expression for the sum of 1 and twice a number n . If you let n be any odd number, will the result always be an odd number?
97. **Critical Thinking** Will the value of $11x$ for $x = -5$ be greater than 11 or less than 11? Explain.
98. Match Column I with Column II in the following:

Column I	Column II
1. The difference of 3 and a number squared	(a) $4 - 2x$
2. 5 less than twice a number squared	(b) $n^2 - 3$
3. Five minus twice the square of a number	(c) $2n^2 - 5$
4. Four minus a number multiplied by 2	(d) $5 - 2n^2$
5. Seven times the sum of a number and 1	(e) $3 - n^2$
6. A number squared plus 6	(f) $2(n + 6)$

7. 2 times the sum of a number and 6 (g) $7(n + 1)$

8. Three less than the square of a number (h) $n^2 + 6$

99. At age of 2 years, a cat or a dog is considered 24 “human” years old. Each year, after age 2 is equivalent to 4 “human” years. Fill in the expression $[24 + \square (a - 2)]$ so that it represents the age of a cat or dog in human years. Also, you need to determine for what ‘ a ’ stands for. Copy the chart and use your expression to complete it.

Age	$[24 + \square (a - 2)]$	Age (Human Years)
2		
3		
4		
5		
6		

100. Express the following properties with variables x , y and z .

- (i) Commutative property of addition
- (ii) Commutative property of multiplication
- (iii) Associative property of addition
- (iv) Associative property of multiplication
- (v) Distributive property of multiplication over addition

(D) Application

1. Game

Think of a number, multiply the number by 8, divide by 2, add 5, and then subtract 4 times the original number.

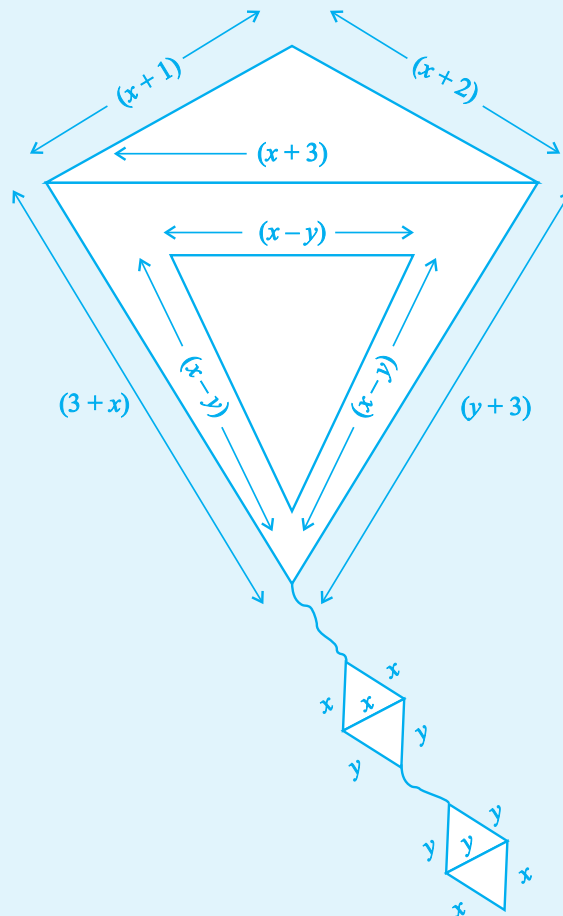
No matter what number you choose the answer will always be 5.

Here's how

	What you say	What the person think	Role of Mathematics
(i)	Pick any number	6 (for example)	n
(ii)	Multiply by 8	$8(6) = 48$	$8n$
(iii)	Divide by 2	$48 \div 2 = 24$	$8n \div 2 = 4n$
(iv)	Add 5	$24 + 5 = 29$	$4n + 5$
(v)	Subtract 4 times the original number	$29 - 4(6) = 4n + 5 - 4n = 5$ $29 - 24 = 5$	

Invent your own Math magic thinking that has atleast five steps. Try it with your friend!

2. Colour the scalene triangle with Yellow, Isosceles with Green and equilateral with Red in the given adjoining figure.



3. Cross Number Puzzle

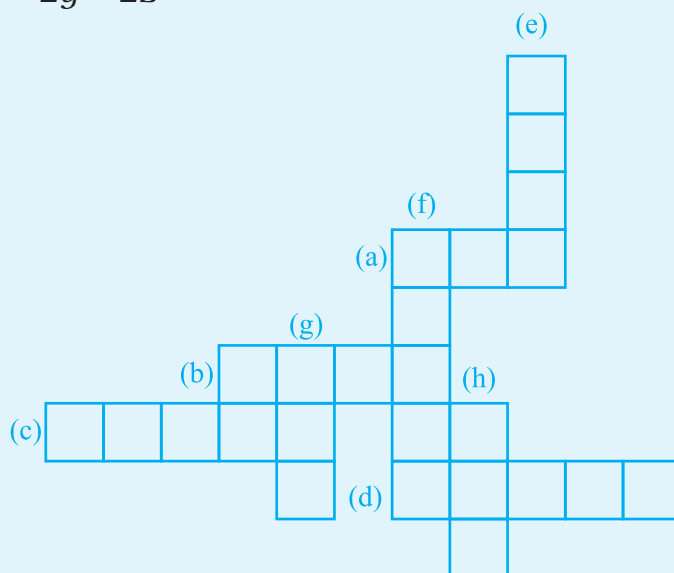
Rohit has to solve the given cross number puzzle to qualify for the next round of Mathematics quiz competition. Help him by evaluating the values of given expression at $x = 0$, $y = 1$, $z = 2$. Also help him to fill the cross number along Across and Downward with the help of given clues, (Numbers to be written in words)

Across

- (a) $xy + yz + zx$
- (b) $x^2y^2 + z^2 - 2xyz$
- (c) $8 - (x+y)$
- (d) $x^2y^3 + y^2z^3 + z^2x^3$

Down

- (e) $x^2 - 2xy(y-z)$
- (f) $\frac{x^3 + y^3 + z^3}{3}$
- (g) $x^3 + y^3 + z^3 - 2yz^2$
- (h) $2x + 2y + 2z$



UNIT 11

EXPONENTS AND POWERS

(A) Main Concepts and Results

- Exponents are used to express large numbers in shorter form to make them easy to read, understand, compare and operate upon.
- $a \times a \times a \times a = a^4$ (read as 'a' raised to the exponent 4 or the fourth power of a), where 'a' is the base and 4 is the exponent and a^4 is called the exponential form. $a \times a \times a \times a$ is called the expanded form.
- For any non-zero integers 'a' and 'b' and whole numbers m and n ,
 - (i) $a^m \times a^n = a^{m+n}$
 - (ii) $a^m \div a^n = a^{m-n}$, $m > n$
 - (iii) $(a^m)^n = a^{mn}$
 - (iv) $a^m \times b^m = (ab)^m$
 - (v) $a^m \div b^m = \left(\frac{a}{b}\right)^m$
 - (vi) $a^0 = 1$
 - (vii) $(-1)^{\text{even number}} = 1$
 - (viii) $(-1)^{\text{odd number}} = -1$
- Any number can be expressed as a decimal number between 1.0 and 10.0 (including 1.0) multiplied by a power of 10. Such form of a number is called its standard form or scientific notation.

(B) Solved Examples

In Examples 1 to 3, there are four options, out of which one is correct. Write the correct one.

Example 1: Out of the following, the number which is not equal to $\frac{-8}{27}$ is

(a) $-\left(\frac{2}{3}\right)^3$

(b) $\left(\frac{-2}{3}\right)^3$

(c) $-\left(\frac{-2}{3}\right)^3$

(d) $\left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right)$

Solution: Correct answer is (c).

Example 2: $(-7)^5 \times (-7)^3$ is equal to

(a) $(-7)^8$

(b) $-(7)^8$

(c) $(-7)^{15}$

(d) $(-7)^2$

Solution: Correct answer is (a).

Example 3: For any two non-zero integers x any y , $x^3 \div y^3$ is equal to

(a) $\frac{x^3}{y^3}$

(b) $\left(\frac{x}{y}\right)^3$

(c) $\frac{x^6}{y^6}$

(d) $\frac{x^9}{y^9}$

Solution: Correct answer is (b).

MULTIPLYING POWERS WITH THE SAME BASE

Words	Numbers	Algebra
To multiply powers with the same base, keep the base and add the exponents.	$3^5 \times 3^8 = 3^{5+8} = 3^{13}$	$b^m \times b^n = b^{m+n}$

In Examples 4 and 5, fill in the blanks to make the statements true.

Example 4: $(5^7 \div 5^6)^2 = \underline{\hspace{2cm}}$

Solution: 5^2

Example 5: $\frac{a^7b^3}{a^5b} = \underline{\hspace{2cm}}$

Solution: $(ab)^2$

In Examples 6 to 8, state whether the statements are True or False:

Example 6: In the number 7^5 , 5 is the base and 7 is the exponent.

Solution: False

A power is written in **base and exponent form** as follows:

The base is the number that is being repeated as a factor in the multiplication.

5^2

The exponent tells you how many times the base is repeated as a factor in the multiplication

For example $7 \cdot 7 = 7^2$, $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$.

Example 7: $\frac{a^4}{b^3} = \frac{a+a+a+a}{b+b+b}$

Solution: False

Example 8: $a^b > b^a$ is true, if $a = 3$ and $b = 4$; but false, if $a = 2$ and $b = 3$.

Solution: True

Example 9: By what number should we multiply 3^3 so that the product may be equal to 3^7 ?

Solution: Let 3^3 be multiplied by x so that the product may be equal to 3^7 .

According to question,

$$3^3 \times x = 3^7$$

$$\text{or } x = 3^7 \div 3^3$$

$$= (3)^{7-3}$$

$$(\text{Using } a^m \div a^n = (a)^{m-n})$$

$$= 3^4$$

$$= 81$$

Therefore, 3^3 should be multiplied by 81 so that the product is equal to 3^7 .

Example 10: Find x so that $\left(\frac{5}{3}\right)^5 \times \left(\frac{5}{3}\right)^{11} = \left(\frac{5}{3}\right)^{8x}$

Solution: Given $\left(\frac{5}{3}\right)^5 \times \left(\frac{5}{3}\right)^{11} = \left(\frac{5}{3}\right)^{8x}$

$$\text{So, } \frac{5^5}{3^5} \times \frac{5^{11}}{3^{11}} = \left(\frac{5}{3}\right)^{8x} \quad \left\{ \text{Using } \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \right\}$$

$$\text{or } \frac{5^5 \times 5^{11}}{3^5 \times 3^{11}} = \left(\frac{5}{3}\right)^{8x}$$

$$\text{or } \frac{(5)^{16}}{(3)^{16}} = \left(\frac{5}{3}\right)^{8x} \quad \{ \text{Using } a^m \times a^n = (a)^{m+n} \}$$

$$\text{or } \left(\frac{5}{3}\right)^{16} = \left(\frac{5}{3}\right)^{8x}$$

$$\text{or } 16 = 8x$$

$$\text{Thus, } 8x = 16$$

$$\text{Therefore, } x = 2$$

Example 11: Express 648 in exponential notation.

Solution: $648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
 $= 2^3 \times 3^4$

Example 12: Express 2,36,00,000 in standard form.

Solution: $236,00,000$
 $= \frac{236,00,000}{100,00,000} \times 100,00,000$
 $= 2.36 \times 10^7$

2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

Example 13: Which of the two is larger : 3^{12} or 6^6 ?

Solution: $3^{12} = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 531441$
 $6^6 = 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 46656$
 So, 3^{12} is greater.

Application on Problem Solving Strategy



Example 14

Find x such that $\frac{1}{5}^5 \times \frac{1}{5}^{19} = \frac{1}{5}^{8x}$

Solution:



Understand and Explore the Problem

- What are you trying to find?
The value of x for the given equation.



Plan a Strategy

- You know the laws of exponents. Apply those laws in the given equation to find the value of x .



Solve

- Given, $\frac{1}{5}^5 \times \frac{1}{5}^{19} = \frac{1}{5}^{8x}$

Using the law of exponents, $a^m \times a^n = a^{m+n}$, we get

$$\frac{1}{5}^{5+19} = \frac{1}{5}^{8x}$$

$$\frac{1}{5}^{24} = \frac{1}{5}^{8x}$$

On both the sides, powers have the same base. So, their exponents must be equal

$$\text{Therefore, } 24 = 8x$$

$$\text{or } x = \frac{24}{8} = 3$$

Hence, the value of x is 3.

**Revise**

- Substitute the value of x in the equation and check if it satisfies the equation.

$$\text{LHS} = \frac{1}{5}^5 \times \frac{1}{5}^{19} = \frac{1}{5}^{5+19} = \frac{1}{5}^{24}$$

$$\text{RHS} = \frac{1}{5}^{8x} = \frac{1}{5}^{8 \times 3} = \frac{1}{5}^{24}$$

$$\text{LHS} = \text{RHS}$$

Hence, the equation is satisfied with $x = 3$. So, our answer is correct.

Think and Discuss

- Try to find the value of x given in the question by changing $\frac{1}{5}$ to $\frac{3}{2}$. What difference do you find in the value of x ? What do you infer from your answer?
- Can you find the value of x if the equation is changed to $(5)^x \div (5)^2 = (5)^3$?

(C) Exercise

In questions 1 to 22, there are four options, out of which one is correct. Write the correct one.

- $[(-3)^2]^3$ is equal to
 (a) $(-3)^8$ (b) $(-3)^6$ (c) $(-3)^5$ (d) $(-3)^{23}$
- For a non-zero rational number x , $x^8 \div x^2$ is equal to
 (a) x^4 (b) x^6 (c) x^{10} (d) x^{16}
- x is a non-zero rational number. Product of the square of x with the cube of x is equal to the
 (a) second power of x (b) third power of x
 (c) fifth power of x (d) sixth power of x

4. For any two non-zero rational numbers x and y , $x^5 \div y^5$ is equal to
 (a) $(x \div y)^1$ (b) $(x \div y)^0$ (c) $(x \div y)^5$ (d) $(x \div y)^{10}$
5. $a^m \times a^n$ is equal to
 (a) $(a^2)^{mn}$ (b) a^{m-n} (c) a^{m+n} (d) a^{mn}
6. $(1^0 + 2^0 + 3^0)$ is equal to
 (a) 0 (b) 1 (c) 3 (d) 6
7. Value of $\frac{10^{22} + 10^{20}}{10^{20}}$ is
 (a) 10 (b) 10^{42} (c) 101 (d) 10^{22}
8. The standard form of the number 12345 is
 (a) 1234.5×10^1 (b) 123.45×10^2
 (c) 12.345×10^3 (d) 1.2345×10^4
9. If $2^{1998} - 2^{1997} - 2^{1996} + 2^{1995} = K \cdot 2^{1995}$, then the value of K is
 (a) 1 (b) 2 (c) 3 (d) 4
10. Which of the following is equal to 1?
 (a) $2^0 + 3^0 + 4^0$ (b) $2^0 \times 3^0 \times 4^0$
 (c) $(3^0 - 2^0) \times 4^0$ (d) $(3^0 - 2^0) \times (3^0 + 2^0)$
11. In standard form, the number 72105.4 is written as 7.21054×10^n where n is equal to
 (a) 2 (b) 3 (c) 4 (d) 5
12. Square of $\left(\frac{-2}{3}\right)$ is
 (a) $\frac{-2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{-4}{9}$ (d) $\frac{4}{9}$

DIVIDING POWERS WITH THE SAME BASE

Words	Numbers	Algebra
To divide powers with the same base, keep the base and subtract the exponents.	$\frac{6^9}{6^4} = 6^{9-4} = 6^5$	$\frac{b^m}{b^n} = b^{m-n}$

Key Concept

Product of Powers Property

Words	To multiply powers with the same base, add their exponents.	
Algebra	$a^m \cdot a^n = a^{m+n}$	Numbers $5^6 \cdot 5^3 = 5^{6+3} = 5^9$

13. Cube of $\left(\frac{-1}{4}\right)$ is

- (a) $\frac{-1}{12}$ (b) $\frac{1}{16}$ (c) $\frac{-1}{64}$ (d) $\frac{1}{64}$

14. Which of the following is not equal to $\left(\frac{-5}{4}\right)^4$?

- (a) $\frac{(-5)^4}{4^4}$ (b) $\frac{5^4}{(-4)^4}$
 (c) $-\frac{5^4}{4^4}$ (d) $\left(-\frac{5}{4}\right) \times \left(-\frac{5}{4}\right) \times \left(-\frac{5}{4}\right) \times \left(-\frac{5}{4}\right)$

15. Which of the following is not equal to 1 ?

- (a) $\frac{2^3 \times 3^2}{4 \times 18}$ (b) $\left[(-2)^3 \times (-2)^4\right] \div (-2)^7$
 (c) $\frac{3^0 \times 5^3}{5 \times 25}$ (d) $\frac{2^4}{(7^0 + 3^0)^3}$

16. $\left(\frac{2}{3}\right)^3 \times \left(\frac{5}{7}\right)^3$ is equal to

- (a) $\left(\frac{2}{3} \times \frac{5}{7}\right)^9$ (b) $\left(\frac{2}{3} \times \frac{5}{7}\right)^6$ (c) $\left(\frac{2}{3} \times \frac{5}{7}\right)^3$ (d) $\left(\frac{2}{3} \times \frac{5}{7}\right)^0$

17. In standard form, the number 829030000 is written as $K \times 10^8$ where K is equal to

- (a) 82903 (b) 829.03 (c) 82.903 (d) 8.2903

18. Which of the following has the largest value?

- (a) 0.0001 (b) $\frac{1}{10000}$ (c) $\frac{1}{10^6}$ (d) $\frac{1}{10^6} \div 0.1$

19. In standard form 72 crore is written as

- (a) 72×10^7 (b) 72×10^8 (c) 7.2×10^8 (d) 7.2×10^7

20. For non-zero numbers a and b , $\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n$, where $m > n$, is equal to

- (a) $\left(\frac{a}{b}\right)^{mn}$ (b) $\left(\frac{a}{b}\right)^{m+n}$ (c) $\left(\frac{a}{b}\right)^{m-n}$ (d) $\left(\left(\frac{a}{b}\right)^m\right)^n$

21. Which of the following is not true?

- (a) $3^2 > 2^3$ (b) $4^3 = 2^6$ (c) $3^3 = 9$ (d) $2^5 > 5^2$

22. Which power of 8 is equal to 2^6 ?

- (a) 3 (b) 2 (c) 1 (d) 4

In questions 23 to 39, fill in the blanks to make the statements true.

23. $(-2)^{31} \times (-2)^{13} = (-2)^{\quad}$

24. $(-3)^8 \div (-3)^5 = (-3)^{\quad}$

25. $\left(\frac{11}{15}\right)^4 \times (\quad)^5 = \left(\frac{11}{15}\right)^9$

26. $\left(\frac{-1}{4}\right)^3 \times \left(\frac{-1}{4}\right)^{\quad} = \left(\frac{-1}{4}\right)^{11}$

COPY AND COMPLETE THE TABLE

Expression	Expression Written Using Repeated Multiplication	Number of Factors	Simplified Expression
$2^2 \cdot 2^4$	$(2 \cdot 2) \times (2 \cdot 2 \cdot 2 \cdot 2)$	6	2^6
$3^5 \cdot 3^5$	$(3 \cdot 3 \cdot 3) \times (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$	_____	_____
$a^2 \cdot a^3$	_____	_____	_____

RAISING A POWER TO A POWER

Words	Numbers	Algebra
To raise a power to a power, keep the base and multiply the exponents.	$(9^4)^5 = 9^4 \cdot 5 = 9^{20}$	$(b^m)^n = b^{m \cdot n}$

27. $\left[\left(\frac{7}{11}\right)^3\right]^4 = \left(\frac{7}{11}\right)^{\text{---}}$

28. $\left(\frac{6}{13}\right)^{10} \div \left[\left(\frac{6}{13}\right)^5\right]^2 = \left(\frac{6}{13}\right)^{\text{---}}$

29. $\left[\left(\frac{-1}{4}\right)^{16}\right]^2 = \left(\frac{-1}{4}\right)^{\text{---}}$

30. $\left(\frac{13}{14}\right)^5 \div (\text{---})^2 = \left(\frac{13}{14}\right)^3$

31. $a^6 \times a^{5 \times} a^0 = a^{\text{---}}$

32. 1 lakh = $10^{\text{---}}$

33. 1 million = $10^{\text{---}}$

34. $729 = 3^{\text{---}}$

35. $432 = 2^4 \times 3^{\text{---}}$

36. $53700000 = \text{---} \times 10^7$

37. $88880000000 = \text{---} \times 10^{10}$

38. $27500000 = 2.75 \times 10^{\text{---}}$

39. $340900000 = 3.409 \times 10^{\text{---}}$

40. Fill in the blanks with <, > or = sign.

(a) 3^2 _____ 15

(b) 2^3 _____ 3^2

(c) 7^4 _____ 5^4

(d) $10,000$ _____ 10^5

(e) 6^3 _____ 4^4

In questions 41 to 65, state whether the given statements are True or False.

41. One million = 10^7

42. One hour = 60^2 seconds

43. $1^0 \times 0^1 = 1$

44. $(-3)^4 = -12$

45. $3^4 > 4^3$

46. $\frac{-3}{5}^{100} = \frac{-3}{-5}^{100}$

47. $(10 + 10)^{10} = 10^{10} + 10^{10}$

48. $x^0 \times x^0 = x^0 \div x^0$ is true for all non-zero values of x .

- 49.** In the standard form, a large number can be expressed as a decimal number between 0 and 1, multiplied by a power of 10.
- 50.** 4^2 is greater than 2^4 .
- 51.** $x^m + x^m = x^{2m}$, where x is a non-zero rational number and m is a positive integer.
- 52.** $x^m \times y^m = (x \times y)^{2m}$, where x and y are non-zero rational numbers and m is a positive integer.
- 53.** $x^m \div y^m = (x \div y)^m$, where x and y are non-zero rational numbers and m is a positive integer.
- 54.** $x^m \times x^n = x^{m+n}$, where x is a non-zero rational number and m, n are positive integers.
- 55.** 4^9 is greater than 16^3 .
- 56.** $\left(\frac{2}{5}\right)^3 \div \left(\frac{5}{2}\right)^3 = 1$
- 57.** $\left(\frac{4}{3}\right)^5 \times \left(\frac{5}{7}\right)^5 = \left(\frac{4}{3} + \frac{5}{7}\right)^5$
- 58.** $\left(\frac{5}{8}\right)^9 \div \left(\frac{5}{8}\right)^4 = \left(\frac{5}{8}\right)^4$
- 59.** $\left(\frac{7}{3}\right)^2 \times \left(\frac{7}{3}\right)^5 = \left(\frac{7}{3}\right)^{10}$
- 60.** $5^0 \times 25^0 \times 125^0 = (5^0)^6$

COPY AND COMPLETE THE TABLE

Expression	Expression Written Using Repeated Multiplication	On Multiplying Fractions	Quotient of Powers
$\left(\frac{2}{3}\right)^4$	$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3 \cdot 3}$	$\frac{2^4}{3^4}$
$\left(\frac{-3}{y}\right)^3$	$\frac{-3}{y} \cdot \frac{-3}{y} \cdot \frac{-3}{y}$	$\frac{(-3)(-3)(-3)}{y \cdot y \cdot y}$	_____
$\left(\frac{a}{b}\right)^5$	_____	_____	_____

Key Concept**To be noted****Power of a Product Property**

In Words

To simplify a power of a product, find the power of each factor and multiply.

In Numbers $(5 \cdot 2)^4 = 5^4 \cdot 2^4$ In Algebra $(ab)^m = a^m \cdot b^m$

$$61. \quad 876543 = 8 \times 10^5 + 7 \times 10^4 + 6 \times 10^3 + 5 \times 10^2 + 4 \times 10^1 + 3 \times 10^0$$

$$62. \quad 600060 = 6 \times 10^5 + 6 \times 10^2$$

$$63. \quad 4 \times 10^5 + 3 \times 10^4 + 2 \times 10^3 + 1 \times 10^0 = 432010$$

$$64. \quad 8 \times 10^6 + 2 \times 10^4 + 5 \times 10^2 + 9 \times 10^0 = 8020509$$

$$65. \quad 4^0 + 5^0 + 6^0 = (4 + 5 + 6)^0$$

66. Arrange in ascending order :

$$2^5, 3^3, 2^3 \times 2, (3^3)^2, 3^5, 4^0, 2^3 8^1$$

67. Arrange in descending order :

$$2^{2+3}, (2^2)^3, 2 \times 2^2, \frac{3^5}{3^2}, 3^2 \times 3^0, 2^3 \times 5^2$$

68. By what number should $(-4)^5$ be divided so that the quotient may be equal to $(-4)^3$?

69. Find m so that $\left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^6 = \left(\frac{2}{9}\right)^{2m-1}$

70. If $\frac{p}{q} = \left(\frac{3}{2}\right)^2 \div \left(\frac{9}{4}\right)^0$, find the value of $\left(\frac{p}{q}\right)^3$.

71. Find the reciprocal of the rational number $\left(\frac{1}{2}\right)^2 \div \left(\frac{2}{3}\right)^3$

72. Find the value of :

(a) 7^0

(b) $7^7 \div 7^7$

(c) $(-7)^{2 \times 7 - 6 - 8}$

(d) $(2^0 + 3^0 + 4^0)(4^0 - 3^0 - 2^0)$

(e) $2 \times 3 \times 4 \div 2^0 \times 3^0 \times 4^0$

(f) $(8^0 - 2^0) \times (8^0 + 2^0)$

73. Find the value of n , where n is an integer and

$$2^{n-5} \times 6^{2n-4} = \frac{1}{12^4 \times 2}.$$

74. Express the following in usual form:

(a) 8.01×10^7

(b) 1.75×10^{-3}

75. Find the value of

(a) 2^5

(b) (-3^5)

(c) $-(-4)^4$

76. Express the following in exponential form :

(a) $3 \times 3 \times 3 \times a \times a \times a \times a$

(b) $a \times a \times b \times b \times b \times c \times c \times c \times c$

(c) $s \times s \times t \times t \times s \times s \times t$

77. How many times of 30 must be added together to get a sum equal to 30^7 ?

78. Express each of the following numbers using exponential notations:

(a) 1024

(b) 1029

(c) $\frac{144}{875}$

79. Identify the greater number, in each of the following:

(a) 2^6 or 6^2

(b) 2^9 or 9^2

(c) 7.9×10^4 or 5.28×10^5

COPY AND COMPLETE THE TABLE

Expression	Expression Written Using Repeated Multiplication	Number of Factors	Simplified Expression
$(4^3)^2$	$(4^3) (4^3) = (4 \cdot 4 \cdot 4) (4 \cdot 4 \cdot 4)$	6	4^6
$(7^2)^3$	$(7^2) (7^2) (7^2) = (7 \cdot 7) (7 \cdot 7) (7 \cdot 7)$	_____	$7^{\text{---}}$
$(x^5)^4$	_____	_____	_____

Key Concept**To be noted****Power of a Power Property**

Words To simplify a power of a power, multiply exponents.

Numbers $(5^4)^2 = 5^4 \cdot 2 = 5^8$

Algebra $(a^m)^n = a^{mn}$

80. Express each of the following as a product of powers of their prime factors:

- (a) 9000 (b) 2025 (c) 800

81. Express each of the following in single exponential form:

- (a) $2^3 \times 3^3$ (b) $2^4 \times 4^2$ (c) $5^2 \times 7^2$ (d) $(-5)^5 \times (-5)$
 (e) $(-3)^3 \times (-10)^3$ (f) $(-11)^2 \times (-2)^2$

82. Express the following numbers in standard form:

- (a) 76,47,000 (b) 8,19,00,000
 (c) 5, 83,00,00,00,000 (d) 24 billion

83. The speed of light in vaccum is 3×10^8 m/s. Sunlight takes about 8 minutes to reach the earth. Express distance of Sun from Earth in standard form.

84. Simplify and express each of the following in exponential form:

- (a) $\left[\left(\frac{3}{7} \right)^4 \times \left(\frac{3}{7} \right)^5 \right] \div \left(\frac{3}{7} \right)^7$ (b) $\left[\left(\frac{7}{11} \right)^5 \div \left(\frac{7}{11} \right)^2 \right] \times \left(\frac{7}{11} \right)^2$
 (c) $(3^7 \div 3^5)^4$ (d) $\left(\frac{a^6}{a^4} \right) \times a^5 \times a^0$
 (e) $\left[\left(\frac{3}{5} \right)^3 \times \left(\frac{3}{5} \right)^8 \right] \div \left[\left(\frac{3}{5} \right)^2 \times \left(\frac{3}{5} \right)^4 \right]$ (f) $(5^{15} \div 5^{10}) \times 5^5$

Division of Powers Rule

When you are dividing two powers with the same base, subtract the second exponent from the first to give you the exponent of the answer.

$$(a^m \div a^n = a^{(m-n)})$$

COPY AND COMPLETE THE TABLE

Expression	Expression Written Using Repeated Multiplication	Simplified Expression	Quotient as a Power
$\frac{3^8}{3^3}$	$\frac{3.3.3.3.3.3.3.3}{3.3.3}$	$3.3.3.3.3$	3^5
$\frac{6^5}{6^3}$	$\frac{6.6.6.6.6}{6.6.6}$	—	6^2
$\frac{a^7}{a^4}$	$\frac{a.a.a.a.a.a.a}{a.a.a.a}$	—	—

85. Evaluate

(a) $\frac{7^8 \times a^{10} b^7 c^{12}}{7^6 \times a^8 b^4 c^{12}}$

(b) $\frac{5^4 \times 7^4 \times 2^7}{8 \times 49 \times 5^3}$

(c) $\frac{125 \times 5^2 \times a^7}{10^3 \times a^4}$

(d) $\frac{3^4 \times 12^3 \times 36}{2^5 \times 6^3}$

(e) $\left(\frac{6 \times 10}{2^2 \times 5^3} \right)^2 \times \frac{25}{27}$

(f) $\frac{15^4 \times 18^3}{3^3 \times 5^2 \times 12^2}$

(g) $\frac{6^4 \times 9^2 \times 25^3}{3^2 \times 4^2 \times 15^6}$

Look for a pattern in the table to extend what you know about exponents to find more about negative exponents.

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
10×10	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
100	10	1	$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	$\frac{1}{1000} = 0.001$

$\div 10$ $\div 10$ $\div 10$ $\div 10$ $\div 10$

Any Number Raised to the Power 0 is 1

Any number that has an exponent of 0 is equal to **1**.

So, $2^0 = 1$, $3^0 = 1$, $10^0 = 1$, $\left(\frac{1}{2}\right)^0 = 1$.

For any number $a \neq 0$, $a^0 = 1$.

You can show this by using the **division of powers rule**.

If you start with 1000, and keep **dividing by 10**, you get this pattern:

$1000 = 10^3$	Now divide by 10 : $10^3 \div 10^1 = 10^{(3-1)} = 10^2$
$100 = 10^2$	Now divide by 10 : $10^2 \div 10^1 = 10^{(2-1)} = 10^1$
$10 = 10^1$	Now divide by 10 : $10^1 \div 10^1 = 10^{(1-1)} = 10^0$
$1 = 10^0$	

When you divide 10 by 10, you have $10^1 \div 10^1 = 10^{(1-1)} = 10^0$.

You also know that 10 divided by 10 is 1. So you can see that $10^0 = 1$.

This pattern works for any base.

For instance, $6^1 \div 6^1 = 6^{(1-1)} = 6^0$, and 6 divided by 6 is 1. $6^0 = 1$.

- 86.** Express the given information in Scientific notation (standard form) and then arrange them in ascending order of their size.

Sl.No.	Deserts of the World	Area (Sq. Kilometres)
1.	Kalahari, South Africa	932,400
2.	Thar, India	199,430
3.	Gibson, Australia	155,400
4.	Great Victoria, Australia	647,500
5.	Sahara, North Africa	8,598,800

Think and Discuss

- 1. Explain** why the exponents cannot be added in the product $14^3 \times 18^3$.
- 2. List** two ways to express 4^5 as a product of powers.

87. Express the given information in Scientific notation and then arrange them in descending order of their size.

Sl.No.	Name of the Planet	Mass (in kg)
1.	Mercury	330000000000000000000000
2.	Venus	487000000000000000000000
3.	Earth	598000000000000000000000
4.	Mars	642000000000000000000000
5.	Jupiter	1900000000000000000000000
6.	Saturn	569000000000000000000000
7.	Uranus	869000000000000000000000
8.	Neptune	102000000000000000000000
9.	Pluto	131000000000000000000000

Think and Discuss

1. **Explain** the difference between $(-5)^2$ and -5^2 .
2. **Compare** 3×2 , 3^2 and 2^3 .
3. **Show** that $(4 - 11)^2$ is not equal to $4^2 - 11^2$.

The $\frac{1}{4}$ th of a cube unit contains about 97,700,000,000,000,000,000,000 atoms. The average size of an atom is about 0.00000003 centimetre across.

Scientific notation is a shorthand way of writing such numbers.

To express any number in scientific notation, write it as the product of a power of ten and a number greater than or equal to 1 but less than 10.

In scientific notation, the number of atoms in a quarter is 9.77×10^{22} , and the size of each atom is 3.0×10^{-8} centimetres across.

Real-Life Math

- 1. Explain** the benefit of writing numbers in scientific notation.
- 2. Describe** how to write 2.977×10^6 in normal form.
- 3. Determine** which measurement would be least likely to be written in scientific notation: size of bacteria, speed of a car, or number of stars in a galaxy.

88. Write the number of seconds in scientific notation.

Sl. No.	Unit	Value in Seconds
1.	1 Minute	60
2.	1 Hour	3,600
3.	1 Day	86,400
4.	1 Month	2,600,000
5.	1 Year	32,000,000
6.	10 Years	3,20,000,000

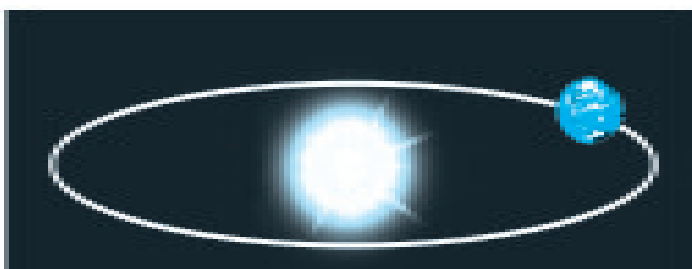
89. In our own planet Earth, 361,419,000 square kilometre of area is covered with water and 148,647,000 square kilometre of area is covered by land. Find the approximate ratio of area covered with water to area covered by land by converting these numbers into scientific notation.

Real-Life Math

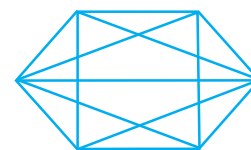
Astronomical Figures The distances from the sun to each of the nine planets in our solar system varies from about 57904280 km to 5899855100 km! These distances are easier to write in shorthand: 5.79×10^7 km and 5.899×10^9 km. The distance from the sun to the star nearest to it, Proxima Centauri, is about 40233600000000 km. It would be much easier for an astronomer to write this distance as 4.023×10^{13} km.

Mars, the fourth planet in our solar system, is 2.269×10^8 km from the sun.

90. If $2^{n+2} - 2^{n+1} + 2^n = c \times 2^n$, find the value of c .
91. A light year is the distance that light can travel in one year.
 1 light year = 9,460,000,000,000 km.
- (a) Express one light year in scientific notation.
- (b) The average distance between Earth and Sun is 1.496×10^8 km.
 Is the distance between Earth and the Sun greater than, less than or equal to one light year?



92. **Geometry Application :** The number of diagonals of an n -sided figure is $\frac{1}{2}(n^2 - 3n)$.
 Use the formula to find the number of diagonals for a 6-sided figure (hexagon).



93. **Life Science :** Bacteria can divide in every 20 minutes. So 1 bacterium can multiply to 2 in 20 minutes. 4 in 40 minutes, and so on. How many bacteria will there be in 6 hours? Write your answer using exponents, and then evaluate.



Most bacteria reproduce by a type of simple cell division known as binary fission. Each species reproduce best at a specific temperature and moisture level.

Writing Strategy**Write a Convincing Argument**

Your ability to write a convincing argument proves that you have understanding of the concept. An effective argument should include the following four parts:

1. A goal
2. A response to the goal
3. Evidence to support the response
4. A summary statement

Write about it

Compare 10^2 and 2^{10} . For any two numbers, which usually gives the greater number, using the greater number as the base or as the exponent? Give at least one exception.

Step 1 : Identify the goal

For any two numbers, explain whether using the greater number as the base or as the exponent will generally result in a greater number. Find one exception.

Step 2 : Provide a response to the goal

Using the greater number as the exponent usually gives the greater number.

Step 3 : Provide evidence to support your response

For the number 10 and 2. Using the greater number, 10, as the exponent will result in a greater number.

$$10^2 = 100$$

$$2^{10} = 1024$$

$$100 < 1024$$

$$10^2 < 2^{10}$$

Exception for the numbers 2 and 3, using the greater number, 3, as the exponent will not result in a greater number.

$$3^2 = 9$$

$$2^3 = 8$$

$$9 > 8$$

$$3^2 > 2^3$$

Step 4 : Summarise your argument

Generally, for any two numbers, using the greater number as the exponent instead of as the base will result in a greater number.

- 94. Blubber** makes up 27 per cent of a blue whale's body weight. Deepak found the average weight of blue whales and used it to calculate the average weight of their blubber. He wrote the amount as $2^2 \times 3^2 \times 5 \times 17$ kg. Evaluate this amount.



- 95. Life Science Application :** The major components of human blood are red blood cells, white blood cells, platelets and plasma. A typical red blood cell has a diameter of approximately 7×10^{-6} metres. A typical platelet has a diameter of approximately 2.33×10^{-6} metre. Which has a greater diameter, a red blood cell or a platelet?
- 96.** A googol is the number 1 followed by 100 zeroes.
 (a) How is a googol written as a power?
 (b) How is a googol times a googol written as a power?
- 97. What's the error?**
 A student said that $\frac{3^5}{9^5}$ is the same as $\frac{1}{3}$. What mistake has the student made?

(D) Application

1. Cross Word Puzzle

Solve the given crossword and then fill up the given boxes in 1 and 2. Clues are given below for across as well as downward fillings. Also for across and down clues, clue number is written at the corner of boxes. Answers of clues have to fill up in their respective boxes.

Down 1 : In 10^6 , 10 is the base and 6 is _____.

Down 2 : $a^n = 1$ only if $n =$ _____.

Down 3 : Very large numbers can be expressed in standard form, also known as _____ notation.

Down 4 : The place of 6 in 5.632 is _____.

Down 5 : In 10^{-5} , -5 is the exponent and 10 is the _____.

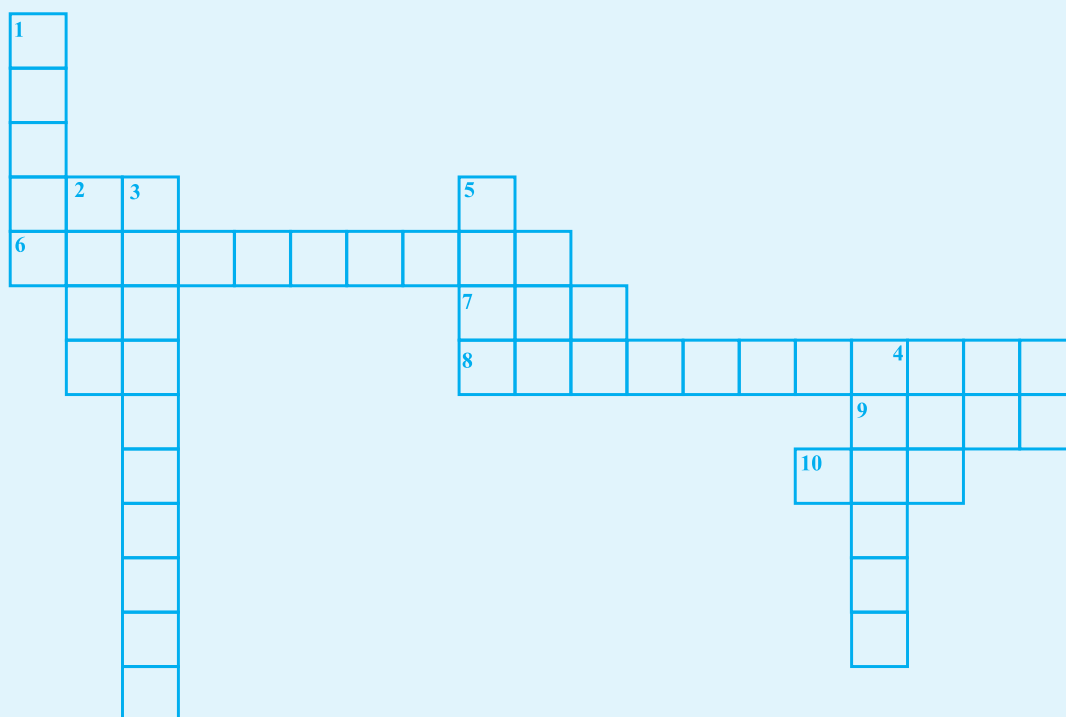
Across 6 : a^{-m} is the _____ of a^m .

Across 7 : $a^m \times a^n = a^x$, where x is the _____ of m and n .

Across 8 : 10^3 is called the _____ form of 1000.

Across 9 : $(-1)^p = 1$ is valid, where p is an _____ integer.

Down 10 : $(1)^n = 1$ is valid for _____ value of n .



2. Cross Number Puzzle

Across

1. 5.724×10^3 is the standard form of _____.

2. The value of $\frac{21^3 \times 10^5 \times 125}{2^5 \times 3^3 \times 5^8}$ is _____.

3. The value of $2^{5 \times 2 - 3 - 2}$ is _____.

4. The value of $11^2 \times 3^2 - 11$ is _____.
5. The number 10^3 is the exponential form of _____.

Down

1. In 2^5 , the exponent is _____.
6. The value of 3^5 is _____.
7. The value of $4 \times 10^4 + 3 \times 10^3 + 2 \times 10^2 + 7 \times 10$ is _____.
8. The cube of 8 is _____.
9. Square of -11 is _____.
10. The value of $(11)^2$ is _____.

	1		6	7
8		2		
10	9		3	
4				
	5			

UNIT 12

PRACTICAL GEOMETRY SYMMETRY AND VISUALISING SOLID SHAPES

(A) Main Concepts and Results

- Let a line ' l ' and a point P not lying on it be given. By using properties of a transversal and parallel lines, a line which passes through the point P and parallel to ' l ', can be drawn.
- A triangle can be drawn if any one of the following sets of measurements are given :
 - (i) Three sides (SSS).
 - (ii) Two sides and the angle between them (SAS).
 - (iii) Two angles and a side (AAS) or (ASA).
 - (iv) The hypotenuse and a leg in the case of a right-angled triangle (RHS).
- A figure has line symmetry, if there is a line about which the figure may be folded so that the two parts of the figure will coincide with each other.
- Regular polygons have equal sides and equal angles. They have multiple (i.e., more than one) lines of symmetry.
- Each regular polygon has as many lines of symmetry as it has sides.
- Mirror reflection leads to symmetry, under which the left-right orientation have to be taken care of.

- Rotation turns an object about a fixed point. This fixed point is called the centre of rotation.
- The angle by which the object rotates is the angle of rotation. Rotation may be clockwise or anti-clockwise.
- A half-turn means rotation by 180° . A quarter-turn means rotation by 90° .
- If, after a rotation, a figure or an object coincides with the original position, we say that it has a rotational symmetry.
- In a complete turn (of 360°), the number of times the figure coincides with its original position is called its order of rotational symmetry.
- Every figure has a rotational symmetry of order 1 (i.e. a rotational symmetry of angle 360°). In such a case it is considered that the figure has no rotational symmetry.
- Some shapes have only one line of symmetry, like the letter E; some have only rotational symmetry, like the letter S; and some have both vertical and horizontal lines of symmetry, like the letter H.
- Plane figures are of two-dimensions (2-D) and the solid shapes are of three-dimensions (3-D).
- The corners of a solid shape are called its vertices, the line segments/curves which form its skeleton are its edges and its flat surfaces are its faces.

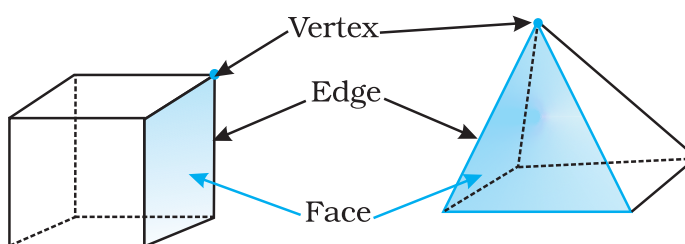


Fig. 12.1

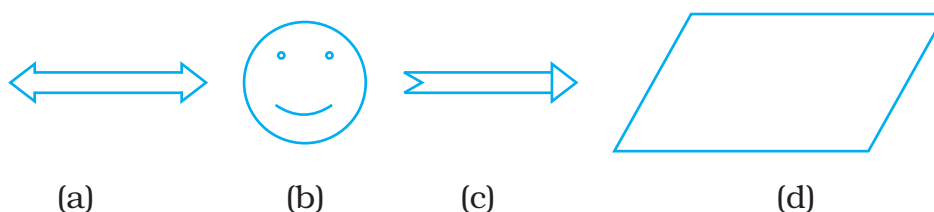
- A net is a skeleton-outline of a solid that can be folded to make the solid.
- Solid shapes can be drawn on a flat surface. This is called a 2-D representation of a 3-D solid (shape).
- Two types of sketches of a solid are possible:
 - (i) An oblique sketch which does not have proportional measurements.
 - (ii) An isometric sketch which is drawn on an isometric dot paper. In this sketch of the solid, the measurements are kept proportional.

- Different sections of a solid can be viewed in many ways:
 - By cutting or slicing, the shape, which would result in the cross-section of the solid.
 - By observing a 2-D shadow of a 3-D shape.
 - By looking at the shape from different positions-the front-view, the side-view and the top-view.

(B) Solved Examples

In Examples 1 to 3, there are four options, out of which one is correct. Choose the correct one.

Example 1: Which of the following is not a symmetrical figure?



Solution: Correct answer is (d).

Example 2: In the word “MATHS” which of the following pairs of letters shows rotational symmetry

- (a) M and T (b) H and S (c) A and S (d) T and S

Solution: Correct answer is (b).

Example 3: The angle of rotation for the figure 12.2 is

- (a) 45° (b) 60°
(c) 90° (d) 180°

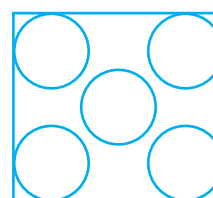


Fig. 12.2

Solution: Correct answer is (C)

In Examples 4 to 6, fill in the blanks to make it a true statement.

Example 4: The figure 12.3 has _____ vertices, _____ edges and _____ faces.

Solution: 10, 15, 7

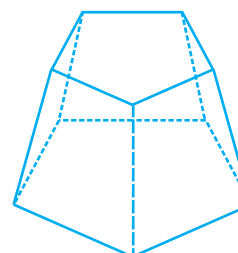


Fig. 12.3

Example 5: The adjoining net in Fig. 12.4 represents a _____.

Solution: Cube

Example 6: Rotation turns an object about a fixed point. This fixed point is called _____.

Solution: centre of rotation.

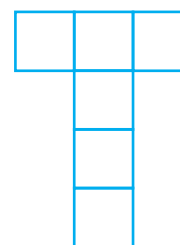


Fig. 12.4

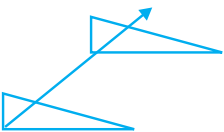

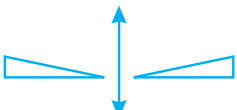
In Examples 7 to 9, state whether the statements are True or False.

Example 7: A net of a 3-D shape is a sort of skeleton - outline in 2-D, which, when folded results in the 3-D shape.

Solution: True

Example 8: A regular pentagon has no lines of symmetry.

Solution: False

Translation	Rotation	Reflection
		
A translation slides a figure along the direction of a line without turning.	A rotation turns a figure around a point, called the centre of rotation .	A reflection flips a figure across a line to create a mirror image.

Example 9: Order of rotational symmetry for the figure 12.5 is 4.

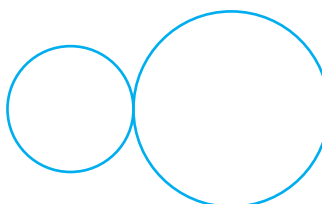
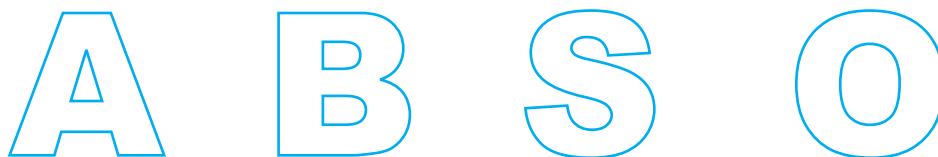


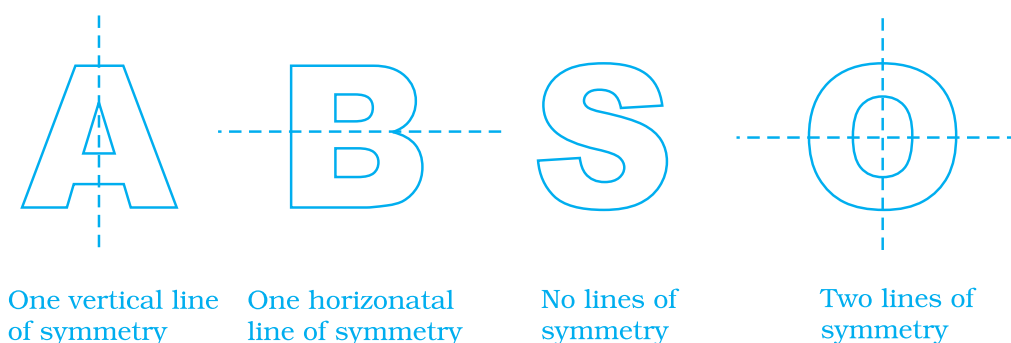
Fig. 12.5

Solution: False

Example 10: Draw all the lines of symmetry for the following letters if they exist.



Solution

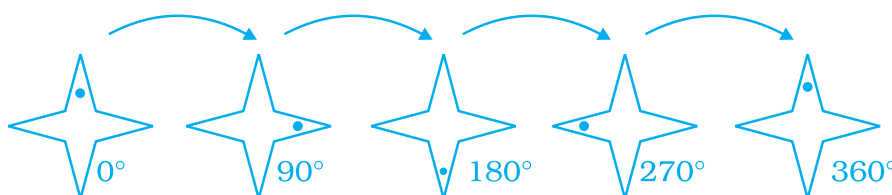


Example 11: State whether the figure 12.6 shows rotational symmetry. If yes, then what is the order of rotational symmetry?



Fig. 12.6

Solution: The given figure shows rotational symmetry. The order of symmetry = 4, which is clear from the following figure:



Note: The dot is placed just to identify different positions of the figure.

Example 12: Identify the following figures:

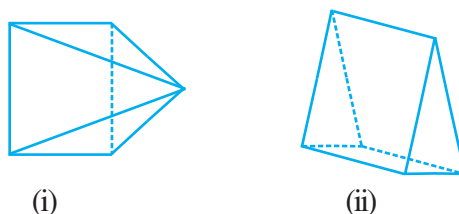


Fig. 12.7

- Solution:**
- (i) Rectangular Pyramid
 - (ii) Triangular Prism

Example 13: Construct a triangle PQR such that $PQ = 6$ cm, $QR = 7$ cm and $PR = 4.5$ cm.

Solution

- Steps:**
- (i) Draw a line segment PQ of length 6 cm.
 - (ii) With P as centre, draw an arc of radius 4.5 cm.
 - (iii) With Q as centre, draw an arc of radius 7 cm which intersects the previous arc at R.
 - (iv) Join PR and QR.

Then $\triangle PQR$ is the required triangle (Fig. 12.8).

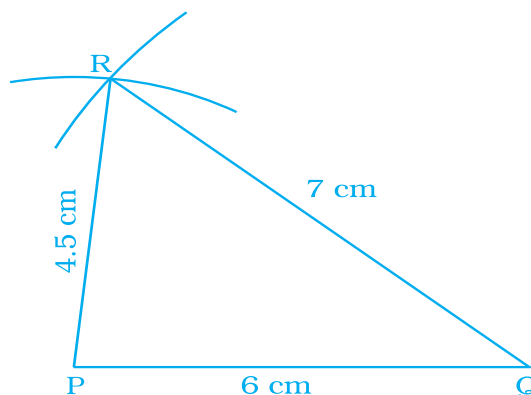


Fig. 12.8

Example 14: Draw the top, the front and the side views of the following solid figure made up of cubes.

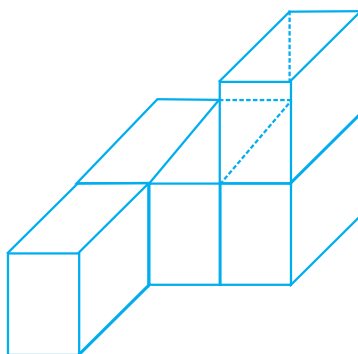


Fig. 12.9

Solution: Desired views are shown in Fig. 12.10 below

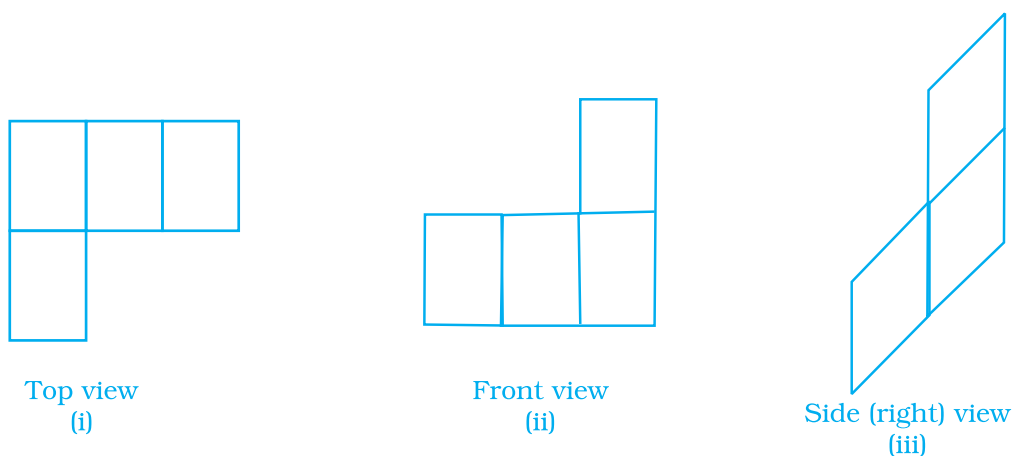


Fig. 12.10

Nature provides many beautiful examples of *symmetry*, such as the wings of a butterfly and a peacock or the petals of a flower. Symmetric objects have parts that are congruent.

A figure has **line symmetry** if you can draw a line through it so that the two sides are mirror images of each other. The line is called the **line of symmetry**.



Example 15: Given a line l and a point M on it draw a perpendicular MP to l where $MP = 5.2\text{cm}$ and a line q parallel to l through P .

Solution

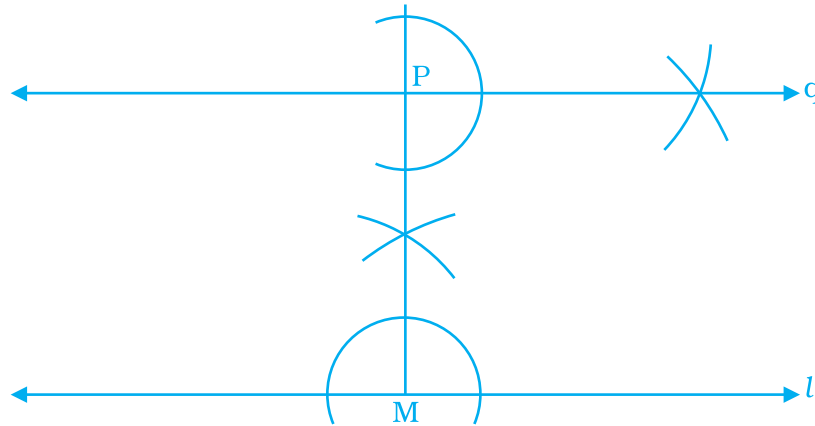


Fig. 12.11

- Steps :**
- (i) Draw a line l .
 - (ii) Take a point M on it.
 - (iii) Draw an angle of 90° at M with l which is perpendicular to l at M .
 - (iv) With M as centre and radius 5.2 cm, draw an arc which intersects the above perpendicular at point P . MP is the required perpendicular
 - (v) At P , draw an angle of 90° with PM and produce to make a line q .

Line q is the required line parallel to line l .

Application on Problem Solving Strategy



Example 16

Determine the number of edges, vertices and faces in the Fig. 12.12.

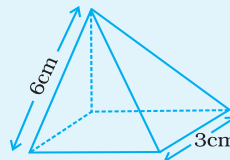


Fig. 12.12

Solution:



Understand and Explore the Problem

- What information is given in the question?
A square pyramid.
- What are you trying to find?
The number of edges, vertices and faces.
- Is there any information that is not needed?
The measure of the edges are not needed.



Plan a Strategy

- Recall the definitions of edges, vertices and faces of a 3-D figure and try to co-relate them to the figure given above.



Solve

- The different plain regions are called faces. Hence, there are 5 faces.
- The line segments formed, where the faces meet are called edges. Hence, there are 8 edges.
- Edges meet at a point which are called vertices. Hence, there are 5 vertices.
- Therefore, a square pyramid has 5 faces, 5 vertices and 8 edges.



Revise

- Try to find the number of vertices and edges of a cuboid.
Can you see a pattern emerging based on your findings?
you can observe that

$$F + V = E + 2$$

Where F,V,E denote number of faces, number of vertices and number of edges respectively of such solids. This is known as 'EULER's FORMULA'. You'll study this concept in your next class.

Think and Discuss

Try to find the number of edges, vertices and faces in some more solids and explore the pattern, if any.

A figure has **rotational symmetry** if you can rotate the figure around some point so that it coincides with itself. The point is the centre of rotation, and the amount of rotation must be less than one full turn, or 360° .



7-fold and 6-fold rotational symmetry mean that the figures coincide with themselves 7 times and 6 times respectively, within one full turn.

7-fold rotational symmetry 6-fold rotational symmetry

(C) Exercise

In each of the Questions 1 to 26, there are four options, out of which one is correct. Choose the correct one.

1. A triangle can be constructed by taking its sides as:

(a) 1.8 cm, 2.6 cm, 4.4 cm	(b) 2 cm, 3 cm, 4 cm
(c) 2.4 cm, 2.4 cm, 6.4 cm	(d) 3.2 cm, 2.3 cm, 5.5 cm
2. A triangle can be constructed by taking two of its angles as:

(a) 110° , 40°	(b) 70° , 115°	(c) 135° , 45°	(d) 90° , 90°
------------------------------	------------------------------	------------------------------	-----------------------------
3. The number of lines of symmetry in the figure given below is:

(a) 4	(b) 8
(c) 6	(d) Infinitely many

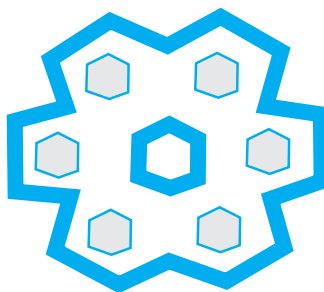


Fig. 12.13

Think and Discuss

1. **Explain** what it means for a figure to be symmetric.
2. **Tell** which capital letters of the alphabet have line symmetry.
3. **Tell** which capital letters of the alphabet have rotational symmetry.

4. The number of lines of symmetry in Fig. 12.14 is

- (a) 1 (b) 3
(c) 6 (d) Infinitely many



Fig. 12.14

5. The order of rotational symmetry in the Fig. 12.15 given below is

- (a) 4 (b) 8 (c) 6 (d) Infinitely many



Fig. 12.15

6. The order of rotational symmetry in the figure 12.16 given below is

- (a) 4 (b) 2
(c) 1 (d) Infinitely many



Fig. 12.16

7. The name of the given solid in Fig 12.17 is:

- (a) triangular pyramid (b) rectangular pyramid
(c) rectangular prism (d) triangular prism

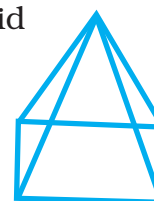


Fig. 12.17

Think and Discuss

1. **Give** a situation in which the front and side views of a figure would be the same.
2. **Explain** whether it is possible for all of the views of a figure to be congruent rectangles.

8. The name of the solid in Fig. 12.18 is:

- (a) triangular pyramid (b) rectangular prism
(c) triangular prism (d) rectangular pyramid

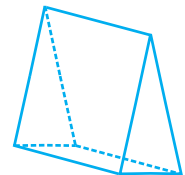


Fig. 12.18

9. All faces of a pyramid are always:

- (a) Triangular (b) Rectangular
(c) Congruent (d) None of these

10. A solid that has only one vertex is

- (a) Pyramid (b) Cube (c) Cone (d) Cylinder

11. Out of the following which is a 3-D figure?

- (a) Square (b) Sphere (c) Triangle (d) Circle

12. Total number of edges a cylinder has

- (a) 0 (b) 1 (c) 2 (d) 3

13. A solid that has two opposite identical faces and other faces as parallelograms is a

- (a) prism (b) pyramid (c) cone (d) sphere

14. The solid with one circular face, one curved surface and one vertex is known as:

- (a) cone (b) sphere (c) cylinder (d) prism

15. If three cubes each of edge 4 cm are placed end to end, then the dimensions of resulting solid are:

- (a) $12\text{ cm} \times 4\text{ cm} \times 4\text{ cm}$ (b) $4\text{ cm} \times 8\text{ cm} \times 4\text{ cm}$
(c) $4\text{ cm} \times 8\text{ cm} \times 12\text{ cm}$ (d) $4\text{ cm} \times 6\text{ cm} \times 8\text{ cm}$

16. When we cut a corner of a cube as shown in the figure 12.19, we get the cutout piece as :

- (a) square pyramid (b) trapezium prism
(c) triangular pyramid (d) a triangle



Fig. 12.19

17. If we rotate a right-angled triangle of height 5 cm and base 3 cm about its height a full turn, we get

- (a) cone of height 5 cm, base 3 cm
(b) triangle of height 5 cm, base 3 cm
(c) cone of height 5 cm, base 6 cm
(d) triangle of height 5 cm, base 6 cm

18. If we rotate a right-angled triangle of height 5 cm and base 3 cm about its base, we get:

- (a) cone of height 3 cm and base 3 cm
(b) cone of height 5 cm and base 5 cm
(c) cone of height 5 cm and base 3 cm
(d) cone of height 3 cm and base 5 cm

19. When a torch is pointed towards one of the vertical edges of a cube, you get a shadow of cube in the shape of

- (a) square (b) rectangle but not a square
(c) circle (d) triangle

20. Which of the following sets of triangles could be the lengths of the sides of a right-angled triangle:

- (a) 3 cm, 4 cm, 6 cm (b) 9 cm, 16 cm, 26 cm
(c) 1.5 cm, 3.6 cm, 3.9 cm (d) 7 cm, 24 cm, 26 cm

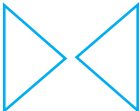
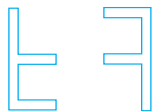


21. In which of the following cases, a unique triangle can be drawn

- (a) $AB = 4 \text{ cm}$, $BC = 8 \text{ cm}$ and $CA = 2 \text{ cm}$
- (b) $BC = 5.2 \text{ cm}$, $\angle B = 90^\circ$ and $\angle C = 110^\circ$
- (c) $XY = 5 \text{ cm}$, $\angle X = 45^\circ$ and $\angle Y = 60^\circ$
- (d) An isosceles triangle with the length of each equal side 6.2 cm .

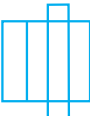
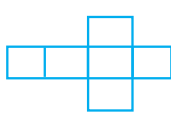

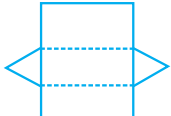
22. Which of the following has a line of symmetry?

- (a) 
- (b) 
- (c) 
- (d) 


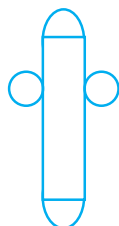
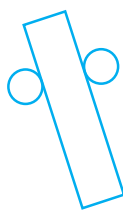
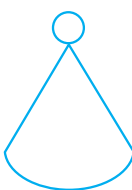
23. Which of the following are reflections of each other?

- (a) 
- (b) 
- (c) 
- (d) 

24. Which of these nets is a net of a cube?

- (a) 
- (b) 
- (c) 
- (d) 

25. Which of the following nets is a net of a cylinder?

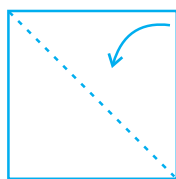
- (a) 
- (b) 
- (c) 
- (d) 

26. Which of the following letters of English alphabets have more than 2 lines of symmetry?

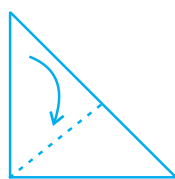
- (a) **Z**
- (b) **O**
- (c) **E**
- (d) **H**

27. Take a square piece of paper as shown in figure (1). Fold it along its diagonals as shown in figure (2). Again fold it as shown in figure (3). Imagine that you have cut off 3 pieces of the form of congruent

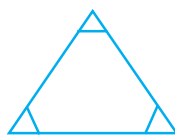
isosceles right-angled triangles out of it as shown in figure 4.



(1)



(2)

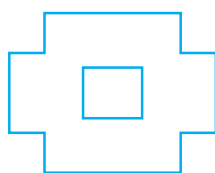


(3)

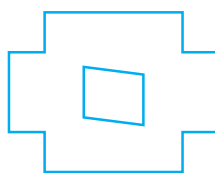


(4)

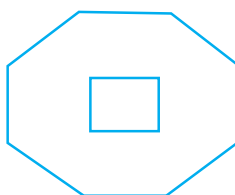
On opening the piece of paper which of the following shapes will you get?



(a)



(b)

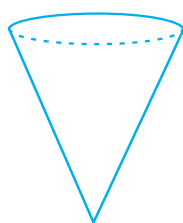


(c)



(d)

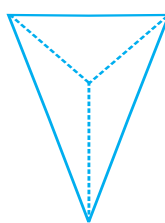
- 28.** Which of the following 3-dimensional figures has the top, side and front as triangles?



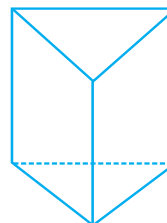
(a)



(b)



(c)



(d)

In Questions 29 to 58, fill in the blanks to make the statements true.

- 29.** In an isosceles right triangle, the number of lines of symmetry is _____.
- 30.** Rhombus is a figure that has _____ lines of symmetry and has a rotational symmetry of order _____.
- 31.** _____ triangle is a figure that has a line of symmetry but lacks rotational symmetry.
- 32.** _____ is a figure that has neither a line of symmetry nor a rotational symmetry.

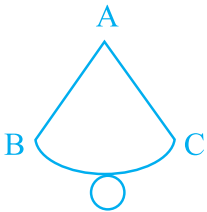
33. _____ and _____ are the capital letters of English alphabets that have one line of symmetry but they interchange to each other when rotated through 180° .
34. The common portion of two adjacent faces of a cuboid is called _____.
35. A plane surface of a solid enclosed by edges is called _____.
36. The corners of solid shapes are called its _____.
37. A solid with no vertex is _____.
38. A triangular prism has _____ faces, _____ edges and _____ vertices.
39. A triangular pyramid has _____ faces, _____ edges and _____ vertices.
40. A square pyramid has _____ faces, _____ edges and _____ vertices.
41. Out of _____ faces of a triangular prism, _____ are rectangles and _____ are triangles.
42. The base of a triangular pyramid is a _____.
43. Out of _____ faces of a square pyramid, _____ are triangles and _____ is/are squares.
44. Out of _____ faces of a rectangular pyramid _____ are triangles and base is _____.
45. Each of the letters H, N, S and Z has a rotational symmetry of order _____.
46. Order of rotational symmetry of a rectangle is _____.
47. Order of rotational symmetry of a circle is _____.
48. Each face of a cuboid is a _____.
49. Line of symmetry for an angle is its _____.
50. A parallelogram has _____ line of symmetry.

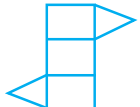
51. Order of rotational symmetry of  is _____.

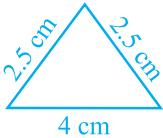
52. A _____ triangle has no lines of symmetry.

53. Cuboid is a rectangular _____.

54. A sphere has _____ vertex, _____ edge and _____ curved surface.

55.  is a net of a _____.
 → Circumference of circle = _____.

56.  is a net of a _____.

57. Order of rotational symmetry of  is _____.

58. Identical cubes are stacked in the corner of a room as shown below.
 The number of cubes that are not visible are _____.

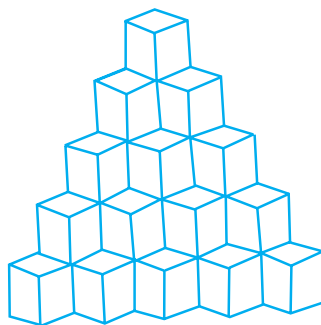


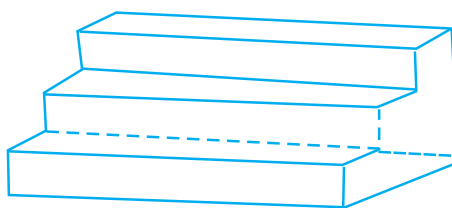
Fig. 12.20

In Questions from 59 to 92, state whether the statements are True or False.

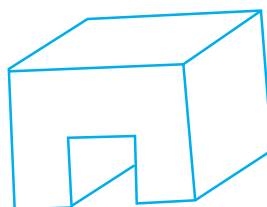
- 59.** We can draw exactly one triangle whose angles are 70° , 30° and 80° .
- 60.** The distance between the two parallel lines is the same everywhere.
- 61.** A circle has two lines of symmetry.
- 62.** An angle has two lines of symmetry.
- 63.** A regular hexagon has six lines of symmetry.
- 64.** An isosceles trapezium has one line of symmetry.
- 65.** A parallelogram has two lines of symmetry.
- 66.** Order of rotational symmetry of a rhombus is four.
- 67.** An equilateral triangle has six lines of symmetry.
- 68.** Order of rotational symmetry of a semi circle is two.
- 69.** In oblique sketch of the solid, the measurements are kept proportional.
- 70.** An isometric sketch does not have proportional length.
- 71.** A cylinder has no vertex.
- 72.** All the faces, except the base of a square pyramid are triangular.
- 73.** A pyramid has only one vertex.
- 74.** A triangular prism has 5 faces, 9 edges and 6 vertices.
- 75.** If the base of a pyramid is a square, it is called a square pyramid.
- 76.** A rectangular pyramid has 5 rectangular faces.
- 77.** Rectangular prism and cuboid refer to the same solid.
- 78.** A tetrahedron has 3 triangular faces and 1 rectangular face.
- 79.** While rectangle is a 2-D figure, cuboid is a 3-D figure.
- 80.** While sphere is a 2-D figure, circle is a 3-D figure.
- 81.** Two dimensional figures are also called plane figures.
- 82.** A cone is a polyhedron.

83. A prism has four bases.
84. The number of lines of symmetry of a regular polygon is equal to the vertices of the polygon.
85. The order of rotational symmetry of a figure is 4 and the angle of rotation is 180° only.
86. After rotating a figure by 120° about its centre, the figure coincides with its original position. This will happen again if the figure is rotated at an angle of 240° .
87. Mirror reflection leads to symmetry always.
88. Rotation turns an object about a fixed point which is known as centre of rotation.
89. Isometric sheet divides the paper into small isosceles triangles made up of dots or lines.
90. The circle, the square, the rectangle and the triangle are examples of plane figures.
91. The solid shapes are of two-dimensional.
92. Triangle with length of sides as 5 cm, 6 cm and 11 cm can be constructed.
93. Draw the top, side and front views of the solids given below in Figures 12.21 and 12.22:

(i)

*Fig. 12.21*

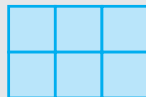
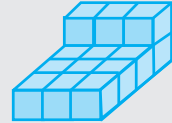
(ii)

*Fig. 12.22*

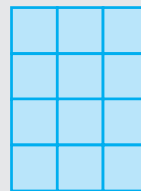
Three-dimensional figures often look different from different points of view. You can use centimetre cubes to help you visualize and sketch three-dimensional figures.

Activity

1. Use centimetre cubes to build the three-dimensional figure at right.
2. Now view the figure from the front and draw what you see. Then view the figure from the top and draw what you see. Finally, view the figure from the side and draw what you see.



Front



Top

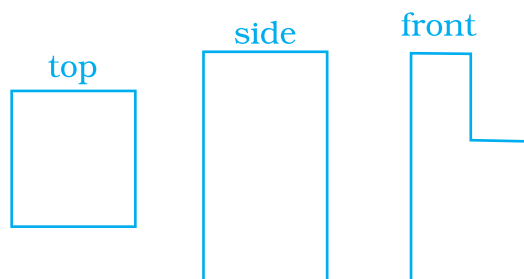


Side

Think and Discuss

1. How many cubes did you use to build the three-dimensional figure?
2. How could you add a cube to the figure without changing the top view?
3. How could you remove a cube from the figure without changing the side view?

94. Draw a solid using the top, side and front views as shown below. [Use Isometric dot paper].



95. Construct a right-angled triangle whose hypotenuse measures 5 cm and one of the other sides measures 3.2 cm.
96. Construct a right-angled isosceles triangle with one side (other than hypotenuse) of length 4.5 cm.

97. Draw two parallel lines at a distance of 2.2 cm apart.
98. Draw an isosceles triangle with each of equal sides of length 3 cm and the angle between them as 45° .
99. Draw a triangle whose sides are of lengths 4 cm, 5 cm and 7 cm.
100. Construct an obtuse angled triangle which has a base of 5.5 cm and base angles of 30° and 120° .
101. Construct an equilateral triangle ABC of side 6 cm.
102. By what minimum angle does a regular hexagon rotate so as to coincide with its original position for the first time?
103. In each of the following figures, write the number of lines of symmetry and order of rotational symmetry.

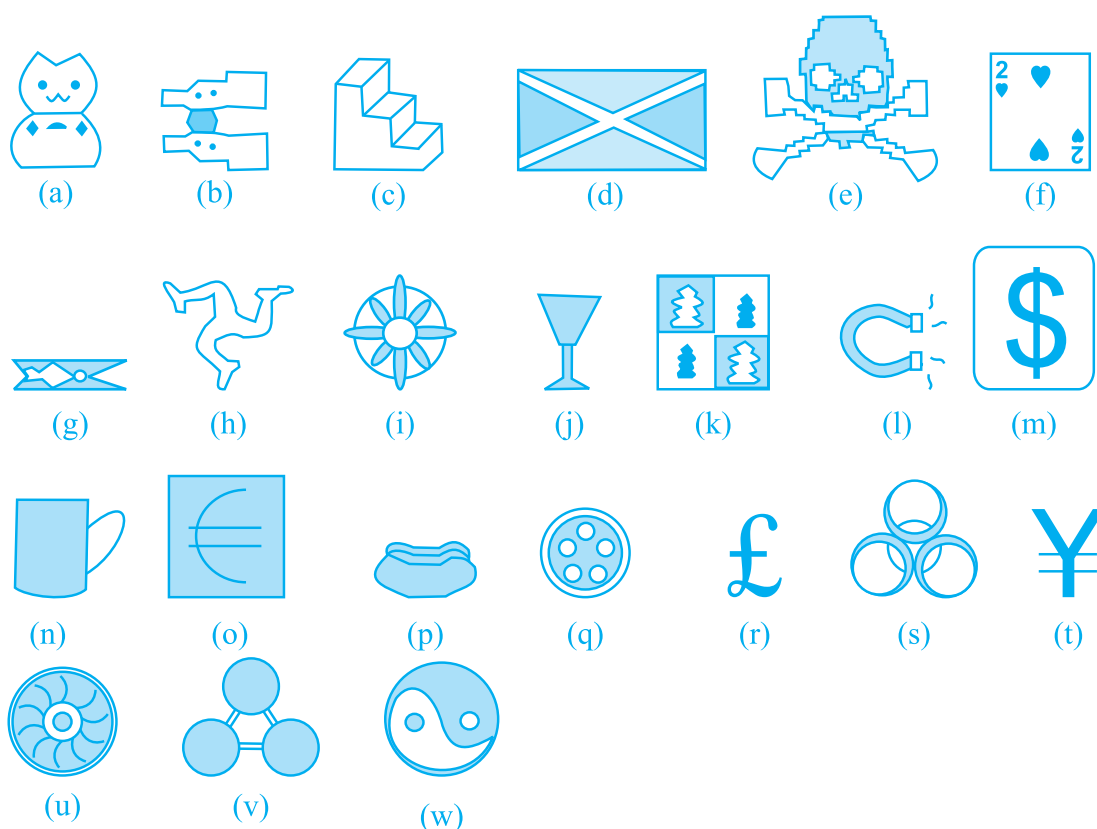


Fig. 12.23

[Hint: Consider these as 2-D figures not as 3-D objects.]

104. In the figure 12.24 of a cube,
 - (i) Which edge is the intersection of faces EFGH and EFBA?
 - (ii) Which faces intersect at edge FB?

- (iii) Which three faces form the vertex A?
- (iv) Which vertex is formed by the faces ABCD, ADHE and CDHG?
- (v) Give all the edges that are parallel to edge AB.
- (vi) Give the edges that are neither parallel nor perpendicular to edge BC.
- (vii) Give all the edges that are perpendicular to edge AB.
- (viii) Give four vertices that do not all lie in one plane.

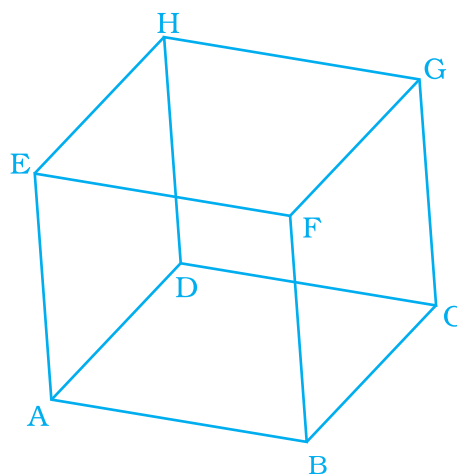


Fig. 12.24

- 105.** Draw a net of a cuboid having same breadth and height, but length double the breadth.
- 106.** Draw the nets of the following:
- (i) Triangular prism
 - (ii) Tetrahedron
 - (iii) Cuboid
- 107.** Draw a net of the solid given in the figure 12.25:

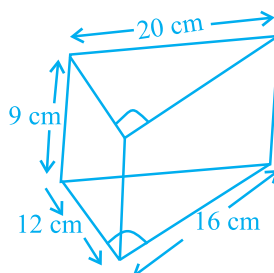


Fig. 12.25

- 108.** Draw an isometric view of a cuboid $6\text{ cm} \times 4\text{ cm} \times 2\text{ cm}$.
- 109.** The net given below in Fig. 12.26 can be used to make a cube.
- (i) Which edge meets AN?
 - (ii) Which edge meets DE?

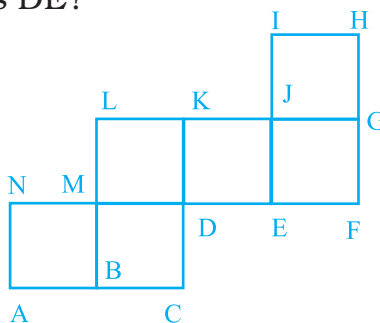


Fig. 12.26

- 110.** Draw the net of triangular pyramid with base as equilateral triangle of side 3 cm and slant edges 5 cm.
- 111.** Draw the net of a square pyramid with base as square of side 4 cm and slant edges 6 cm.
- 112.** Draw the net of rectangular pyramid with slant edge 6 cm and base as rectangle with length 4 cm and breadth 3 cm.

Activity

- 1.** Use centimetre cubes to build a figure that has the front, tops and side views shown.



- 2.** You can build the figure by first making a simple figure that matches the front views.



- 3.** Now add cubes so that the figure matches the top view.



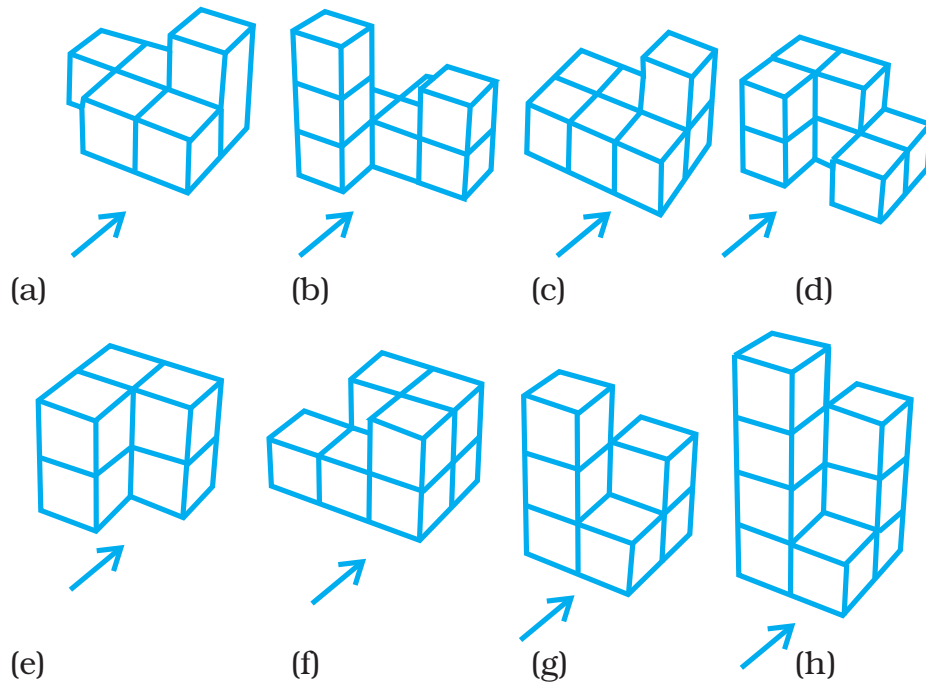
- 4.** Finally, remove cubes so that the figure matches the side view. Check that the front and top views are still correct for the figure that you built.



Think and Discuss

Discuss whether there is another step-by-step method for building the above figure. If so, is the final result the same.

- 113.** Find the number of cubes in each of the following figures and in each case give the top, front, left side and right side view (arrow indicating the front view).

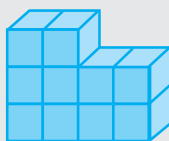


- 114.** Draw all lines of symmetry for each of the following figures as given below:



Try This

- 1.** Use centimetre cubes to build each three-dimensional figure given below. Then sketch the front, top and side views.



(i)



(ii)



(iii)



(iv)

115. How many faces does Fig. 12.27 have?

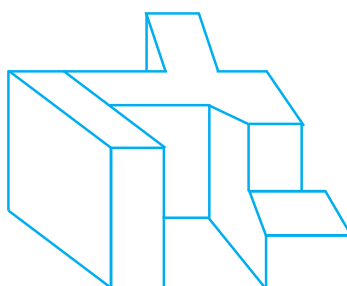
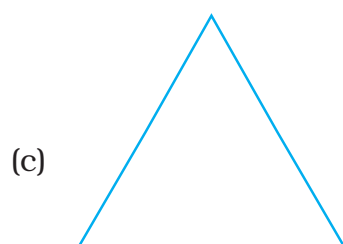
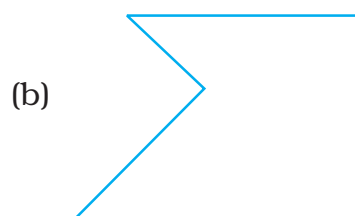
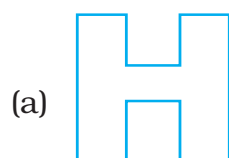
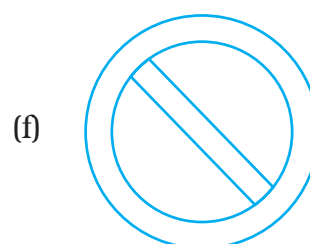
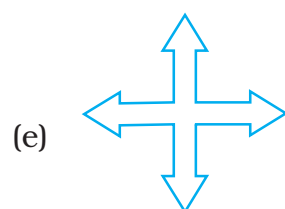
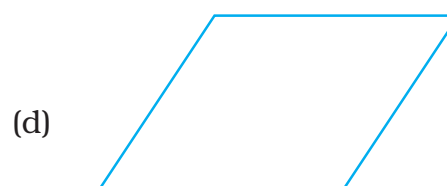
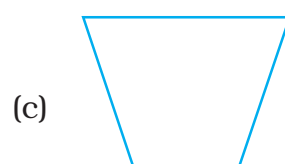
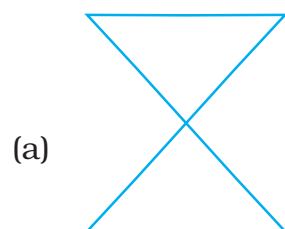


Fig. 12.27

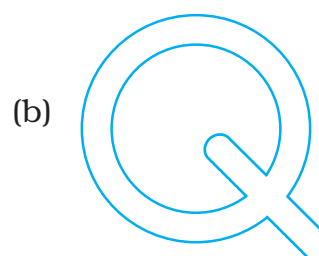
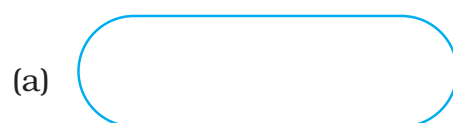
116. Trace each figure. Then draw all lines of symmetry, if it has.

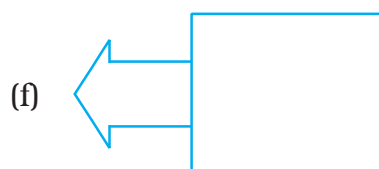
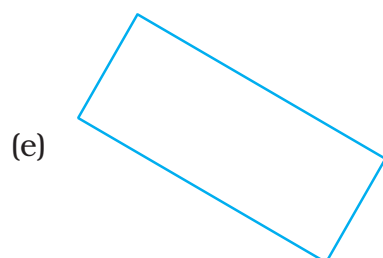
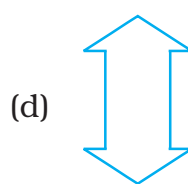
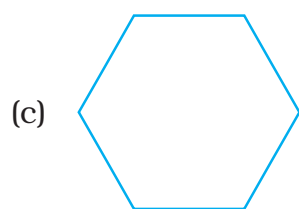


117. Tell whether each figure has rotational symmetry or not.

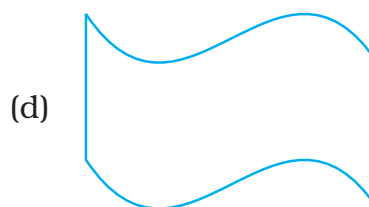
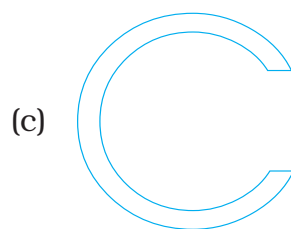
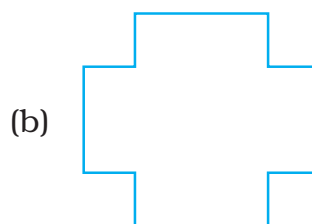
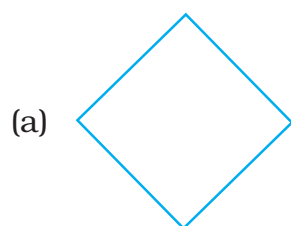


118. Draw all lines of symmetry for each of the following figures.





119. Tell whether each figure has rotational symmetry. Write yes or no.



120. Does the Fig. 12.28 have rotational symmetry?

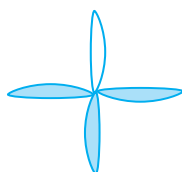


Fig. 12.28

- 121.** The flag of Japan is shown below. How many lines of symmetry does the flag have?

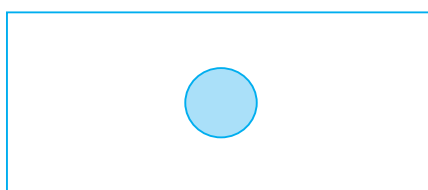
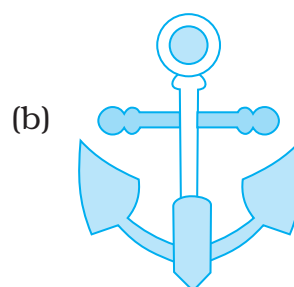
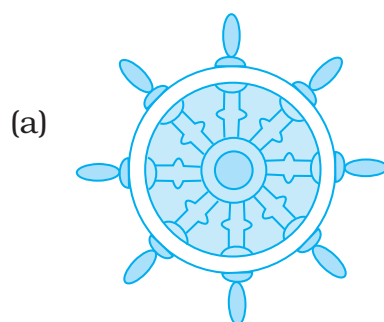


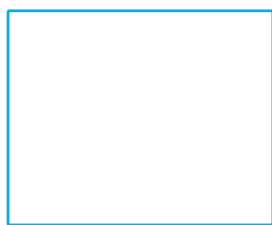
Fig. 12.29

- 122.** Which of the figures given below have both line and rotational symmetry?

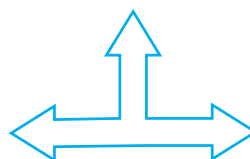


123. Which of the following figures do not have line symmetry?

(a)



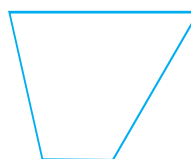
(b)



(c)



(d)



124. Which capital letters of English alphabet have no line of symmetry?

(D) Application

1. Crossword Puzzle

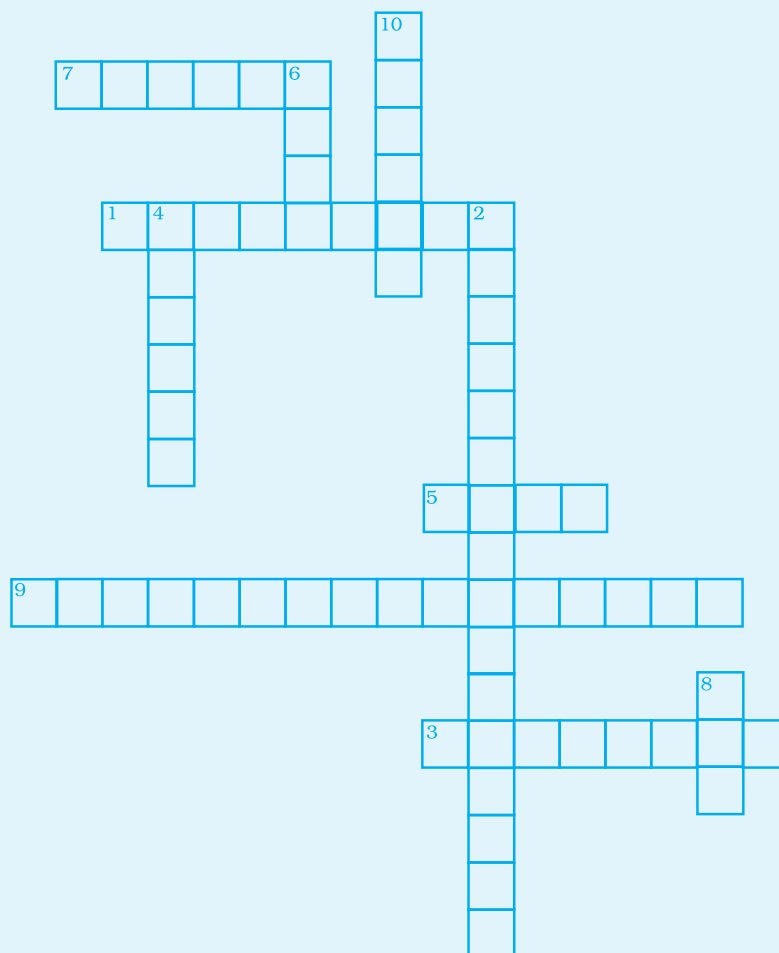
Solve the crossword and fill the given box across, downward as per the mentioned clue in the boxes.

Across

1. The sketch of a solid in which the measurements are kept proportional.
3. Two or more lines which remain apart at a constant distance, even if extended indefinitely.
5. The 3-D figure which has a Joker's cap.
7. A 2-D figure which has unlimited lines of symmetry and an infinite order of rotation.
9. The solid which has 5 faces- 3 of which are rectangles and 2 are triangles.

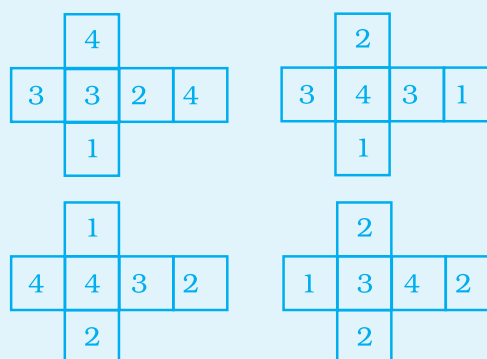
Down

2. The fixed point around which the object is rotated.
4. The solid shape which does not have a vertex or edge.
6. The line where two faces of a 3-D figure meet.
8. The skeleton 2-D figure which when folded results in a 3-D shape.
10. Shadow of a cube.



2. Crazy Cubes

Make four cubes with paper and tape, numbering each face as shown.



The goal is to line up the cubes so that 1, 2, 3 and 4 can be seen along the top, bottom, front and back of the row of cubes. They can be in any order, and the numbers do not have to be right side up.

ANSWERS

Unit 1

1. (d) 2. (c) 3. (d) 4. (d) 5. (b) 6. (c)
7. (c) 8. (c) 9. (a) 10. (b) 11. (a) 12. (c)
13. (d) 14. (a) 15. (b) 16. (d) 17. (c) 18. (c)
19. (a) 20. (a) 21. (d) 22. (c) 23. (b) 24. (d)
25. (a) 26. (c) 27. (d) 28. (c) 29. (d) 30. (b)
31. a 32. 0 33. 3140 34. -3, 8, (-8), 8 35. D
36. y, x, z 37. 0 38. 3 39. 11, 5, -55 40. -180
41. 23 42. Whole, Negative 43. Even 44. Positive
45. Negative 46. 1 47. (-1) 48. 50 49. -210
50. 45 51. 12, 5 52. 23, 1, -100, 1 53. 35 54. -47
55. -1 56. -1 57. -2 58. 40 59. Minus
60. Negative integer 61. Multiplication 62. -5 63. 10
64. -45 65. 83 66. -75 67. -1 68. -113 69. -1
70. -1 71. 1 72. True 73. False 74. True 75. False
76. False 77. True 78. True 79. True 80. False 81. True
82. True 83. True 84. False 85. False 86. False 87. True
88. False 89. True 90. False 91. False 92. True 93. True
94. False 95. True 96. True 97. False 98. True 99. True
100. False 101. False 102. True 103. True 104. True 105. False
106. False 107. True 108. False
109. (a) $-5 \times 2 = \underline{-10} = -15 - (-5)$
 $-5 \times 1 = \underline{-5} = \underline{-10} - \underline{(-5)}$

$$-5 \times 0 = 0 = \underline{-5} - \underline{(-5)}$$

$$-5 \times -1 = 5 = \underline{0} - \underline{(-5)}$$

$$-5 \times -2 = \underline{10} = \underline{5} - \underline{(-5)}$$

$$(b) 7 \times 3 = \underline{21} = \underline{28} - \underline{7}$$

$$7 \times 2 = \underline{14} = \underline{21} - \underline{7}$$

$$7 \times 1 = \underline{7} = \underline{14} - \underline{7}$$

$$7 \times 0 = \underline{0} = \underline{7} - \underline{7}$$

$$7 \times \underline{-1} = \underline{-7} = \underline{0} - \underline{7}$$

$$7 \times -2 = \underline{-14} = \underline{-7} - \underline{7}$$

$$7 \times -3 = \underline{-21} = \underline{-14} - \underline{7}$$

110. (a) 0 (b) +1 (c) -1 **111.** -1, -10, +3, -2

112. (a) 725 years (b) 71 years (c) 1383BC (d) Archimedes

113. Antarctica, Asia, N. America, Europe, S. America, Africa, Australia.

114. -2, 6 **115.** $-5 \rightarrow 3$, $6 \rightarrow -2$, $-7 \rightarrow 1$, $8 \rightarrow -1$, **116.** -3, 12

117. (a) \rightarrow (vi), (b) \rightarrow (iii), c \rightarrow (v), d \rightarrow (vii), e \rightarrow (viii), f \rightarrow (iv)
g \rightarrow (ii), h \rightarrow (ix), i \rightarrow (i)

118. ₹ $[500 + 200 + 150 - 120 - 240] = ₹ 490$

119. (a) A number of solutions can be possible e.g., $4 + (-6) = -2$

(b) A number of solutions can be possible e.g., $8 + (-2) = 6$

(c) A number of solutions can be possible e.g., $-7 - (2) = -9$

(d) A number of solutions can be possible e.g., $4 - (-3) = 7$

(e) A number of solutions can be possible e.g., $-12 - (-7) = -5$

(f) A number of solutions can be possible e.g., $-4 + (-7) = -11 < -10$

(g) A number of solutions can be possible e.g., $-1 - 4 = -5 < -4$

(h) A number of solutions can be possible e.g., $-8 - (-9) = 1 > -6$

(i) A number of solutions can be possible e.g., $-2 - (-10) = 8$

(j) A number of solutions can be possible e.g., $-20 - (-9) = -11$

(k) A number of solutions can be possible e.g., $-3 \times 5 = -15$

(l) A number of solutions can be possible e.g., $4 \times 6 = 24$.

120. Ramu went wrong in solving $-(-3)$ and took it as -3 only.

121. Reeta went wrong in sloving $+(-6)$ and took it as $+6$.

122. (a) C (b) D (c) A, C, B, D **123.** 356 m. **124.** (i) -3561

(ii) -4300 (iii) 5300 (iv) -1360 **125.** (i) 49 (ii) 28

126. (i) $4 \Delta (-3) = 21$, $(-3) \Delta 4 = 28$, No

(ii) $(-7) \Delta (-1) = -6$, $(-1) \Delta (-7) = 42$, No

127. (a) $v = 1$

(b) $w = 0$

(c) $x = 4$

128. 2500m

129. Hydrogen -259°C , Krypton -157°C ,

Oxygen -223°C , Helium -272°C , Argon -189°C **130.** Fatima.

131. Net profit ₹ 27 **132.** (i) 10 (ii) 30 **133.** Since Yash scored 94 marks So, Minimum correct responses = $94 \div (+2) = 47$, Two possibilities are there:

1. Correct answer 47, unattempted 3

2. Correct answer 48, unattempted, wrong answer 1

134. 60 sec or 1 min

135. 23rd January

136. 19,759 m

(D)

Puzzle 1

(i)

-1	-9	4
3	-2	-7
-8	5	-3

(ii)

7	-2	-6	-1
-4	-3	1	4
0	-2	-3	3
-5	5	6	-8

Puzzle 2

(i) -10

(iv) -3

(ii) 8

(v) -33

(iii) 7

(vi) 18

Increasing order

$$-33 < -10 < -3 < 7 < 8 < 18$$

E U C L I D

Puzzle 3

Solution: September

Puzzle 4

(a)

Puzzle 5

(a) 6 (b) -2 (c) -8

Puzzle 6

Arrange -12 in the centre and -2, 4, -5, 50, -25, 20 in clockwise order.

Unit 2

1. (b) 2. (c) 3. (c) 4. (b) 5. (d) 6. (a)
7. (c) 8. (b) 9. (d) 10. (c) 11. (b) 12. (d)
13. (c) 14. (d) 15. (c) 16. (b) 17. (d) 18. (a)
19. (c) 20. (b) 21. $\frac{1}{7}$ 22. $\frac{7}{3}$ 23. 18 24. 36
25. $\frac{76}{3}$ or $25\frac{1}{3}$ 26. $\frac{15}{7}$ 27. $\frac{2}{15}$ 28. $\frac{17}{9}$ or $1\frac{8}{9}$ 29. $\frac{1}{5}$
30. 10 31. X 32. 32 33. 25400 34. 9350 35. 0.47
36. 0.047 37. 0.0047 38. Less 39. multiply, reciprocal 40. 4
41. 100 42. X 43. X 44. 667 45. False 46. False
47. False 48. False 49. True 50. True 51. True 52. True
53. False 54. False 55. Yes, increase
56. The value of fraction would increase 57. D 58. 26.25 59. $\frac{2}{5}$
60. $\frac{5}{12}$ part 61. 24 pages 62. $\frac{5}{14}$ 63. Greater than 1.5
64. convert both into (1) decimals (2) fractions
65. (a) $\frac{16}{25}$ gram (b) $\frac{2}{5}$ gram 66. (a) 1 tsp (b) $1\frac{1}{2}$ tsp (c) 2 tsp
67. 24 boxes 68. 142 book marker 69. (a) 11.74 cm (approximatly)
- (b) 11.14cm (approximatly) 70. (a) 10.15 cm (b) 6.10 cm

71. (a) 58.718 cm (b) 40.506 cm 72. ₹ 1471.25 73. (a) D, (b) E
- (c) $\frac{3}{6}$ or $\frac{1}{2}$ or middle 74. 741.6 km (approximety) 75. 1
76. $\frac{27}{125}$ 77. $\frac{18}{31}$ 78. 2 79. 64 80. ₹ 114.30 81. 4.5°F
82. (i) 1964, 1965, 1978, 1958, 2002
(ii) 1946 should fall between 1965 and 1978
83. (a) 14.9920 (b) 11.9970 (c) 2.9950
84. Ravi + 0.01 cm, Kamal - 0.08 cm, Tabish - 0.06 cm
85. 7.41 86. 70720 87. ₹ 104625 88. $\frac{1}{4}$ m 89. 90 bricks
90. $14\frac{1}{4}$ m 91. first usher 92. ₹ 23.15
93. 3.27 minutes 94. 11 days 95. 0.93 kg
96. (a) 90 (b) 74 (c) 50 97. $\frac{7}{8}$ L
98. $\frac{1}{6}$ part of work, $\frac{5}{6}$ part of work, complete work
99. $\frac{1}{5}$, $\frac{23}{25}$, $\frac{7}{10}$ 100. 5 pillows
101. 4 shirts 102. 3 hours 103. 600 km 104. ₹ 200
105. (i) (a) $\frac{5}{13}$ (b) $\frac{10}{13}$ (ii) (c) 7 tonnes 106. 5.1875
107. (1) → (d) (2) → (f) (3) → (c) (4) → (b) (5) → (a) (6) → (e)
108. 0.05 109. 2.4 110. 24.15 111. $\frac{20}{3}$ cm or $6\frac{2}{3}$ cm 112. $\frac{1}{3}$
113. $305\frac{19}{25}$ cm² 114. ₹ 300 115. 76 m 116. 10.816
117. Greater than 1: $\frac{2}{3} \div \frac{1}{2}$, $6 \div \frac{1}{4}$, $4\frac{1}{3} \div 3\frac{1}{2}$, $\frac{2}{3} \times 8\frac{1}{2}$

Less than 1: $\frac{2}{3} \div \frac{2}{1}$, $\frac{1}{5} \div \frac{1}{2}$

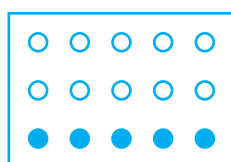
118. 37.5 **119.** $\frac{7}{648}$ **120.** $\frac{3}{2}$ **121.** 500 **122.** 0.00001

123. Error $-0.30 > -0.25$

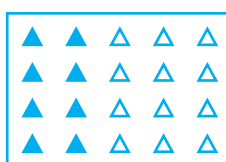
124. mixed fractions are not converted into improper fraction. **125.** $\frac{1}{7}$

(D)

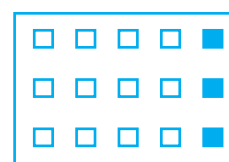
1.



(a)



(b)



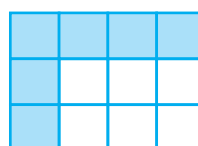
(c)

2.

(i) $\frac{1}{4} \times \frac{1}{3}$



$\frac{1}{3}$



$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

(ii) $\frac{1}{3} \times \frac{1}{5}$

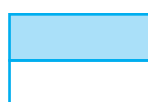


$\frac{1}{5}$



$\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

(iii) $\frac{1}{2} \times \frac{1}{5}$

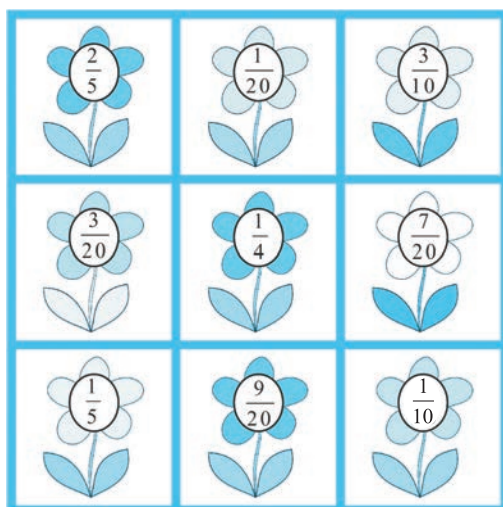


$\frac{1}{5}$



$\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$

3.

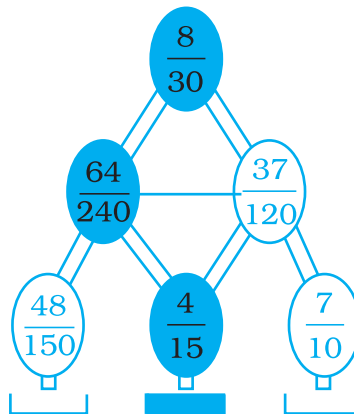


4. Sleep→8hrs, Study→3hrs, Meals→2hrs, School→7hrs and Personal time 4 hrs.

5.

Sl.No.	Ingredients	Given for One Cake	Triple Amount of Cake	Half Amount of Cake
(a)	Sugar	2 Cups	6 Cups	1 Cup
(b)	Milk	$\frac{3}{4}$ Cup	_____	_____
(c)	Coconut	1 Cup	_____	_____
(d)	Salt	$\frac{1}{8}$ Teaspoon	_____	_____
(e)	Cocopowder	1 Tablespoon	_____	_____
(f)	Butter	$1\frac{1}{2}$ Tablespoon	_____	_____
(g)	Eggs	2	_____	_____

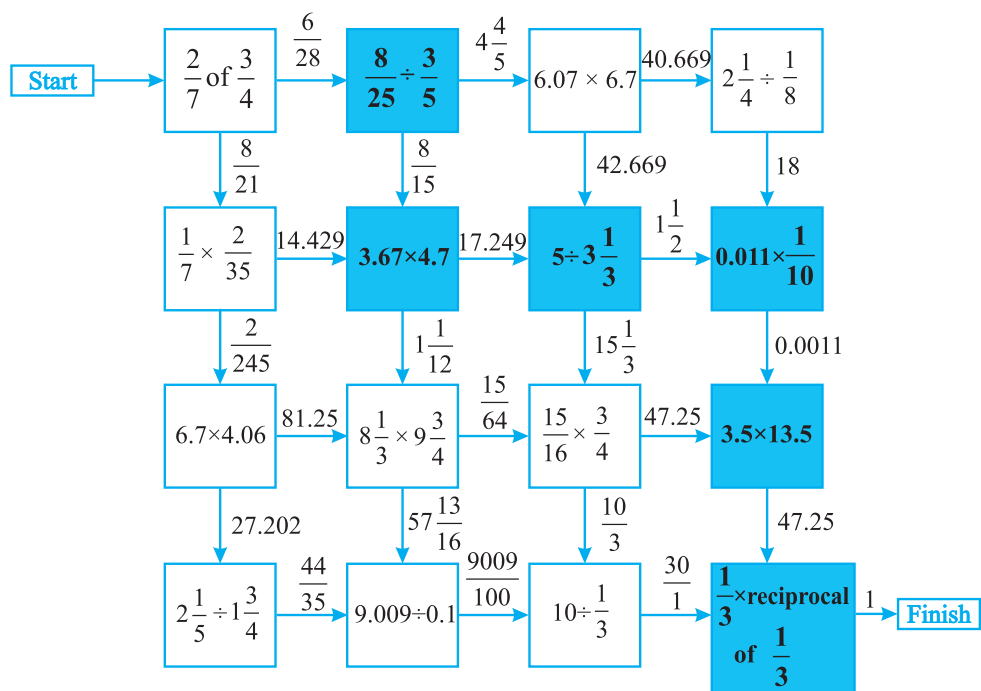
7.



8.

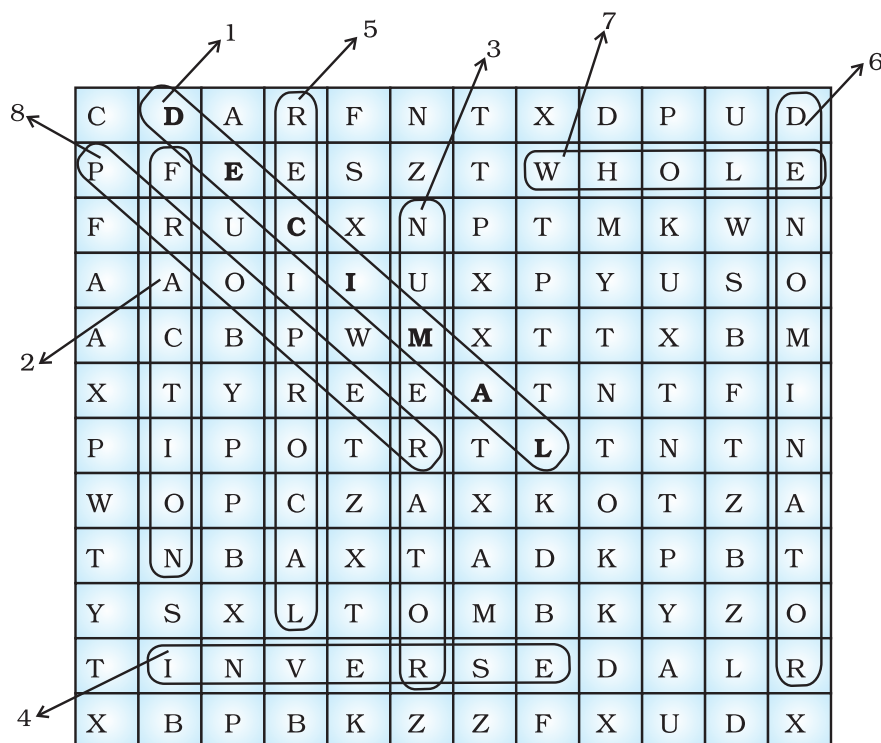
Box 1	Box 2	Box 3
0.096	0.376	1.808
0.001	0.4200	0.987
0.066	0.62	11.00
0.0864	0.578	0.888

9.



10. 1. 20 cm
2. $\frac{40}{9}$ cm
3. Length of bottom of vertical support = 9 cm
Length of upper of vertical support = 3 cm

11.



12.

- Across** : 1. Proper
2. Denominator
3. Equivalent
4. Greater
5. Improper
6. One

- Down** : 1. Product
2. Decimal
8. Reciprocal
9. Fraction

13.

- (i) $\frac{1}{2} + \frac{1}{4}$ (ii) $\frac{1}{8} + \frac{1}{2}$ (iii) $\frac{1}{2} + \frac{1}{3} + \frac{1}{12}$ (iv) $\frac{1}{3} + \frac{1}{11} + \frac{1}{231}$ (v) $\frac{1}{1} + \frac{1}{5} + \frac{1}{15}$

Unit 3

1. (b) 2. (c) 3. (a) 4. (a) 5. (c) 6. (b)
7. (c) 8. (b) 9. (c) 10. (b) 11. (b) 12. (d)
13. (a) 14. (d) 15. (c) 16. (d) 17. Range
18. $\frac{\text{Sum of all observations}}{\text{Number of observations}}$ 19. Mode 20. Median
21. Central tendency 22. 1 23. 0 24. 1 25. 6
26. A double bar graph 27. Bar graph 28. 3
29. Minimum, Maximum 30. Odd 31. 52–55 32. False 33. True
34. False 35. False 36. True 37. True 38. False 39. True
40. False 41. False 42. True 43. True 44. False 45. False
46. False 47. True 48. False 49. False 50. 10, 10, 10, Yes
51. 11
52. (Mode is the observation that occurs most frequently in a set of observation).
53. (a) Black (b) Mode 54. (a) 25 (b) 30.41 (c) 33 55. (a) 65.6
- (b) 4 (c) 30 56. (a) 1 (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) 0 57. 4 58. 4.5 59. One
60. Blue 61. $\frac{4}{7}$ 62. $\frac{1}{6}$
63. (a) Impossible to happen.
(b) May or may not happen.
(c) May or may not happen.
(d) Certain to happen.
(e) Impossible to happen.
(f) Certain to happen.
64. Mean = 3.13, Median = 3, Mode = 2 65. 14 66. 10
67. 11.14 68. 8
69. (a) 154 cm
(b) 128 cm
(c) 26 cm
(d) 142 cm

- 70.** (a) 8 or 17 or 16 (except 15)
 (b) Two times 15
 (c) Three times 17
- 71.** (a) Group A Mode = 7 and 10
 Range = 3
 Group B
 Mode = 12
 Range = 5
 (b) Range = 5, Mode = 7 and 12
- 72.** (a) Production of motor bikes by XYZ Automobiles Ltd. during January to June.
 (b) 2100 (c) 300
 (d) June, 500 (e) 767 bikes (nearest whole numbers)
- 73.** (a) 4 (b) 18
 (c) 4 (d) 10
 (e) 42
- 74.** (a) The production of rice (in million tonnes) by a country during the years 2005 to 2009.
 (b) 2006 (c) 2006
 (d) 54 million tonnes (e) 10 million tonnes
- 75.** (a) Marks obtained by a students in different subjects.
 (b) Maths (c) 68.2
 (d) Hindi, Maths (e) 68.2%
- 76.** (a) 1800 (b) 300
 (c) Tamil (d) 2300
- 77.** (a) Cricket (b) 17
 (c) 65 (d) Cricket
 (e) 4 sports (hockey, football, tennis, badminton)
 (f) 14 : 7 or 2 : 1
- 78.** (a) Comparison of sales of brand A and brand B during the month of January to June.
 (b) March (c) 3 Lakh
 (d) 41.8 Lakh (e) April, June
 (f) 31 : 36
- 79.** (a) Comparison of minimum temperature during the months November to February for the years 2008 and 2009.
 (b) 18 :15 or 6:5 (c) Two February and November
 (d) 11.25 (e) February

- 80.** Give the double bar graph here
- 81.** (a) number of students (boys and girls) in different section of Class VII.
 (b) 110 boys (c) Sections VII A and VII D
 (d) VII B (e) VII C
- 82.** (a) Give the double bar graph here
 (b) Thursday (c) 200
- 83.** (a) Give the double bar graph here
 (b) VIII (c) X
 (d) 13 :14 (e) 10%
- 84.** (a) Give the bar graph here
 (b) Saturday (c) 267
 (d) 9:20 (e) 44.5
 (f) 4 days (Monday, Tuesday, Thursday, Saturday)
- 85.**
- | (a) Building | Height No. of stories | Height of each storey | |
|------------------------|-----------------------|-----------------------|------|
| MVRDC | 156 | 35 | 4.45 |
| Oberoi woods tower II | 170 | 40 | 4.25 |
| Oberoi woods tower III | 170 | 40 | 4.25 |
| RNA Nirage | 180 | 40 | 4.25 |
| Planet Godrej | 181 | 51 | 3.5 |
| UB Tower | 184 | 20 | 9.2 |
| Ashok Tower | 193 | 49 | 3.9 |
| The Imperial I | 249 | 60 | 4.15 |
| The Imperial II | 249 | 60 | 4.15 |
- 86.** (a) Give bar graph here
 (b) 84% (c) 81.6%
 (d) 34 : 35
 (e) Three subjects(English, Hindi and S.Sc.)
 (f) Soni, 11 marks
 (g) In English and Science 14 marks.
- 87.** (a) Give the double bar graph here
 (b) 210 (c) Electronics
 (d) Yoga (e) Yoga, Dramatics
 (f) Fine Arts

- 88.** (a) Give the double bar graph here
 (b) In year 2007 (c) 4420
 (d) May (e) August
 (f) February
- 89.** (a) Give the double bar graph here
 (b) Town D (c) Town A
- 90.** (a) Give double bar graph here
 (b) Mussoorie (c) Manali
 (d) Manali, Nainital, Mussoorie, Kullu
- 91.** (a) Give double bar graph here
 (b) Butterscotch (c) 46 (d) 21 (e) 5 : 6

Unit 4

- 1.** (c) **2.** (a) **3.** (d) **4.** (d) **5.** (c) **6.** (d)
7. (b) **8.** (d) **9.** (a) **10.** (a) **11.** (c) **12.** (d)
13. (c) **14.** (c) **15.** (b) **16.** (a) **17.** (c) **18.** (b)
19. (a) $60 - x$ (b) $60 - 2x$ (c) $-2x = 30$
 (d) 15 (e) 45, 15
20. (a) $81 - x$ or $2x$ (b) $2x = 81 - x$ (c) $x = 27$ (d) 54, 27
21. (a) $2x$ (b) $4x + 3x = 280$ (c) $x = 40$ (d) 80
22. (a) $2x$ (b) $6x$ or $2(2x + x)$ (c) $6x = 60$ (d) $x = 10$
23. (a) ₹ $5x$ (b) ₹ $2x$ (c) $5x + 2x = 70$ (d) 10, 10
24. (a) $30 - x$ (b) $2000x + 1000(30 - x)$ (c) $1000x + 30000 = 52000$
 (d) $x = 22$ (e) 22, 8 **25.** 2 **26.** $x = 3$ **27.** $x = -1$ **28.** 5
29. No **30.** No **31.** No **32.** One **33.** $3x + 5 = 4x - 7$
34. $x = 3$ **35.** 4 **36.** 0 **37.** -3 **38.** 4
39. Satisfies, root **40.** sign **41.** 2 **42.** 7 **43.** 0
44. 0 **45.** 75 **46.** 25 **47.** 72 **48.** $\frac{7}{4}$ **49.** True
50. False **51.** False **52.** False **53.** True **54.** False **55.** False
56. (i) \leftrightarrow (C) (ii) \leftrightarrow (E) (iii) \leftrightarrow (F) (iv) \leftrightarrow (D) (v) \leftrightarrow (B) (vi) \leftrightarrow (A)

57. $2x - 13 = 3$ 58. $\frac{x}{5} = x - 5$ 59. $x = 7 + \frac{x}{3}$ 60. $6x = 10 + x$

61. $\frac{x}{2} - 10 = 4$ 62. $p - 5 = 2$ 63. $5x + 7 = 27$ 64. $x + (x + 3) = 43$

65. $\frac{1}{2}(x-1)=7$ 66. $\frac{x}{2}+5=9$ 67. $2x+4=18$ 68. 9 years

69. 30, 42 70. 2 71. ₹ 20 72. ₹ 425 73. 560

74. 2 75. 2 76. 6 77. $6\frac{1}{4}$ years

78. 5 years 79. 18 years 80. 18 81. 16 kg, 64 kg 82. 72

83. 6 84. 4, 8 85. 1, 2, 3 86. 36 87. 16 m

88. 6 cm, 12 cm, 12 cm 89. 8, 10 90. 35° , 55° 91. 50, 100

92. 45, 15 93. 9 94. 50 95. 180 km 96. 9.6

97. 6 98. 11 years, 39 years 99. width = 30 cm, length = 60 cm

100. ₹ 30 101. 1867 102. ₹ 13740 103. 16

104. (a) $X - V = V$ (b) $VI + IV = X$, $VI + V = XI$ 105. $i = 1$, $u = 4$,

$a = 5$, $q = 3$, $t = 2$, $s = 8$, $p = 9$, $c = 6$, $k = 7$ 106. $\Delta = 7$, $* = 4$

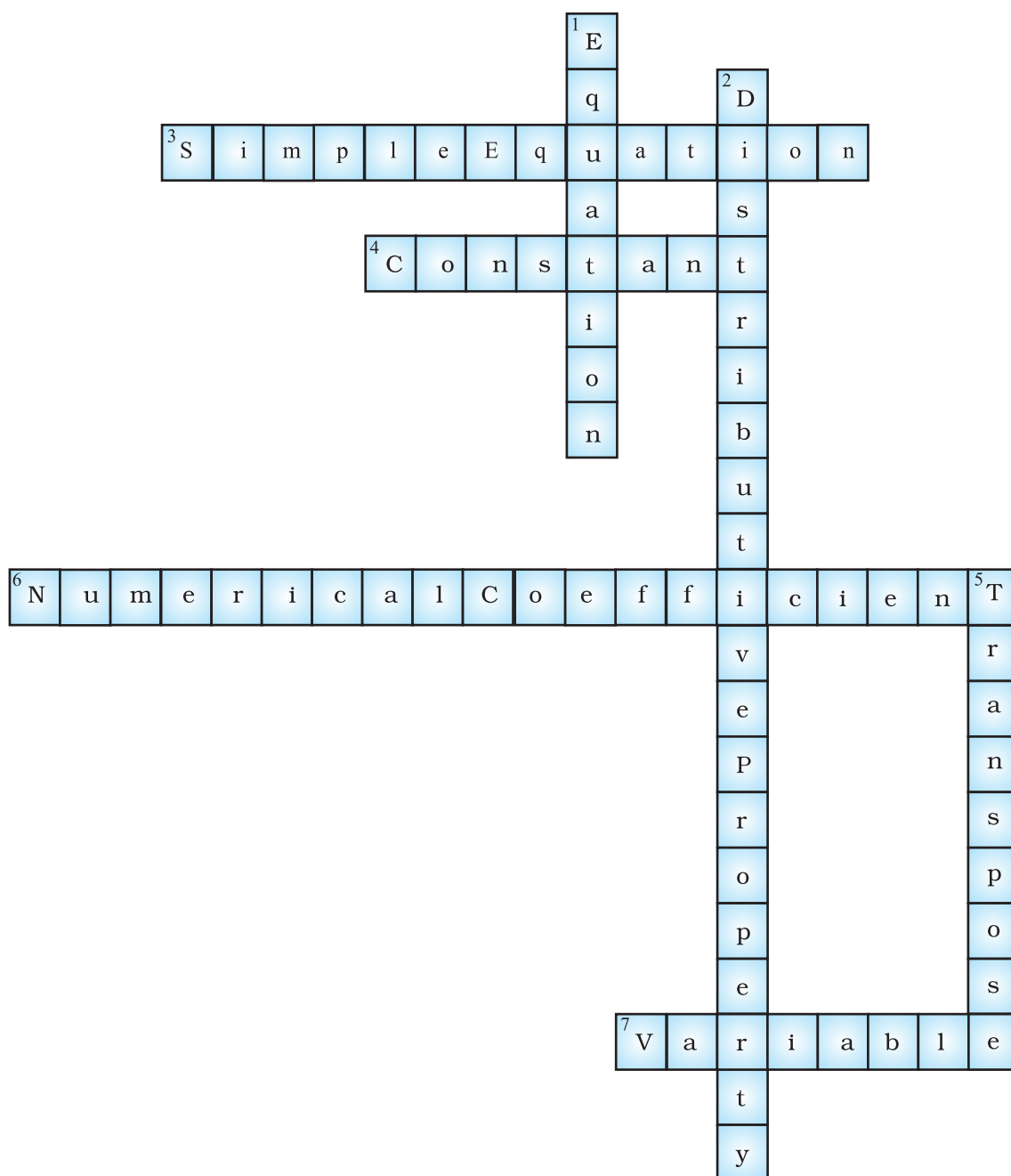
107.  = 6 kg,  = 10 kg

(D)

1.

⁶ 1					
¹ 9			¹⁰ 7	8	⁴ 3
	⁵ 5	² 5			0
	³ 1	0	⁹ 8		0
⁸ 4			⁷ 4	9	0
3		¹¹ 1			
	¹² 1	2	5		

2.



3.

First you must split the pearls into equal groups. Place any three pearls on one side of the scale and any other three on the other side. If one side weighs less than the other, then the fake pearl is on that side. But you are

not done yet! You still need to find the imitation, and you can use the scale only once more. Take any of the two pearls from the lighter pan, and weigh them against each other. If one pan is lighter, then that pan contains the fake pearl. If they balance, then the leftover pearl of the group is the fake.

If the scale balances during the first weighing, then you know the fake is in the third group. Then you can choose two pearls from that group for the second weighing. If the scale balances, the fake is the one left. If it is unbalanced, the false pearl is the lighter one.

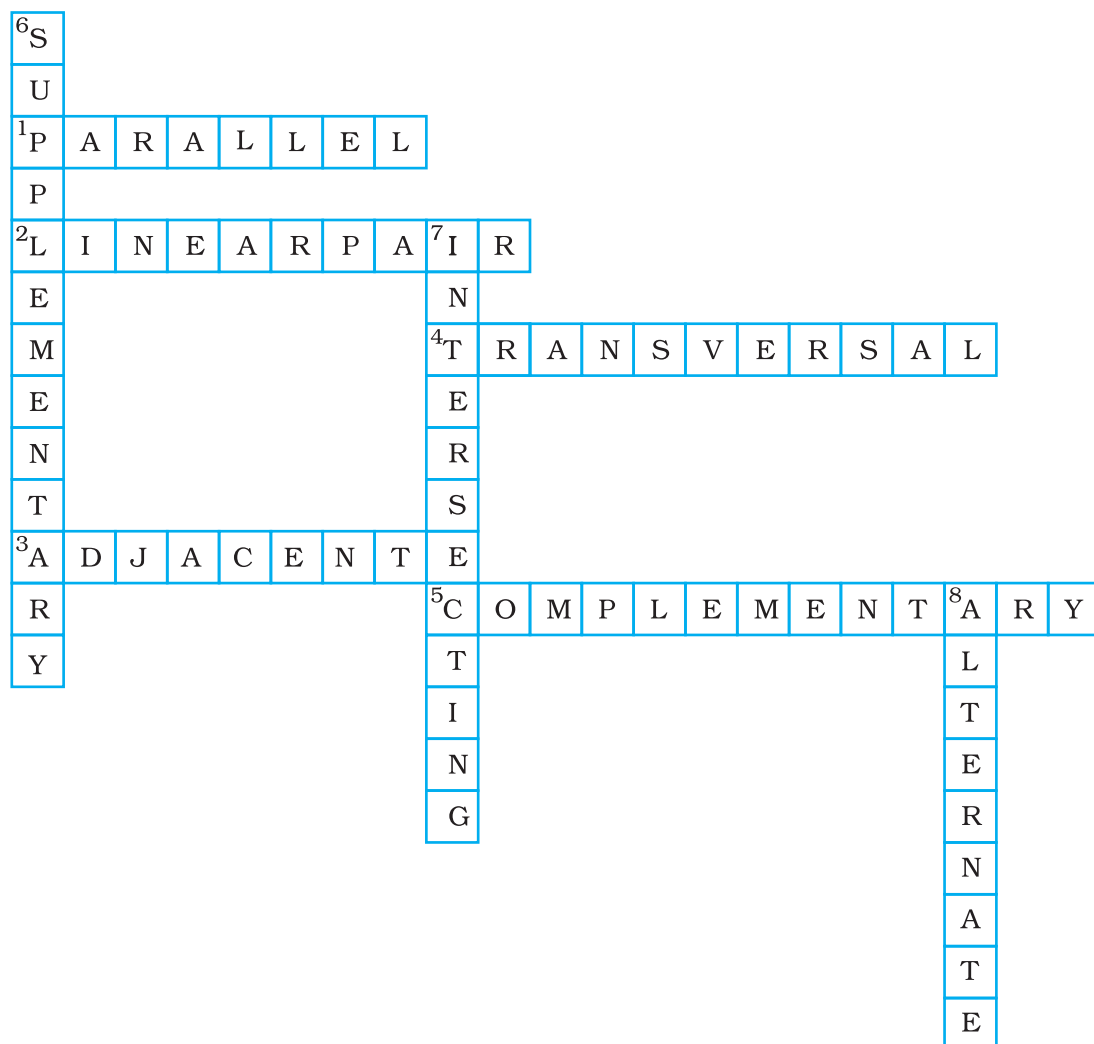
Unit 5

- | | | | | | |
|---|--------------------------|------------------------------|---------------------|-----------------------|------------------|
| 1. (b) | 2. (c) | 3. (b) | 4. (b) | 5. (b) | 6. (d) |
| 7. (b) | 8. (d) | 9. (d) | 10. (c) | 11. (d) | 12. (a) |
| 13. (a) | 14. (a) | 15. (b) | 16. (c) | 17. (b) | 18. (a) |
| 19. (a) | 20. (c) | 21. (a) | 22. (b) | 23. (b) | 24. (a) |
| 25. (d) | 26. (d) | 27. (c) | 28. (d) | 29. (b) | 30. (b) |
| 31. (c) | 32. (d) | 33. (a) | 34. (a) | 35. (c) | 36. (b) |
| 37. (d) | 38. (d) | 39. (c) | 40. (b) | 41. (a) | |
| 42. Complementary | 43. Supplementary | | 44. Distinct | | |
| 45. 180° | 46. Arm | 47. Same | 48. Opposite | 49. Parallel | |
| 50. Linear | 51. Obtuse | 52. Right angle | 53. Acute | 54. 90° | |
| 55. 45° | 56. 60° | 57. False | 58. False | 59. False | 60. True |
| 61. True | 62. True | 63. False | 64. True | 65. True | 66. False |
| 67. False | 68. True | 69. False | 70. True | 71. False | |
| 72. | | | | | |
| (i) (a) $\angle AOB, \angle BOC$ | | (b) $\angle AOB, \angle BOD$ | | | |
| (c) $\angle BOC, \angle COD$ | | (d) $\angle AOC, \angle COD$ | | | |
| (ii) (a) $\angle PQR, \angle PQT$ | | (b) $\angle SPR, \angle RPQ$ | | | |
| (c) $\angle PRQ + \angle QRU$ | | | | | |
| (iii) (a) $\angle TSV, \angle VSU$ | | (b) $\angle SVU, \angle SVT$ | | | |
| (iv) (a) $\angle AOC, \angle AOD$ | | (b) $\angle AOD, \angle BOD$ | | | |
| (c) $\angle BOD, \angle BOC$ | | (d) $\angle BOC, \angle AOC$ | | | |
| 73. (a) (i) $\angle 1, \angle 3; \angle 2, \angle 4; \angle 5, \angle 7; \angle 6, \angle 8$ | | | | | |

- (ii) $\angle 1, \angle 2$; $\angle 2, \angle 3$; $\angle 3, \angle 4$; $\angle 4, \angle 1$; $\angle 5, \angle 6$; $\angle 6, \angle 7$;
 $\angle 7, \angle 8$; $\angle 8, \angle 5$
- (b) (i) NIL (ii) NIL
- (c) (i) _____ NIL
- (ii) $\angle ABD, \angle DBC$; $\angle ABE, \angle EBC$
- (d) (i) $\angle ROQ, \angle POS$; $\angle ROP, \angle QOS$
(ii) $\angle ROP, \angle POS$; $\angle ROT, \angle TOS$; $\angle QOS, \angle SOP$; $\angle QOT, \angle TOP$;
 $\angle ROQ, \angle QOS$; $\angle ROQ, \angle ROP$
- 74.** (i) $\angle AOD, \angle AOC$; $\angle AOC, \angle BOC$; $\angle BOC, \angle BOD$; $\angle AOD, \angle BOD$
(ii) $\angle POS, \angle SOQ, \angle POR, \angle QOR$
(iii) $\angle 1, \angle 2$; $\angle 3, \angle 4$; $\angle 5, \angle 6$
- 75.** $\angle QUR = 138^\circ$ **76.** (a) 4 (b) 4 (c) (i) $45^\circ, 45^\circ$ (ii) $60^\circ, 30^\circ$
- 77.** 83° **78.** 90°
- 79.** (a) $\angle TQS, \angle SQR$
(b) $\angle SQR, \angle SQP$; $\angle TQR, \angle TQP$;
(c) $\angle SQR, \angle SQT$; $\angle TQR, \angle TQP$; $\angle SQT, \angle TOP$; $\angle PQS, \angle SQR$
- 80.** (i) $\angle x, \angle y$; $\angle x + \angle y, \angle z$; $\angle y, \angle z, \angle y + \angle z, \angle x$
(ii) $\angle x = \angle y = \angle z, \angle x, \angle y, \angle y, \angle z, \angle z, \angle x$
- 81.** (a) 13
(b) Linear pair, Supplementary, Vertically opposite. Angles, Adjacent angles.
(c) Vertically opposite angles – (1, 3); (2, 4)
Linear Pairs: 1,2; 2,3; 3,4; 4,1.
- 82.** (a) Yes (b) No (c) No (d) No
- 83.** $\angle 7, \angle 2$; $\angle 1, \angle 8$; $\angle 5, \angle 6$; $\angle 6, \angle 3$; $\angle 3, \angle 4$; $\angle 4, \angle 5$
- 84.** (a) obtuse
(b) acute
(c) right angle
- 85.** No **86.** $\angle 1, \angle 2$; $\angle 2, \angle 3$; $\angle 3, \angle 4$; $\angle 4, \angle 1$. **87.** 152°
- 88.** $\angle a = 30^\circ, \angle b = 150^\circ, \angle c = 150^\circ$ **89.** $\angle x = 35^\circ, \angle y = 145^\circ$
- 90.** (i) 30° (ii) 105° (iii) 75° (iv) 75°
- 91.** $\angle x = 60^\circ, \angle y = 120^\circ, \angle z = 60^\circ$ **92.** $\angle EFD = 70^\circ$
- 93.** $\angle AOD = 139^\circ$ **94.** 110° **95.** $44^\circ, 46^\circ$ **96.** $100^\circ, 80^\circ$

- 97.** $45^\circ, 135^\circ$ **98.** $89^\circ, 91^\circ$ **99.** $60^\circ, 120^\circ$ **100.** 40°
101. $67^\circ, 48^\circ$ **102.** 396° **103.** $65^\circ, 70^\circ$ **104.** 100°
105. (i) 142° (ii) 45° **106.** 281° **107.** $114^\circ, 132^\circ$
108. $20^\circ, 40^\circ, 30^\circ$ **109.** $m \parallel n$ **110.** (i) No, (ii) yes **111.** $EF \parallel GH$
113. $110^\circ, 100^\circ$

(D) 2.



Unit 6

- 1.** (d) **2.** (c) **3.** (b) **4.** (c) **5.** (d) **6.** (c)
7. (c) **8.** (c) **9.** (c) **10.** (a) **11.** (c) **12.** (b)
13. (b) **14.** (c) **15.** (c) **16.** (d) **17.** (a) **18.** (d)
19. (b) **20.** (c) **21.** (c) **22.** (c) **23.** (b) **24.** (a)
25. (c) **26.** (b) **27.** (d) **28.** (b) **29.** (c) **30.** (d)
31. (c) **32.** (b) **33.** (a) **34.** (d) **35.** (d) **36.** (b)
37. (b) **38.** (d) **39.** (d) **40.** (c) **41.** (b) **42.** (b)
43. (b) **44.** (d) **45.** (b) **46.** (c) **47.** (a) **48.** (b)
49. (c) **50.** Obtuse **51.** a right angle **52.** hypotenuse
53. Altitude **54.** 60° **55.** equal **56.** equal **57.** 90° **58.** two
59. equal **60.** congruent **61.** Length and breadth **62.** side
63. (i) $\angle Z$ (ii) XZ (iii) $\angle Y$ (iv) XY (v) X (vi) ZY **64.** $\triangle XZY$
65. $\triangle RSP$ **66.** $\triangle DRQ$ **67.** $\triangle PQO$ **68.** (i) $\triangle ADC$, (ii) DC, (iii) $\angle DCA$,
(iv) $\angle BAD$ and $\angle BCD$ **69.** (i) $\angle PQR + \angle PRQ$ (ii) $\angle QRP + \angle QPR$
70. False **71.** False **72.** True **73.** False **74.** False **75.** False
76. False **77.** True **78.** False **79.** False **80.** True **81.** False
82. False **83.** True **84.** False **85.** False **86.** False **87.** True
88. False **89.** True **90.** False **91.** True **92.** True **93.** True
94. False **95.** True **96.** True **97.** True **98.** False **99.** True
100. False **101.** True **102.** False **103.** False **104.** True **105.** False
106. False **107.** $100^\circ, 60^\circ, 20^\circ$ **108.** 35° **109.** (i) $a = 20^\circ$,
 $b = 130^\circ, c = 50^\circ$, (ii) $a = 65^\circ, b = 115^\circ, c = 25^\circ$ **110.** $y = 30^\circ$
111. $\angle A = 30^\circ$ **112.** Triangle, Obtuse angled triangle **113.** 10 km
114. 40 m **115.** $\angle Q = 75^\circ, \angle R = 75^\circ$ **116.** $\angle x = 75^\circ, \angle y = 135^\circ$
117. $\angle PON = 90^\circ, \angle NPO = 20^\circ$ **118.** $x = 70^\circ, y = 80^\circ$ **119.** 50°

120. 55° **121.** $20^\circ, 80^\circ, 80^\circ$ **122.** $36^\circ, 54^\circ, 90^\circ$ **123.** 360°

124. $\angle B = 40^\circ, \angle A = 70^\circ, \angle C = 70^\circ$ **125.** $x = 80^\circ, y = 75^\circ$

126. $x = 20^\circ$ **127.** $70^\circ, 60^\circ, 50^\circ$ **128.** $x = 30^\circ, y = 40^\circ, z = 110^\circ$

129. $x = 60^\circ, y = 120^\circ, z = 30^\circ$ **130.** 40° and 80°

131. $\angle P = 80^\circ, \angle Q = 60^\circ, \angle R = 40^\circ$ **132.** (i) $\angle F = 50^\circ$ (ii) $\angle EOF = 120^\circ$

133. $\angle S = 90^\circ$

134. (a) not possible (b) $\triangle ABD \cong \triangle CDB$

(c) $\triangle XYZ \cong \triangle LMN$ (d) not possible

(e) $\triangle MNO \cong \triangle PON$ (f) $\triangle AOD \cong \triangle BOC$

135. (a) $\triangle ABD \cong \triangle ACD$ (b) $\triangle XYZ \cong \triangle UZY$

(c) $\triangle ACE \cong \triangle BDE$ (d) $\triangle ABC \cong \triangle CDE$

(e) not possible (f) $\triangle LOM \cong \triangle CDE$

136. $x = 50^\circ$ **137.** $y = 51^\circ, x = 129^\circ$ **138.** Yes **139.** 100cm **140.** $z = 160^\circ$

141. 49 cm, sum of two sides should be greater than third side.

142. (a) $\angle S = \angle D, \angle T = \angle E, \angle U = \angle F, ST = DE, TU = EF, SU = DF$

(b) $\angle A = \angle L, \angle B = \angle M, \angle C = \angle N, AB = LM, BC = MN, AC = LN$

(c) $\angle Y = \angle P, \angle Z = \angle Q, \angle X = \angle R, YZ = PQ, ZX = QR, YX = PR$

(d) $\angle X = \angle M, \angle Y = \angle L, \angle Z = \angle N, XY = ML, YZ = LN, XZ = MN$

143. (a) $\triangle ABC \cong \triangle NLM$ (b) $\triangle LMN \cong \triangle GHI$

(c) $\triangle LMN \cong \triangle LON$ (d) $\triangle ZYX \cong \triangle WXY$

(e) $\triangle AOB \cong \triangle DOE$ (f) $\triangle STU \cong \triangle SVU$

(g) $\triangle PSR \cong \triangle RQP$ (h) $\triangle STU \cong \triangle PQR$

144. (a) $AB = AC, BD = CD, AD = AD$

(b) Yes, (SSS)

145. (a) $LM = ON, LN = OM, MN = NM$

(b) Yes, (SSS)

146. Yes, SSS, $\angle N = 40^\circ$ **147.** Yes, (SSS)

148.(i) $\triangle PQR \cong \triangle TUS$ (ii) Not congruent

(iii) $\triangle BCD \cong \triangle BAE$ (iv) $\triangle STU \cong \triangle XZY$

(v) $\triangle DOF \cong \triangle HOC$ (vi) Not congruent

(vii) $\triangle PSQ \cong \triangle RQS$ (viii) $\triangle LMN \cong \triangle OMN$

149.(i) $\triangle PQR \cong \triangle STU$ (ii) Not congruent

150.(i) Yes, (SAS)

(ii) Yes, CPCT

151. Yes, (SAS) **152.** yes, (ASA)

153.(i) Yes, (ASA)

(ii) Yes, CPCT

(iii) Yes, CPCT

154.(i) Yes, (RHS)

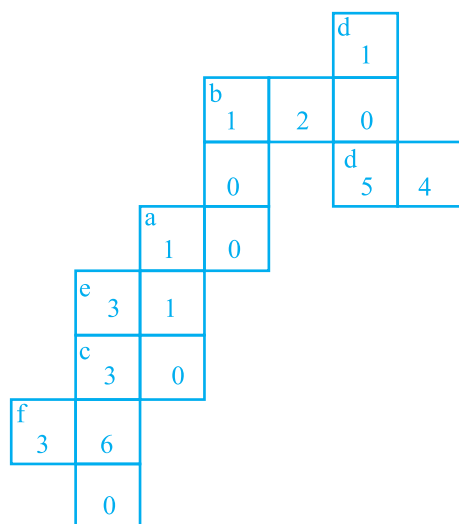
(ii) Yes, CPCT

155. 38m **156.** 12m **157.** 6m

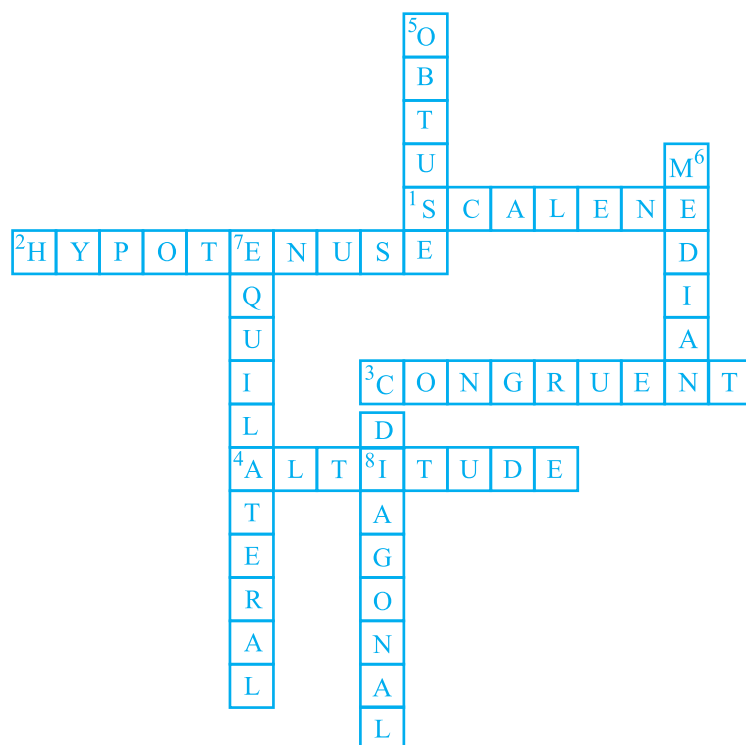
158. $AB = EO$, $\angle ABC = \angle EOD = 90^\circ$, $CA = DE$, yes, (RHS)

(D)

6.



7.



Unit 7

- | | | | | | |
|-----------------------|-----------------------|----------------------|-----------------------|---------------------------|----------------------------|
| 1. (d) | 2. (c) | 3. (a) | 4. (b) | 5. (c) | 6. (a) |
| 7. (c) | 8. (b) | 9. (a) | 10. (b) | 11. (a) | 12. (c) |
| 13. (c) | 14. (d) | 15. (b) | 16. (c) | 17. (d) | 18. (b) |
| 19. (d) | 20. (c) | 21. (c) | 22. (c) | 23. (a) | 24. $66\frac{2}{3}$ |
| 25. 3 : 16 | 26. ₹ 108 | 27. 60 km | 28. 250 | 29. 160 | |
| 30. fraction | 31. 30 | 32. 10 | 33. 46 | 34. 88 | 35. 900 |
| 36. 90 | 37. ₹ 83 | 38. ₹ 96 | 39. ₹ 8100 | 40. $7\frac{1}{7}$ | |
| 41. Profit, 10 | 42. Loss 10 | 43. ₹ 5355000 | 44. Profit, 20 | | |
| 45. Profit, 10 | 46. Profit, 14 | 47. ₹ 900 | 48. ₹ 20800 | | |

- 49.** ₹ 5250 **50.** 0.5 **51.** 60
52. T = Time period, R% = Rate of Interest, P = Principal
53. ₹168 **54.** Multiply **55.** Right **56.** Sum **57.** More
58. attached sheet
59. aattachd seheet
60. True **61.** False
62. False **63.** False **64.** True **65.** False **66.** False
67. False **68.** False **69.** True **70.** False **71.** False **72.** False
73. False **74.** False **75.** True **76.** False **77.** False **78.** False
79. False **80.** (i) 1250, (ii) 1250, (iii) 800, (iv) 900
81. (a) 75 (b) 75 (c) 17;51 (d) 30

82. 16.6% or $\frac{50}{3}\%$ **83.** $22\frac{1}{2}\%$ **84.** 1% **85.** $\frac{4}{5}$ **86.** 1 : 3

87. 1 : 6 **88.** 3 : 2 **89.** 364 **90.** 8.9% **91.** 156%
92. 3 **93.** 6400 **94.** 500 **95.** 50% **96.** 0.069%
97. 100% **98.** 2kg **99.** 125% **100.** 15% **101.** 1200
102. Carbon = 75g, Calcium = 250 g **103.** 96 kg **104.** gain of 18.5%
105. ₹ 7500 **106.** ₹ 800 **107.** ₹ 5760 **108.** 20% by Car,
80% by Train **109.** ₹ 1600 **110.** ₹ 6750 **111.** ₹ 80,000
112. 8 : 25 **113.** ₹ 50,000 **114.** ₹ 12,000 **115.** 82
116. (a) 3:2 (b) 68 mm Hg (c) 259 : 169
117. (a) 9300 cm (b) 36 kg (c) 0.000000085
118. (a) 3 : 2; 3 : 2; 8 : 5; 8 : 3; 9 : 5
(b) 60%; 60%; 61.53% , 72.72%; 64.28%

119. $\frac{6}{10000}$ **120.** $48 \neq 36$ **121.** ₹ 256 **122.** 12.5%

123. 4.5m **124.** Nancy **125.** ₹ 25,000
126. 83% **127.** ₹ 30,000 **128.** ₹ 756
129. (i) \leftrightarrow E, (ii) \leftrightarrow H, (iii) \leftrightarrow O, (iv) \leftrightarrow J, (v) \leftrightarrow G, (vi) \leftrightarrow L, (vii) \leftrightarrow B,
(viii) \leftrightarrow A, (ix) \leftrightarrow F, (x) \leftrightarrow K, (xi) \leftrightarrow D, (xii) \leftrightarrow I, **130.** 25
131. ₹ 6000 and ₹ 4000 **132.** 12.5% **133.** 30 years

134. ₹ 12,000

135. ₹ 5,000

137. 45

138. a) Mean = 1435000km², Median = 475000km², Mode = 3,10,000 km²

b) 4.19

c) 50%

d) 21.1%

139. 44528685 km²

140. Red = 37.5%, Blue 12.5%, Green = 50%

(D)

- | | | | | |
|------------|----------------|----------------|-------------------|-----------|
| (i) | 1. Cost Price, | 2. Interest, | 3. Per cent, | 4. Profit |
| | 5. Principal, | 6. Proportion, | 7. Selling Price, | 8. Amount |

- | | | | | | |
|-------------|---------------|-------------|--------------|---------------|-------------|
| (ii) | Across | Down | (iii) | Across | Down |
| | 1. 20 | 6. 32 | | 1. 50 | 2. 24 |
| | 2. 1520 | 7. 6000 | | 2. 240 | 5. 104 |
| | 3. 72 | 8. 75 | | 3. 5 | 6. 40 |
| | 4. 3000 | 2. 1200 | | 4. 300 | 7. 9 |
| | 5. 25 | 9. 490 | | | |
| | | 10. 9000 | | | |
| | | 4. 385 | | | |
| | | 5. 216 | | | |

Unit 8

- | | | | | | |
|---------------------|---------------------------|--------------------------|---------------------------|---------------------------|---------------------------|
| 1. (d) | 2. (c) | 3. (d) | 4. (b) | 5. (a) | 6. (b) |
| 7. (c) | 8. (c) | 9. (c) | 10. (c) | 11. (b) | 12. (c) |
| 13. negative | 14. positive | 15. $\frac{2}{7}$ | 16. $\frac{-3}{4}$ | 17. left | 18. right |
| 19. smaller | 20. smaller | 21. different | 22. same | 23. $\frac{-2}{3}$ | 24. $-\frac{1}{5}$ |
| 25. - 1 | 26. $\frac{-1}{2}$ | 27. 1 | 28. - 36 | 29. 12 | 30. - 1 |
| 31. < | 32. > | 33. < | 34. < | 35. = | 36. zero |
| 37. 1 | 38. $\frac{9}{49}$ | 39. 0 | 40. 0 | 41. $\frac{-5}{2}$ | 42. - 1 |

- 43.** $b \div m$ **44.** positive, negative **45.** simplest **46.** zero
47. True **48.** True **49.** True **50.** False **51.** True **52.** True
53. True **54.** False **55.** True **56.** True **57.** True **58.** True
59. False **60.** False **61.** True **62.** False **63.** True **64.** False
65. False **66.** (i) \leftrightarrow (c), (ii) \leftrightarrow (e), (iii) \leftrightarrow (a), (iv) \leftrightarrow (b), (v) \leftrightarrow (d)

67. $\frac{-5}{8}, \frac{-15}{28}, \frac{17}{13}$ **68.** (i) $\frac{27}{36}$ (ii) $\frac{-60}{-80}$

69. (i) $\frac{-5}{6}$ (ii) $\frac{-1}{4}$ **70.** (i) $\frac{2}{5}$, (ii) $\frac{-2}{7}$, (iii) $\frac{-3}{7}$, (iv) $\frac{-13}{7}$

- 71.** Yes. Since standard form of $\frac{-8}{28} = -\frac{2}{7}$ and standard form of

$$\frac{32}{-112} = -\frac{2}{7}.$$

72. $\frac{-7}{10}, \frac{2}{-3}, \frac{5}{-8}, \frac{-3}{5}, \frac{-1}{4}.$



74. -20

75. (i) $\frac{-6}{8}, \frac{-9}{12}, \frac{-12}{16}$ (ii) $\frac{14}{22}, \frac{21}{33}, \frac{28}{44}$

76. (i) $\frac{20}{-25}, \frac{24}{30}, \frac{28}{-35}$ (ii) $\frac{-40}{35}, \frac{-48}{42}, \frac{-56}{49}$

77. $\frac{42}{56}, \frac{44}{56}, \frac{46}{56}, \frac{48}{56}$

78. (i) $\frac{127}{143}$, (ii) 1 **79.** (i) $\frac{83}{28}$, (ii) $\frac{9}{13}$

80. (i) $\frac{1}{3}$, (ii) $\frac{42}{11}$ **81.** (i) -13 , (ii) $\frac{3}{7}$

82. (i) $\frac{-55}{49}$ (ii) -2 83. (i) $\frac{7}{8}$ (ii) $3\frac{1}{9}$

84. It has more than one answer like $\frac{-78}{17}, \frac{-79}{18}$.

85. (i) $\frac{-11}{40}, \frac{19}{40}, -\frac{3}{80}, \frac{-4}{15}$

86. (i) $\frac{8}{25}$ (ii) $\frac{4641}{80}$ (iii) $\frac{-4}{15}$ (iv) $\frac{-3}{10}$

87.

+	$-\frac{1}{9}$	$\frac{4}{11}$	$-\frac{5}{6}$
$\frac{2}{3}$	$\frac{5}{9}$	$\frac{34}{33}$	$-\frac{1}{6}$
$-\frac{5}{4}$	$-\frac{49}{36}$	$-\frac{39}{44}$	$-\frac{25}{12}$
$-\frac{1}{3}$	$-\frac{4}{9}$	$\frac{1}{33}$	$-\frac{7}{6}$

88. $\frac{6}{8}, \frac{7}{2}, \frac{1}{1}, \frac{1}{4}, \frac{0}{1}, \frac{5}{3}$ 89. $\frac{m}{n}$

90. (a) $\frac{p}{q} < \frac{r}{s}$, (b) $p \times s = r \times q$, (c) $\frac{p}{q} > \frac{r}{s}$

91. (a) $\frac{-34}{48}$, (b) $\frac{-24}{4}$, (c) $\frac{-5}{17}$, (d) $\frac{1600}{81}$

92. (a) $\frac{7}{20}$, (b) $\frac{6}{5}$, (c) $\frac{-45}{7}$, (d) $\frac{-2}{7}$, (e) $\frac{5}{9}$

93. (a) 0, (b) $\frac{5}{36}$, (c) $\frac{-136}{234}$, (d) $\frac{3}{40}$

94. (a) $\frac{31}{36}$ (b) $\frac{1}{36}$ (c) $\frac{-5}{6}$ (d) $-\frac{48}{45}$ (e) -36

(f) $\frac{3}{20}$ (g) $\frac{-56}{135}$ (h) $-\frac{17}{36}$ (i) $\frac{13}{36}$ (j) $-\frac{56}{135}$

(k) $-\frac{5}{4}$ **95.** $\frac{3}{2}$ **96.** $\frac{1}{3}$ **97.** $\frac{8}{5}$ **98.** $\frac{-1}{2}$ **99.** 16

100. 2.25m **101.** (i) $\frac{-3}{20}, \frac{-6}{40}, \frac{-9}{60}$ (ii) $-5, \frac{-10}{2}, \frac{-15}{3}$

102.

Number	Natural No.	Whole No.	Integer	Fraction	Rational No.
-114			✓		✓
$\frac{19}{17}$				✓	✓
$\frac{623}{1}$	✓	✓	✓	✓	✓
$-19\frac{3}{4}$					✓
$\frac{73}{71}$				✓	✓
0		✓	✓	✓	✓

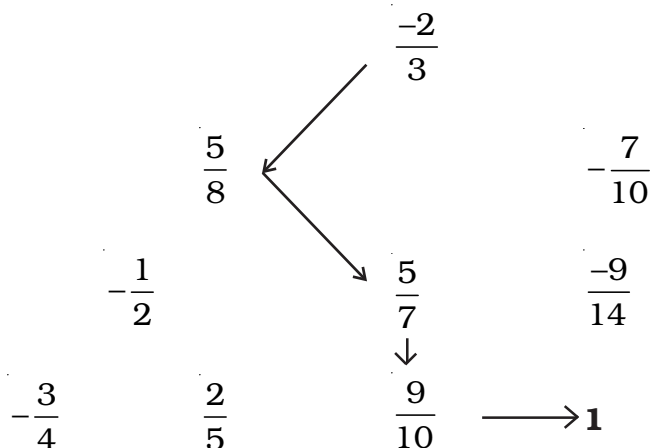
103. $\frac{49}{51}, 99$ **104.** 45 **105.** $7 : 2; \frac{7}{2}$ **106.** (d) **107.** (c)

108. (b) **109.** (a)

110. She divided numerator by 5 but denominator by -5

(D)

1.



2.

$-\frac{1}{4}$		$-\frac{1}{6}$
$-\frac{3}{8}$		$-\frac{11}{60}$
$-\frac{1}{2}$		$-\frac{1}{5}$
$-\frac{5}{12}$		$-\frac{12}{70}$
$-\frac{1}{3}$		$-\frac{1}{7}$

0	$-\frac{1}{2}$	-1
	-2	
	$-\frac{5}{2}$	
	-3	
-4	$-\frac{9}{2}$	-5

3. $-\frac{112}{224}$

4. (Make from graph)

Unit 9

1. (a) 2. (c) 3. (b) 4. (b) 5. (c) 6. (d)
 7. (d) 8. (a) 9. (d) 10. (c) 11. (a) 12. (c)
 13. (c) 14. (a) 15. (a) 16. (c) 17. (a) 18. (b)
 19. (c) 20. (c) 21. (b) 22. (c) 23. (d) 24. (b)

- 25.** (d) **26.** (b) **27.** (b) **28.** (c) **29.** (b) **30.** (c)
31. (b) **32.** (a) **33.** (a) **34.** (b) **35.** (b) **36.** (a)
37. (a) **38.** no. of sides **39.** perimeter, area **40.** 18cm^2
41. 35cm^2 **42.** base **43.** height/altitude **44.** circumference
45. π **46.** 9 **47.** $3.14/\frac{22}{7}$ **48.** π **49.** r **50.** 10000
51. 100 **52.** 10,000 **53.** Height **54.** 10,00,000
55. 3,60,000 **56.** $\frac{1}{1000}$ or 0.001 **57.** True **58.** (a) True (b) False,
(c) False(d) True **59.** False **60.** True **61.** False **62.** True
63. False **64.** True **65.** True **66.** True **67.** False **68.** True
69. False **70.** Flase **71.** True **72.** True **73.** 540
74. 377.1498 **75.** 64m^2 **76.** 16.25m^2 **77.** 24 m **78.** 8cm, 20cm^2
79. $XY = 6\text{ cm}$, $YZ = 8\text{cm}$ **80.** (i) 180m (ii) 2975m^2 **81.** 42 cm^2
82. circular pizza **83.** 33 m **84.** 450m^2 **85.** 30cm^2 **86.** 36 cm
87. 6 cm **88.** 32 cm **89.** $l = 9\text{m}$, and $m = 15\text{m}$, other side = 30m
90. 15 cm and 17 cm **91.** 120 cm **92.** 98 cm^2 **93.** 56cm^2
94. 46.45cm^2 **95.** 82 cm^2 **96.** 55 cm^2 **97.** 227cm^2
98. 308 cm^2 **99.** $149\frac{3}{16}\text{cm}^2$
100. Yes, It increases by 32 cm
101. 64 cm^2 **102.** perimetr = 26 cm, area = 24 cm^2
103. 205cm **104.** 2.97cm^2 , ₹ 72.08 **105.** 28200m^2
106. ₹ 5400 **107.** ₹ 26400 **108.** 88cm, circle **109.** 550 m
110. 31.43 m (app.), 75.43m^2 (app.) , **111.** 6.75m^2 , 13 : 27
112. (a) 188.68m^2 , (b) Rs 67776.80, (c) 62.6m (d) 251
113. (a) $(5x + 65)\text{m}^2$ (b) 44m (c) ₹ 250 ($x + 21$) including lobby between
two bedrooms, ₹ 150 ($x + 35$) excluding lobby between two bedrooms.
(d) ₹ 150 ($15 - x$) (e) 7m

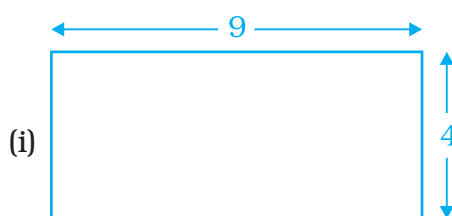
- 114.** 31.5m^2 **115.** 9086m^2 **116.** ₹ 1530 **117.** 1320cm^2
118. 1000cm^2 **119.** Area in both cases is 86 cm^2 **120.** 144
121. 57 m **122.** 35 cm^2 , 2.8 cm **123.** 108 **124.** 40 cm^2
125. (i) ₹ 4440 (ii) ₹ 69600 (iii) 22m^2 **126** (a) (i) 20.10m (ii) 22.68 m
 (iii) 21.78m (iv) 12.16m (v) 10.94m (b) ₹ 1848, ₹ 5929.36, ₹ 1478,
 ₹ 5737.86, ₹ 5008.52 (family room) (c) ₹ 43830
127. 2086 cm^2 **128.** 7550 cm^2 **129.** 7mm **130.** 2411520 km
131. ₹497.64

(D)

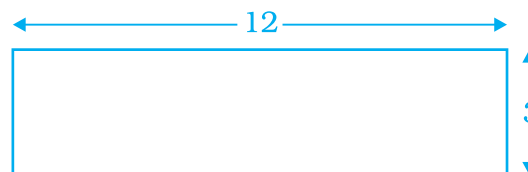
- (i) 87.78 m (ii) 436.64 m^2 (iii) 10.50 m^2 (iv) 2.62 m^2 (v) 7.88m^2
- (i) 39 m (ii) 81.74 m^2 (iii) 12.238 m^2 (iv) 10.26 m^2
- (i) 32 m^2 (ii) 13050 m^2 (iii) 470 m
- (i) 1344.15 m^2 (ii) 293.2 m

5.	Radius	Diameter	Circumference
Foot ball	11.3 cm	22.6 cm	71 cm
Basket ball	12.4 cm	24.8 cm	77.872 cm
Cricket ball	3.66 cm	7.32 cm	23 cm
Volley ball	10.3 cm	20.6 cm	64.684 cm
Hockey ball	3.565 cm	7.13 cm	22.4 cm
Lawn Tennis ball	3.175 cm	6.35 cm	19.939 cm
Shot put	65 mm	130 mm	408.2 mm

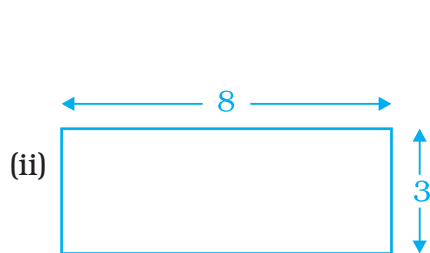
6.



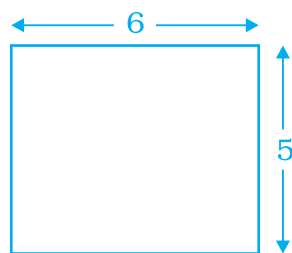
Area = 36
Perimeter = 26



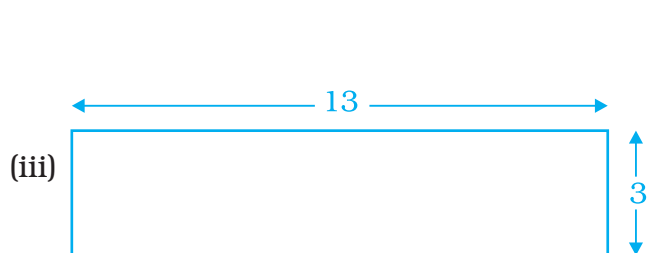
Area = 36
Perimeter = 30



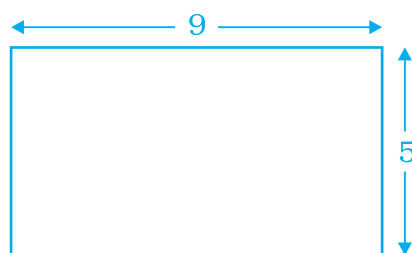
Area = 24
Perimeter = 22



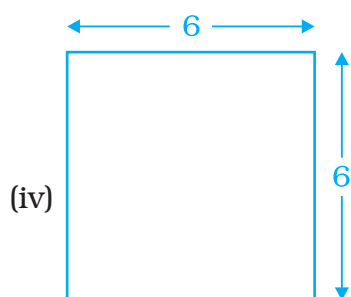
Area = 30
Perimeter = 22



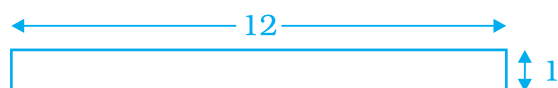
Area = 39
Perimeter = 32



Area = 45
Perimeter = 28

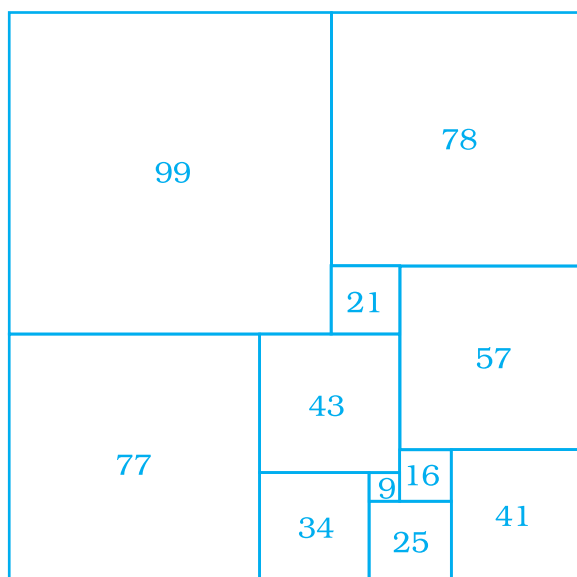


Area = 36
Perimeter = 24



Area = 12
Perimeter = 26

7.



8. (i) Circumference (ii) Perimeter (iii) Area
 (iv) Parallelogram (v) Square (vi) Triangle
 (vii) One (viii) Diameter

Unit 10

1. (c) 2. (c) 3. (a) 4. (c) 5. (d) 6. (b)
 7. (d) 8. (a) 9. (c) 10. (a) 11. (c) 12. (a)
 13. (d) 14. (a) 15. (d) 16. (d) 17. a like term
 18. π 19. like 20. Unlike 21. r 22. one
 23. $n, 6n$ 24. constant 25. $55y$ 26. binomial 27. $2x^2$
 28. $b + c$ 29. $2y, 2y^2$ 30. $2x$ or $-4y^2$ or $-z$ 31. $-23x^2$
 32. $15xy$ 33. T 34. F 35. T 36. F 37. T

38. F 39. F 40. T 41. T 42. F 43. F

44. T 45. F 46. T 47. F 48. F 49. F

50. F 51. T 52. F 53. (a) $x^2 + xy$, Binomial

(b) $r - (3p \times 2q)$, Binomial (c) $p \times 2q \times 3r$, Monomial

(d) $ab + bc + ca$, Trinomial (e) $3x$, Monomial

(f) $2p + 2q$, Binomial (g) $\frac{1}{2}mn$, monomial

(h) x^2 , Monomial (i) $t^3 - s^3$, Binomial

(j) $(x \div 15)x$, Monomial or $\frac{x^2}{15}$ (k) $x^2 + z^3$, Binomial

(l) $q^3 - 2q$, Binomial 54. (i) 1, (ii) -2 (iii) 3 (iv) y^3

55. (i) 1, 1, -3, 5, -7 (ii) 10, -7, -9, 2, 2

56. (a) $4x^2yz^2 + 4xy^2z$ Binomial (b) $x^4 - 3xy^3 + y^4$ Trinomial

(c) $p^3q^2r + pq^2r^3 - 6p^2qr^2$ Trinomial (d) $2a - 2b + 2c$ Trinomial

(e) $60x^3 + 49x + 15$ Trinomial 57. (a) $-2p^2 - 9pq + 6q^2$

(b) $2x^3 - 3x^2y + 2xy^2 - y^3 + 4y$ (c) zero (d) $p^2 + q^2 + r^2$

(e) $x^3y^2 + 4x^2y^3 + x^4 + 7y^4$ (f) $p^2qr - 2pq^2r - pqr^2$ (g) zero

(h) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$ (i) $p^5 + \frac{5}{8}p^4 - p^3 + \frac{25}{8}p^2 - 17p + \frac{31}{4}$

(j) $33t^3 - 6t^2 - 10t - 20$ 58. (a) $4p^2qr$ (b) $a^2 + b^2 + 2ab$

(c) $x^3 + y^3 + 3x^2y + 3xy^2$ (d) $x^4 - 4x^3y^3 + 2y^4$ (e) $-2ab + 2bc + 2ac$

(f) $a^2 + b^2 + 2ab$ (g) $x^4 + y^4 - x^3y^2 + 6xy^3$ (h) $-3ab - 3bc - 3ac$

(i) $-4.5x^5 + 5x^4 + 0.2x^2 - 7.3x - 5.7$ (j) $y^3 - y - 22$

59. (a) $-3x^2y - 3xy^2$ (b) $-3p^2q^2 + pq$ 60. (a) $x^3 - x^2y - xy^2 - y^3$
 (b) $m^2 + 2n^2 - 2mn$ 61. $68a^3 - 47a^2 + 6a + 16$
 62. $y^4 - 17y^3 - 46y^2 + 52y - 54$ 63. $-13p^3 + 98p^2 - 72p + 94$
 64. $-99x^3 + 33x^2 + 13x + 41$ 65. $-9a^2 + 15a - 2$
 66. (A) 1 (B) 25 (C) 1 (D) -125 (E) $\frac{13}{3}$
 (F) $-\frac{5}{3}$ (G) $-\frac{13}{6}$ (H) 6 67. (A) 2 (B) 6 (C) 8 (D) -1
 (E) 14 (F) 9 68. (i) $4x^2 + 6x - 10$ (ii) $6x^2 - 6$
 (c) $12x^2 - 8x - 4$ 69. $a = -2$ 70. $-x^2$ 71. $-3a^2 + 3b^2 - 20ab$
 72. $10x^2 - 8y^2 + x$ 73. (a) $22y + 120$ (b) $8x + 14y$
 74. $y[x - \frac{1}{2}z]$ 75. $\frac{3}{2}m^2$ 76. $8x + 50$
 77. $350 + 50x$ or $50(x + 7)$ 78. $9 + 3x$ 79. $4x + 2y$ 80. $\frac{1}{2}xyz$
 81. $14x + 2y$ 82. ₹ $(10x + 20)$ 83. (a) $4x + 1$ (b) $\frac{1}{3}(4x + 1)$
 84. $11xy^2$ 85. (i) $18r + 6b = 6(3r + b)$, (ii) $6p + 6g$ 200 $(p + g)$
 86. (i) 15 (ii) 66 (iii) 410 87. 385
 88. (a) 385 (b) 550 (c) 1045
 89. (i) $8\frac{7}{2}$ (ii) 1 90. (i) $\frac{-9}{2}$ (ii) $\frac{303}{8}$
 91. Three subtracted from four times 'b'.
 92. Eight times the sum of m and five.
 93. Quotient on dividing seven by the difference of eight and x ($x < 8$).
 94. Seventeen times quotient of sixteen by w.

95. (i) $\frac{1}{4}(x+7)$, $\frac{1}{4}(7+x)$ (ii) $\frac{n-5}{3}$

96. $2n+1$, yes 97. Less than 11

98. $1 \rightarrow (e)$, $2 \rightarrow (c)$, $3 \rightarrow (d)$, $4 \rightarrow (a)$, $5 \rightarrow (g)$
 $6 \rightarrow (h)$, $7 \rightarrow (f)$, $8 \rightarrow (b)$

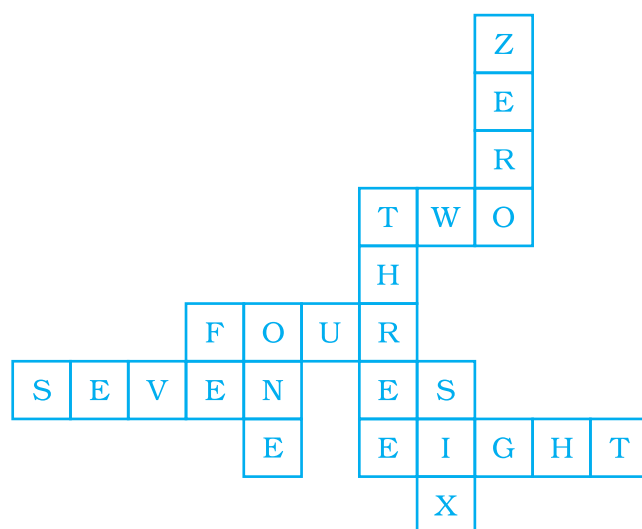
99. Expression : $24 + 4(a-2)$, 'a' stands for the present age of dog or cat

Age	$[24 + 4(a-2)]$	Age (Human Years)
2	$24 + 4(2-2)$	24
3	$24 + 4(3-2)$	28
4	$24 + 4(4-2)$	32
5	$24 + 4(5-2)$	36
6	$24 + 4(6-2)$	40

100. (i) $x+y = y+x$, (ii) $x \times y = y \times x$,
 (iii) $x+(y+z) = (x+y)+z$, (iv) $x \times (y \times z) = (x \times y) \times z$,
 (v) $x \times (y+z) = x \times y + x \times z$

(D)

3.



Unit 11

1. (b) 2. (b) 3. (c) 4. (c) 5. (c) 6. (c)
7. (c) 8. (d) 9. (c) 10. (b) 11. (c) 12. (d)
13. (c) 14. (c) 15. (d) 16. (c) 17. (d) 18. (d)
19. (c) 20. (c) 21. (c) 22. (b) 23. 44 24. 3
25. $\frac{11}{15}$ 26. 8 27. 12 28. 0 29. 32 30. $\frac{13}{14}$
31. 11 32. 5 33. 6 34. 6 35. 3 36. 5.37
37. 8.888 38. 7 39. 8 40. (a) < (b) < (c) > (d) < (e) <
41. False 42. True 43. False 44. False 45. True 46. True
47. False 48. True 49. False 50. False 51. False 52. False
53. True 54. True 55. True 56. False 57. False 58. False
59. False 60. True 61. True 62. False 63. False 64. True
65. False 66. Ascending order: $4^0, 2^3 \times 2, 2^3 \times 3^1, 3^3, 2^5, 3^5 (3^3)^2$
67. Descending order: $2^3 \times 5^2, (2^2)^3, 2^{2+3}, \frac{3^5}{3^2}, 3^2 \times 3^0, 2 \times 2^2$
68. $(-4)^2$ or 16 69. $m = 5$ 70. $729/64$ 71. $\frac{32}{27}$
72. (a) 1, (b) 1, (c) 1, (d) -3, (e) 24, (f) 0 73. $n = 0$
74. (a) 80100000 (b) 0.00175
75. (a) 32, (b) -243, (c) -256 76. (a) $27a^4 = 3^3a^4$ (b) $a^2b^3c^4$ (c) $S^4 \times t^3$
77. 30^6 78. (a) 2^{10} (b) 3×7^3 (c) $\frac{3^2 \times 2^4}{5^3 \times 7}$ 79. (a) 2^6 (b) 2^9 (c) 5.28×10^5
80. (a) $2^3 \times 3^2 \times 5^3$ (b) $3^4 \times 5^2$ (c) $2^5 \times 5^2$
81. (a) 6^3 (b) 4^4 (c) 35^2 (d) 5^6 (e) $(30)^3$ (f) $11^2 \times (-2)^5 = -3872$
82. (a) 7.647×10^6 (b) 8.19×10^7 (c) 5.83×10^{11} (d) 2.4×10^{10}
83. $1.44 \times 10^{11}\text{m}$

- 84.** (a) $(3/7)^2$ (b) $\frac{7}{11}^5$ (c) 3^8
 (d) a^7 (e) $\frac{3}{5}^5$ (f) 5^{10}
- 85.** (a) $49a^2b^3$ (b) 3920 (c) $\frac{25}{8}a^3$ (d) 729
 (e) $1/75$ (f) $6075/2$ (g) 1
- 86.** Gibson, Australia; Thar, India; Great Victoria, Australia; Kalahari, South Africa; Sahara, North Africa.
- 87.** Jupiter, Saturn, Neptune, Uranus, Earth, Venus, Mars, Mercury, Pluto.
- 88.** (1) 6×10^1 (2) 3.6×10^3 (3) 8.64×10^4 (4) 2.6×10^6 (5) 3.2×10^7
 (6) 3.2×10^8
- 89.** 12 : 5 **90.** $c = 3$ **91.** (a) 9.46×10^{12} km, (b) less than
- 92.** 9 **93.** 2^{18} **94.** 3060 kg
- 95.** Red blood cell has a greater diameter than a platelet.
- 96.** (a) 1×10^{100} (b) 10^{200} **97.** He has left power of 3 which is 5.

(D)

Down 1.

2.

3.

4.

5.

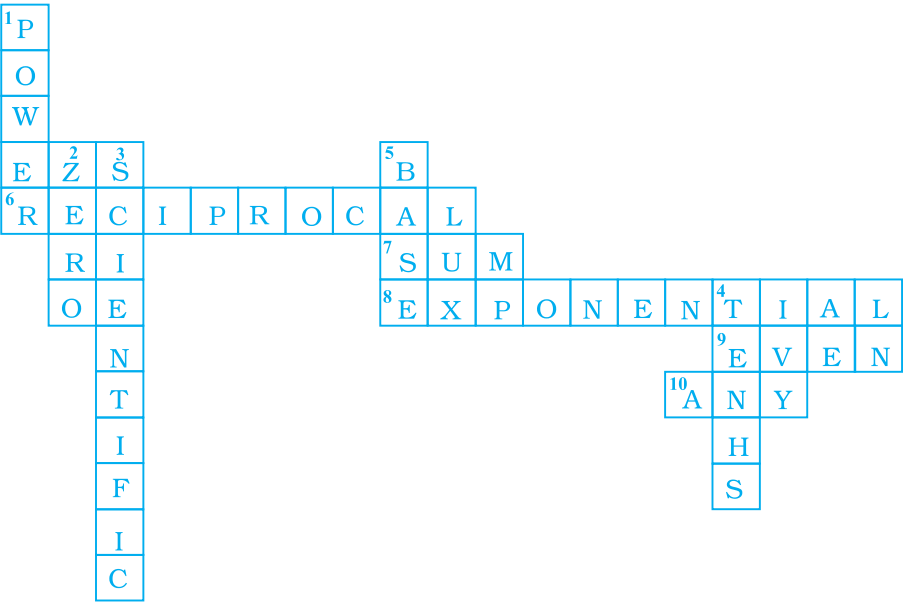
Across 6.

7.

8.

9.

10.

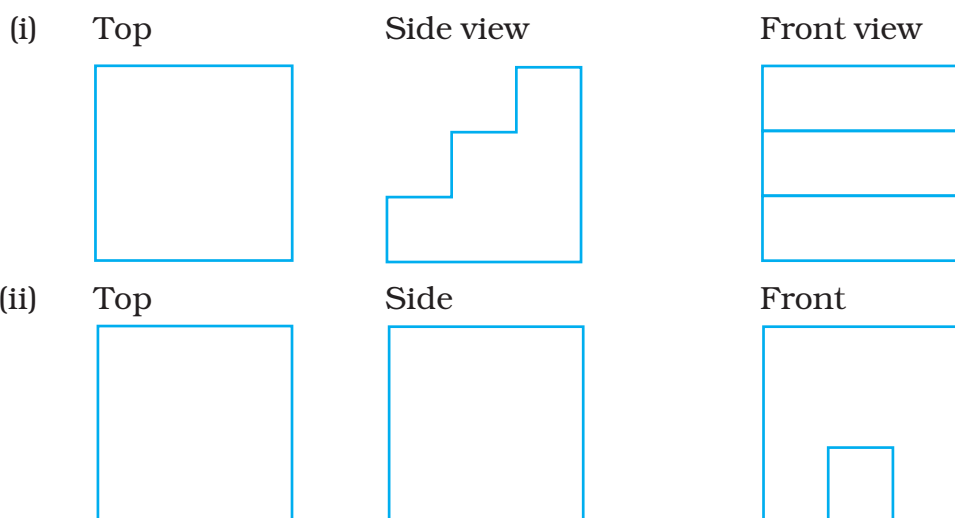


Activities 2

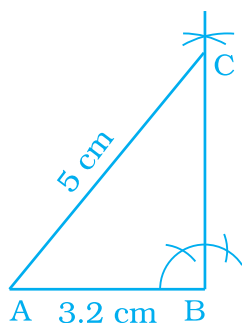
	1	5	7	6	7
			2		4
8	5		2	3	4
					3
10	1	9	1		3
					2
4	2	2	0		7
	5	1	0	0	0

Unit 12

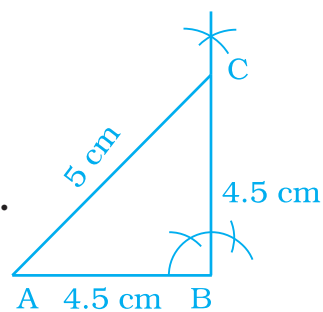
1. (b) 2. (a) 3. (c) 4. (b) 5. (c) 6. (b)
7. (b) 8. (c) 9. (d) 10. (c) 11. (b) 12. (c)
13. (a) 14. (a) 15. (a) 16. (c) 17. (a) 18. (d)
19. (b) 20. (c) 21. (c) 22. (c) 23. (a) 24. (b)
25. (c) 26. (b) 27. (a) 28. (c) 29. one 30. 2,2
31. Isosceles 32. Quadrilateral 33. M and W 34. Edge
35. Face 36. Vertices 37. Sphere 38. 5, 9, 6 39. 4, 6, 4
40. 5, 8, 5 41. 5, 3, 2 42. Triangle 43. 5, 4, 1
44. 5, 4, rectangle 45. 2 46. 2 47. Infinite
48. Rectangle 49. Bisector 50. No 51. 8 52. Scalene
53. Prism 54. 0, 0, 1 55. Cone 56. Triangle Prism
57. 1 58. 10 59. False 60. True 61. False 62. False
63. True 64. True 65. False 66. False 67. False 68. False
69. False 70. False 71. True 72. True 73. False 74. True
75. True 76. False 77. True 78. False 79. True 80. False
81. True 82. False 83. False 84. True 85. False 86. True
87. False 88. True 89. False 90. True 91. False 92. False
- 93.



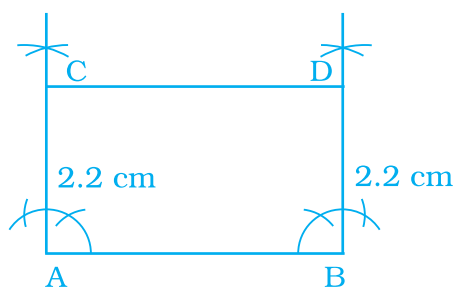
95.



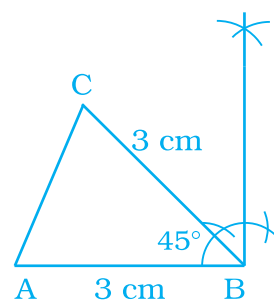
96.



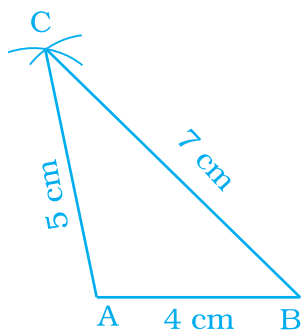
97.



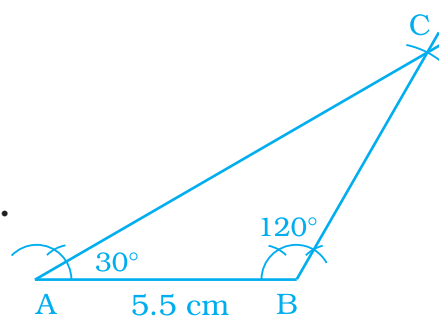
98.



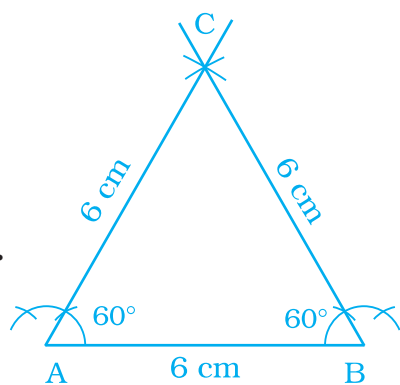
99.



100.



101.



102. 60°

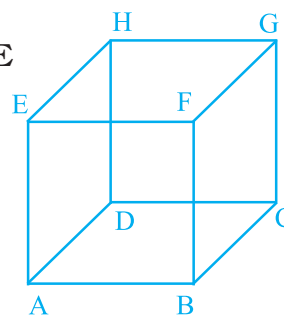
103.

Figure	Number of Lines of Symmetry	Order of Rotation of Symmetry
a	1	1
b	1	1
c	1	1
d	2	2
e	1	2
f	0	1
g	1	1
h	0	3
i	4	4
j	1	1
k	0	1
l	1	1
m	0	2
n	0	1
o	1	1
p	1	1
q	1	1
r	0	3
s	3	3
t	1	1
u	10	10
v	3	3
w	0	1

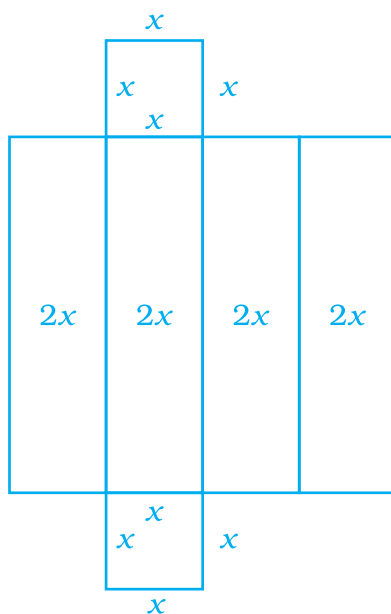
104. (i) EF (ii) ABFE, BFGC (iii) ABEF, ABCD, ADHE

(iv) D (v) CD, EF, GH (vi) AE, EF, GH, HD

(vii) AE, BF, AD, BC (viii) Several group of points like – A, E, C, B

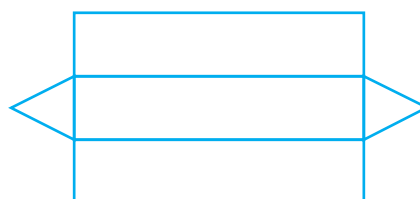


105.

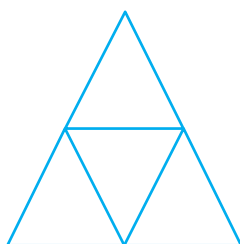


106.

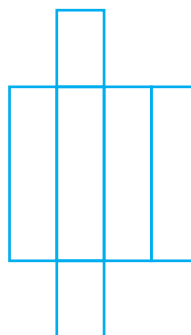
(i) Triangle prism



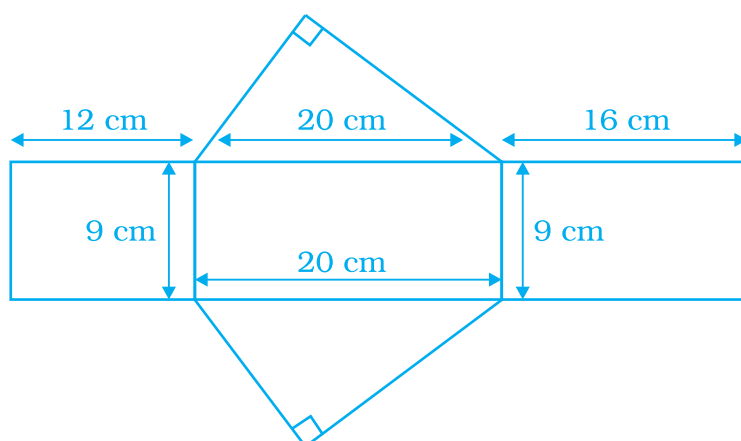
(ii)



(iii)



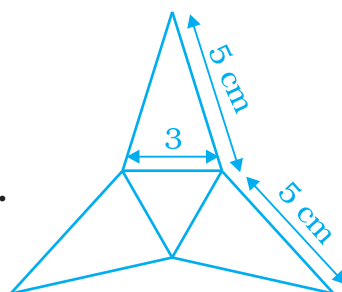
107.



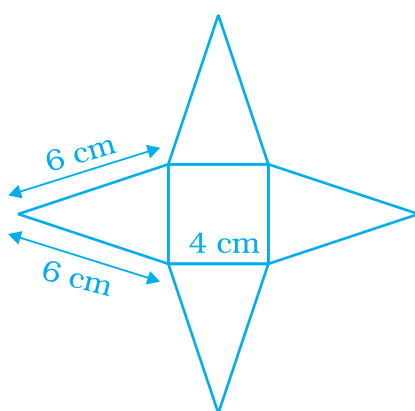
109.

(i) HG (ii) CD

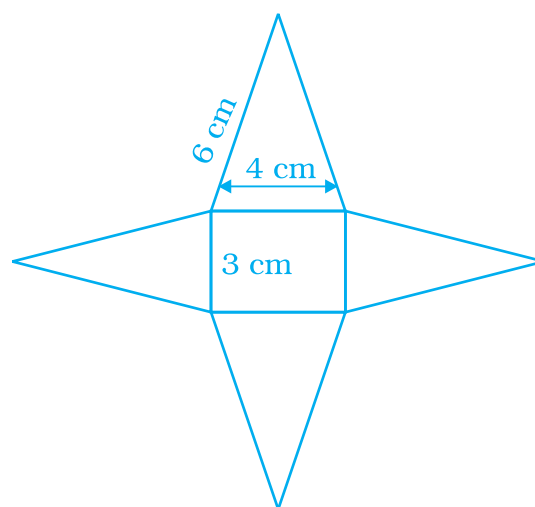
110.



111.



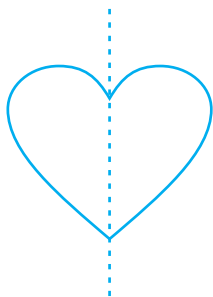
112.



113. (a) 6 (b) 8 (c) 7 (d) 8 (e) 6 (f) 8 (g) 6 (h) 8

114.

(a)



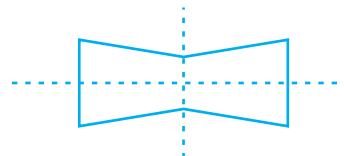
One line of symmetry

(b)



No line of symmetry

(c)

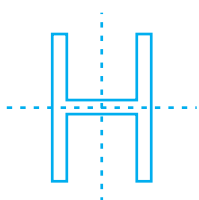


Two line symmetry

115. 16

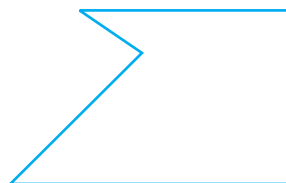
116.

(a)



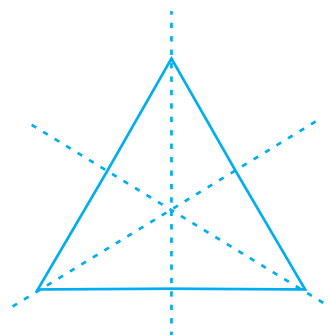
2 lines of symmetry

(b)



No line of symmetry

(c)



3 lines of symmetry

117.

(a) Yes

(b) No

(c) Yes

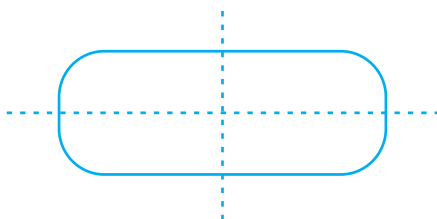
(d) Yes

(e) Yes

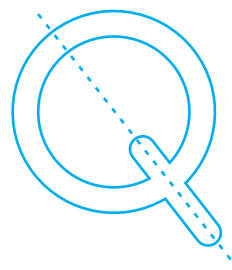
(f) Yes

118.

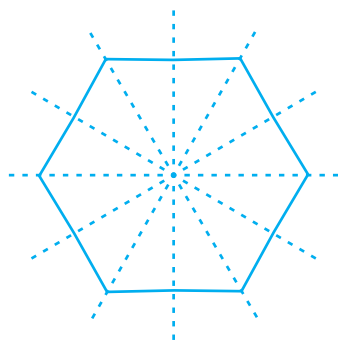
(a)



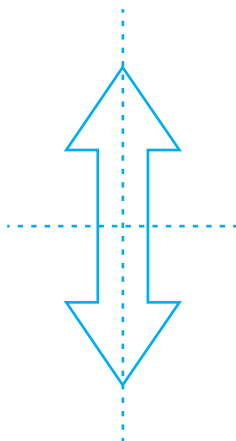
(b)



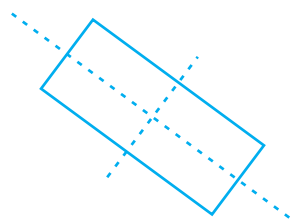
(c)



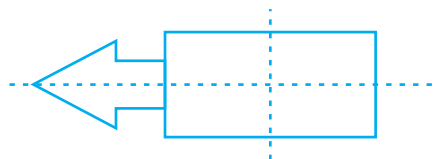
(d)



(e)



(f)



119. (a) Yes (b) Yes (c) No (d) Yes

120. No

121. 2

122. (a) and (c)

123. d

Extra Question:-

Write the name 5 letter of English alphabet which have no line of symmetry

124. F, G, J, L, N, P, Q, R, S, Z,

(D)

Across

1. ISOMETRIC
3. PARALLEL
5. CONE
7. CIRCLE
9. TRIANGULAR PRISM

Down

2. CENTRE OF ROTATION
4. SPHERE
6. EDGE
8. NET
10. SQUARE

